Module 7: Right Triangles

Topic 2 Content: 30°-60°-90° Special Right Triangle Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on the 30°-60°-90° special right triangle. So your knowledge of right triangles and the relationship between the angles and the sides of a triangle is going to come in handy during this topic. You ready to get started? Let's go.

Before we start talking about the 30°-60°-90° special right triangle, let's start back a bit about to the 45°-45°-90° special right triangle. We know that the 45°-45°-90° triangle helped us solve problems involving isosceles right triangles. Where our legs were congruent, and then of course we have the length of our hypotenuse. That formula for this special right triangle is that the length of the hypotenuse is equal to the length of a leg times the square root of 2. Right? Okay so keep that in mind just to kind of warm you up again about special right triangles and let's look here at the 30°-60°-90° special right triangle.

In this case, we don't have an isosceles right triangle that we're working with, we're working with the right triangle that has angle measures that are of course 90 degrees, 30 degrees and then 60 degrees. How this is different from our isosceles right triangle—let me actually flip back to discuss this really quickly—is we knew that—because our legs have the same length—that the angles were opposite of those legs also have the same length.

Now in the case of a 30°-60°-90° special right triangle, our legs do not have the same length because our angle measure here—the angle besides the right angle—do not have the same length. The angle that has a measure of 30 degrees—the leg opposite it—we refer to it as the short leg because its length is going to be shorter than the length of the leg that's opposite the 60-degree angle. We refer to that as the long leg. That throws back to our relationship that we know that when given a triangle, the shortest side iss opposite the smallest angle, the longest side is opposite the longest angle.

In this case, of course, we always know that the right angle is the largest angle of a right triangle, and that's why our hypotenuse is the longest length overall. But in reference to our legs, the sides beside the hypotenuse, the side opposite the 30-degree angle is the short leg and the side opposite the 60-degree angle is the long leg.

Now with 30°-60°-90° special right triangles, there're actually two formulas that you can use to solve for different side lengths. The first one: The hypotenuse is equal to 2 times the short leg. The second one: The long leg is equal to the short leg times the square root of 3. Depending on what part of the triangle—what length you were given—that will affect which one of these formulas you use first. Sometimes you just use one of the formulas. Sometimes you have to use both of the formulas to really get to what it is you're trying to solve. You'll see some examples of that in the next few examples. But these are formulas

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that you're going to have to commit to memory. The hypotenuse is 2 times the short leg. The long leg equals the short leg times the square root of 3.

Let's look at this example. Here we're asked to solve for both *x* and *y*. Before we do anything, I'm going to write those formulas down. I kind of write a shorthand version of them just to help me remember. The hypotenuse is 2 times the short leg, and then the long leg equals the short leg times the square root of 3. That just helps me keep straight what my formulas are. Now that those are written down, let's go ahead and label the parts of our triangle.

We see here—I always like to label the hypotenuse first just so I don't get it mixed up with either of my legs. I know the hypotenuse is opposite the right angle. In this case, it's *y*, and I'm given this 60-degree angle. I know opposite the 60-degree angle—that's my long leg. That means that 5 has to be my short leg. I'll just circle those just so they stand out. In this case I was given the length of the short leg. Now if I take a look at my formulas, both of my formulas involve the short leg. Because I'm trying to solve for *x* and *y*, I'm going to have to use both of these formulas to get to those answers. But it really doesn't matter which one I use first because both formulas involve the short leg.

I'll just start out by using the first formula I wrote down first. The hypotenuse equals 2 times the short leg. My hypotenuse is *y* equals 2 times and my short leg is 5. 2 times 5 is 10, so *y* equals 10. I'm all done with that side, got it solved. Now I'm going to move on. I'm just going to move that out of the way. I'm going to solve for my long leg. The long leg equals the short leg times the square root of 3. My long leg is *x* equals, my short leg is 5 square roots of 3. Then I'm all done with that one. You see how I used both formulas to help me solve for both *x* and *y* in this special right triangle? Keep this in mind and let's go ahead and get to our next example.

Here again, we're asked to solve for *x* and *y*. We're going to start out by writing down my formulas first. The hypotenuse equals 2 times the short leg. The long leg equals the short leg times the square root of 3. Now that my formulas are written down, I'm going to go ahead and label the sides of my triangle. I know opposite the right angle, there's my hypotenuse. This time, I was given the 30-degree angle, so I know opposite from that is the short leg. That means *y* has to be the long leg.

Now looking at this triangle, I see that I was given the length of the hypotenuse first. That means that I'm going to have to use the formula that involves the hypotenuse first. Let's see here. The hypotenuse equals 2 times the short leg. So 6 equals 2 times and the short leg is *x*.

Here I have an equation I have to solve for, 6 equals 2 times x. Let's divide each side by 2. So 3 equals x. We're all done here.

