Module 7: Right Triangles

Topic 1 Content: The Pythagorean Theorem Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on the Pythagorean Theorem. All that you know about right triangles is going to come together for you in this topic. Are you ready to get going? Let's start.

Before we start talking about the Pythagorean Theorem specifically, let's just review the parts of a right triangle. You know, of course, that a right triangle is a triangle that includes a right angle. The side opposite that right angle, we refer to as the hypotenuse. It's always the longest side of a right triangle. If you think about it, if you have a triangle that includes a right angle, that right angle is going to be the largest angle in your triangle. The side opposite the largest angle has to be the largest side. When a right triangle, the side opposite the right angle, we refer to as the hypotenuse, and the other sides we refer to as legs. Right triangle, you're going to have a pair of legs and a hypotenuse.

With the Pythagorean Theorem, let's take a look here; we refer to the hypotenuse as C and the legs as A and B. I could've just as easily as referred to this leg as A, and this leg as B, and orienting those angles a little differently. As long as you know that C is the hypotenuse, and A and B are your legs. What the Pythagorean Theorem tells us is that if triangle ABC is a right triangle, then $A^2 + B^2 = C^2$. What that means is the length of this leg squared, plus the length of this leg squared, will be equal to the length of the hypotenuse squared.

I know you spent some time in eighth-grade math really proving that Pythagorean Theorem and finding out why all those relationships are true. In geometry, what we're going to do is apply that theorem to solve various types of problems. The one thing I want to make sure you know is that, now, I've written out the Pythagorean Theorem as $A^2 + B^2 = C^2$. You're going to often see me write it as $C^2 = A^2 + B^2$. Sometimes I'll flip it around like that. Both mean the same thing, so either way you want write it is fine. We know that because of the law of symmetry. $A^2 + B^2 = C^2$ or $C^2 = A^2 + B^2$, both are acceptable ways to write that part of the Pythagorean Theorem.

Let's go ahead and start applying this theorem to solve these problems. Here, in this first example, we're asked to find the unknown length. Round to the nearest tenth if necessary. What you always want to do when solving right triangles is you want to label your sides. Because I know I'm going to use the Pythagorean Theorem to solve this triangle, I need to label the sides appropriately with A, B, and C. A good rule of thumb is to always label C first. Go ahead and get the hypotenuse out of the way, so that you don't get it confused with your legs. I know the hypotenuse, always across from the right angle, so it's C, right here, and A and B are my legs. Like I said earlier, I could call this leg A, and this leg B, or I could call this leg A, and this one B. It doesn't matter as long as you have C as your hypotenuse. I'm going to call this one A, call this one B.

Let's start using that Pythagorean Theorem to solve this triangle. $C^2 = A^2 + B^2$. Like I said, you're going to often see me write it this way, which is fine. C I know is x. $x^2 = 5^2 + 8^2$. Let's start to simplify this equation. $x^2 = 5^2$ is 25, and 8^2 is 64. Now we have on the right side, 25 + 64, which is 89. I know, right now, that $x^2 = 89$. Now we have to undo the square term. We need to take the square root of both sides. x is going to equal the square root of 89. That is not a perfect square. 89's not a perfect square number. In this case, we are going to have to round to the nearest tenth, like it mentioned.

I'm going to switch to my calculator. Let's get our calculator up. I've already cleared the memory on mine, but just make sure you remember how to do that. You want to hit the second button up in the top left corner, and the plus, the addition sign, right above enter. Then you want to select *Reset*, which for me is number seven, and then *ALL RAM*, the first one, and then *Reset* to confirm. Now my memory is clear. I'm good to go with my calculator.

If we recall, we're trying to take the square root of 89. You want to press your second key again, that blue bottom for me, in the upper left corner. If you look down, go down that column, right under the second button, and you see your x^2 key, right above it is the square root symbol. That's the one we're going to press. Press that x^2 key. What the second key does is it allows you to call for any of those functions that are written above a button. You want the square root of 89. Let's hit *ENTER*. Here, if we round to the nearest tenth, that'll give us 9.4.

Let's go back to our problem. Let's go full screen, and make sure we have our pen. We do. The square root of 89 is approximately 9.4. That means x is about 9.4 units. We're all done with this problem. You see how we apply the Pythagorean Theorem to solve for that unknown side length. All right?

Let's keep going, here. Let's try another triangle. Here again, we're asked to find the unknown length and round to the nearest tenth, if we need to. Let's label the sides. Start with the hypotenuse just because it's a good rule of thumb, so C. We can label A and B however we want. I'm going to go ahead and write the Pythagorean Theorem right over here. $C^2 = A^2 + B^2$. C is 15 in this case, so $15^2 = x^2 + 9^2$.

If you take a look at what we have so far, you'll notice in this case, the hypotenuse is not the side that I'm actually solving for. I already know that my hypotenuse is 15 units. I'm actually trying to determine the length of one of my legs. When you use the Pythagorean Theorem with a problem set up in that way, you have to solve this a little differently. Really, it's like you're solving an equation where you have to isolate your variable. Your variable's not already isolated here. The steps are a little different, but not too, too, different, but it is a little different from the previous example.

Here we go, 15^2. That's 225, so $225 = x^2 + 9^2$ is 81. Now I need to isolate my variable, here. I'm going to subtract 81 from each side, 225 – 81, that's $144 = x^2$. These terms are wiped out. Now, my x-term's on the right, but we're fine with that, because we handled Algebra I. We're just going to go ahead and take the square root of each side. The square root of 144 is 12, and the square root of x^2 is x. Look at there, we're all done with the problem. $x = 12$. We didn't have to round in this case, because 144 is a perfect square. We found, here, that our unknown length is 12 units long exactly.