Now we actually know the length of the short leg. I'm actually going to write here—where I have short leg—I'm going to put that it equals three. Because at first it was x, an unknown value. But now that I know it's 3—because I know I'm going to have to use it in my other formula—I'm just going to go ahead and replace it in my triangle also just to keep myself straight.

Let's use that other formula. I'm going to get this kind of slid over a little bit just so we have a little more workspace right here. The long leg equals the short leg times the square root of 3. The long leg is y equals the short leg, which I know is 3 now, times the square root of 3, and you're all done there. You see how here it was a little different from the first example because we were given the hypotenuse so we had to use that hypotenuse first. But again we just use what we had to work our way through these formulas and solve for x and y.

It's your turn to try one. Go ahead press pause, take a few minutes, work your way through this one, press play when you're ready to check your work.

Let's see how you did here. I'm going to start out by writing my formulas. The hypotenuse equals 2 times the short leg. The long leg equals the short leg times the square root of 3. Now I'm going to label the sides of my triangle. I know opposite the right angle, that's the hypotenuse. And opposite the 60-degree angle, that's the long leg, so that means 10 is going to have be the short leg.

In this problem, I was given the length of the short leg, both formulas involve the short leg, so I can use either one first. It doesn't matter. I think I'll go ahead and use the long leg formula first this time just so you can see that it really doesn't matter which one you use. The long leg equals the short leg times the square root of 3. Our long leg is x equals our short leg is 10 times the square root of 3. We've got x. I've used the long leg formula first. I'm going to go ahead and use the hypotenuse formula. The hypotenuse is y equals 2 times the short leg, which is 10. y equals 2 times 10. That's 20. And you're all done with that one. So x is 10 square roots of 3 and y is 20. Good job on that.

Now before we leave special right triangles, I want to expose you a little bit to some practical problems. Let's take a look at this first one. A ladder rises at an angle of 60 degrees and then rests against the side of a house at a point 24 feet above the ground. What is the exact length of the ladder? In geometry, like I've said before, when you're not given an image—like you're given some situation describing something kind of elaborate, but you're not given an image to give you an idea of what's going on—you're going to want to draw one because 9 times out of 10, the answer to that problem is much easier to find when you have an image to go off of. Just to get a visual of what you're really solving for here.

I'm not an artist, but I'm just going to roughly sketch a house with a ladder leaning against it. Let's see, let's get a little house going here. There's our house, and we have a ladder leaning against it. Let's just get a little ladder leaning against our house here. Let's try to get the ladder going down a little further. We are told that the ladder rises at an angle. Let's go ahead and highlight some of that stuff. It rises at an angle of 60 degrees, and then it rests against the side of a house at a point 24 feet above the ground.

Let's get a visual of this. Ideally—it may not look like it from my sketch—but ideally your house should be sitting straight up and down on the ground. In other words, it should be perpendicular to the ground. The angle formed by the side of the house and the ground—this should be a right angle here—make sure I had red ink. What happens is you have kind of a triangle that forms right on the side. With one side representing the side of the house, one side that just sits between the bottom of the house and the bottom of the ladder. Then another side is represented by the ladder itself. If I pull that triangle out of the problem that's going to help me get to what I'm trying to get to here.

Let's put our information in. It says the ladder rises at an angle of 60 degrees. Well that's the angle right here. It's saying right here, the ladder's rising at an angle of 60 degrees. Let's get 60 degrees right over there. It says it rises at that angle and then it rests against the side of the house at a point 24 feet above the ground. This point here where the ladder touches the house, it's telling you that it's 24 feet above the ground. This side of your triangle is 24 feet. Let's get that 24 over there. What is the exact length of the ladder? That's this side of our triangle, so right here that's x. We use this picture to really just help us get this triangle.

Now that we have the triangle, let's focus on that. We don't have to pay attention to my pretty little sketch over here anymore. We'll put it off to the side. We'll focus on this triangle here. Let's get it back together, there we go. Let's group it together and then slide it to the left a little. If I look here, I know that I have a right angle in this triangle, and I have a 60-degree angle in this triangle. That means I have a 30°-60°-90° special right triangle. I'm going to go ahead and write those formulas down. Matter of fact, let's just erase the house so we can get the formulas in this space here. Or let's get the triangle in that space and write our formulas to the right. There we go.

The hypotenuse equals 2 times the short leg. The long leg equals the short leg times the square root of 3. Here I know my hypotenuse is *x*, going to go ahead and label that with an *h*. Opposite this 60-degree angle, I know that's the long leg. That means this bottom length has to be the short leg. Notice that you're not actually given a length for the short leg. Now that doesn't mean that we won't end up having to figure out what it is in order to get the hypotenuse, but just notice for right now that we don't know what it is. We are asked to find the exact length of the ladder, and again, when you're told or when you're asked to find an exact length—like when we were working with our 45°-45°-90° special right triangle—hat word *exact* is a hint that you're going to use one of your special right triangle formulas to solve because those triangles always give you an exact answer. You're never going to round anything there. Unless you're told to approximate, but when you're told "exact," 9 times out of 10, it's one of these special right triangle formulas you're using.

Here I'm trying to solve for *x*, which I notice is the hypotenuse, so I look and see. Of course, I'm going to need to use that hypotenuse formula to figure out what it is. But when you start digging a little deeper, you don't know the hypotenuse, it's *x* and you also don't know the length of the short leg. Even though you weren't asked to find that length, you're going to have to figure it out in order to figure out what your hypotenuse is. The hypotenuse formula—we're going to use it but not at first. We're going to have to use the formula for the long leg in order to figure out the short leg. Then once we have it, we can go ahead and use that hypotenuse formula.

Let's look here, we're told that the long leg is 24. Let's use that long leg formula. The long leg equals the short leg times the square root of 3. The long leg is 24, the short leg we're just going to continue to call it SL times the square root of 3. We're trying to solve for the short leg. Let's divide both sides by the square root of 3. Get some more space here. I'm going to flip some things around. The short leg equals 24 divided by the square root of 3. Now remember from when we worked with our 45°-45°-90° special right triangle, you cannot leave and irrational number in your denominator. We're going to have to rationalize this denominator. In this case we're going to multiply the numerator and the denominator by the square root of 3. So 24 times the square root of 3. 24—I just wrote the 2—let's fix that. Let's switch to black ink. That's 24 square roots of 3. The square root of 3 times the square root of 3. That's the square root of 9, which is just 3.

Now that I have to this point, let's go ahead and divide right there. So 24 divided by 3, that's 8. The short leg is 8 square roots of 3. Now you may be tempted to stop right there but remember that was just your short leg. You're going to have use that measure to figure out the length of the hypotenuse. I know this problem's got multiple steps to it. The hypotenuse is 2 times the short leg. Let's scroll back down. Get a little space here. The hypotenuse is 2 times the short leg. Our hypotenuse is x equals 2 times the short leg, which we know is 8 square roots of 3. Then remember those algebra skills, just multiply your whole numbers here. So 2 times 8 is 16. So it's 16 square roots of 3. That's x. Now taking this back to original problem to get the units. I think we were asked to find the length of that ladder. The exact length of the ladder, let's scroll back down, it's in feet. It is 16 square roots of 3 feet. Good job on that one. I know it took multiple steps to get to the answer, but I think you did good hanging in there.

Now go ahead and take a few minutes and try this one. Press pause, take your time, work your way through it, press play when you're ready to check your work.

Let's see how you did here. Let's highlight the information as we go. A gymnastics wedge at a local recreation center is in the shape of a right triangle. The wedge is 6 feet tall and rises at an angle of 30 degrees. What is the exact length of the longest side of the wedge?

If you're not familiar with gymnastics, you may not have known what a gymnastics wedge is. Sometimes when you're working your way through problems, through practical problems, you may not know exactly what every single thing in the problem is. That's okay. There will always be enough geometry in there to get to what you really need to get to, even though you don't know or you may not know what a gymnastics wedge is. By the way, it's the kind of incline that gymnasts will use to practice, like doing back handsprings and back flips on when they are learning those. But even if you don't know what it is, you know that it's in the shape of a right triangle. You know enough geometry to know what a right triangle is. Focus on that: that this wedge is in the shape of a right triangle.

It says it's6 feet tall and height's always perpendicular; height's always straight up and down. So 6 feet and it rises at an angle of 30 degrees. What is the exact length of the longest side of the wedge? The longest side in any right triangle is my hypotenuse. Remember not the long leg but the longest side, always the hypotenuse. Let's write our formulas down. The hypotenuse equals 2 times the short leg. The long leg equals the short leg times the square root of 3.

Now let's label the triangle. Opposite the right angle, that's our hypotenuse, 30 degrees so the side opposite that has to be the short leg. We have the long leg down here. We're asked to find the hypotenuse and we're given the short leg. We need to use the hypotenuse formula obviously, but the short leg since we know it we can now dive right into that formula. We're not going to have to use the long leg formula to figure out the short leg first. We already know what it is. The hypotenuse equals 2 times the short leg. The hypotenuse is x equals 2 times and our short leg is 6. So 2 times 6 that's 12. Taking it back to the units in our problem, the wedge the sides of the wedge were measured in feet. The longest is 12 feet. You're all done with that one. Good job there.

Guys, we've reached the conclusion of this topic on the 30°-60°-90° special right triangle. I hope we saw how your knowledge of triangles and special right triangles came in handy during this topic. Bye.