You've seen two different ways that you can use the Pythagorean Theorem to solve for a right triangle. If we flip to that first example, here our hypotenuse was unknown. It was a situation where you were simplifying that right side, here, to solve for x. Here, on our second example, we had a leg that was unknown. You had to do probably what looked more familiar to you like solving an equation from Algebra I. Keep those two strategies, or the two things that we did, in mind while you work your way through this one. Go ahead and press pause, and take a few minutes and try this problem. When you're ready to check your work, go ahead and press play.

All right, let's see how you did here. You should have started out by labeling the sides of your triangle. Know this is my hypotenuse, that's C. I can label A and B however I want. I'll go ahead and write the Pythagorean Theorem right here, so $C^2 = A^2 + B^2$. C is 13, so $13^2 = 12^2 + x^2$. 13^2 is 169, 12^2 is 144, plus x^2. This is a problem where we are solving for the length of one of our legs. It'll probably resemble like solving an equation for you from Algebra I. Let's subtract 144 from each side, 169 – 144 is 25, equals x^2. Then our last step, go ahead and take that square root. Square root of 25 is 5, and the square root of x^2 is x. Our unknown length in this triangle, five units long. Good job on that one.

Before we leave Pythagorean Theorem completely, I want us to take a look at a couple of real-world problems, or a couple of practical problems. Let's look at this one. We are told that a 25-foot ladder touches a house at a point 24 feet above the ground. How far is the bottom of the ladder from the house? Let's see here. We're told that the ladder is 25 feet, then we have a house sitting on the ground. What you have to infer, or assume to be true here, is that your house is perpendicular to the ground. Where the house touches the ground, it forms a right angle. Right here, in theory, you have a right angle. That means that you have the side of your house. You have that distance from the bottom of that house to the bottom of the ladder, and then you have the ladder. You have a right triangle trapped inside of this practical problem.

Looking back at the details we were given, we're told that the ladder is 25 feet. We're told that it touches the house at a point 24 feet above the ground. Right at this point, we're told it's 24 feet above the ground, where this ladder touches the house. Right here, 24. Then we're asked to determine how far is the bottom of the ladder from the house. This distance, right here, x. If we pull that triangle out of the picture, going to get it right underneath, here, this is what we have. We have a right triangle. The hypotenuse is 25. The legs are 24 and x. What we have to do to figure out how far the bottom of the ladder is from the bottom of the house is we need to solve this right triangle.

Let's go ahead and use the Pythagorean Theorem to do that. Let's label all sides. C is the hypotenuse. A and B are the legs. We have, here, let's switch back to black ink, Pythagorean Theorem, so $C^2 = A^2 + B^2$. C^2 is 25, so $25^2 = 24^2 + x^2$.

I think I'm going to switch to my calculator for that one. It's kind of a large number there. Let's get that value, 25^2. Then let's go ahead and get 24^2 while we're here. We have 625 and 576, so $625 = 576 + x^2$. We're going to need to subtract 576 from each side. Cancel out here. Let's put that in the calculator, also, since it's so large, so 625 − 576. 625 − 576, it's 49. Back to our problem, here. Let's scroll to get a little work space. $49 = x^2$. Then our last step, square root of each side. 49's a perfect square. Square root of 49 is 7, and the square root of x^2 is x. What we know is that the bottom side of this triangle is 7 feet. Taking it back to our problem, that tells us how far the bottom of the ladder is from the bottom of the house. It is 7 feet away.

I do believe now that it's your turn. Go ahead and press pause. Take a few minutes. Work your way through this one. Press play when you're ready to check your answer.

Let's see how you did here. You're told that the sides of a rectangular TV screen is determined by the length of its diagonal. What is the size of the TV screen shown below? I'm given the dimensions 18 inches and 24 inches of this rectangular TV screen, and I'm asked to find the length of the diagonal.

What I'm going to do is—because I know that a rectangle has four right angles—is I'm going to focus my attention on this right triangle, apply the Pythagorean Theorem in order to figure out the length of that diagonal, which is essentially the hypotenuse of that right triangle. I'm going to redraw this. I'm going to pull the triangle out of the picture, there. Got a little space, going to have to scroll a little bit for a moment. Let's get the triangle right underneath here.

18, 24, and then there's x. Let's use the Pythagorean Theorem to solve for x in this case. If $C^2 = A^2 + B^2$, so C is our hypotenuse, A and B are the legs. In this case C^2 would be x^2 equals—get a little bit more room, $18^2 + 24^2$. Let's go ahead and switch to our calculator because these numbers are kind of large. Let's get 18^2 and 24^2. 18^2, 324. 24^2, 576. $324 + 576$. Get that out of the way. $324 + 576$. Because those numbers are even larger, let's go ahead and add them in the calculator. Let's switch back. $324 + 576$, that's 900. Back to our work. Let's get a little more room. $x^2 = 900$. We'll take the square root of each side, $x = 30$. Bringing this value back to our word problem here, that means that the sides of this TV screen is 30 inches. We have a 30-inch TV screen here. Good job on that.

All right, guys. We've reached the conclusion of this topic on how to us the Pythagorean Theorem to solve right triangles. We saw how your prior knowledge of the Pythagorean Theorem from your geometry studies in eighth grade helped you get through this topic. Bye.