Module 7: Right Triangles

Topic 2 Content: 45°-45°-90° Special Right Triangle Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on the 45°-45°-90° special right triangle. Your knowledge of isosceles triangles is going to really come in handy during this topic. You ready to get started? Let's go.

Like I said, your knowledge of isosceles triangles is going to come in handy during this case. That's because a 45°-45°-90° special right triangle is an isosceles right triangle. Let me take this apart here and really show you what I mean by that.

Just reviewing the parts of a right triangle. I'm actually going to move this triangle to the center here. Let's get it here. We know that opposite the right angle is hypotenuse, so this side right here I know is the hypotenuse of my right triangle. We know our hypotenuse is always the side with the longest length. We know that our other sides are our legs. Now, in the case of an isosceles right triangle, you know that an isosceles triangle is a triangle with two congruent sides. In the case of an isosceles right triangle, that means your legs are going to have the same length. The length of this leg will be equal to the length of this leg.

How that 45°-45°-90° came into place, that's referring to the angle measures inside of the right triangle. We know that it's a right triangle. Obviously, it has a right angle. Think back to what you remember about the relationships between sides and angles in a triangle. You recall that we discussed that across from your smallest angle would be your shortest side in a triangle. Across from your largest angle would be your largest side in a triangle. Now, what's also true is that if you have a triangle that has two congruent sides, then the angles opposite those sides are congruent. In the case of my isosceles right triangle, that means that this angle has the same measure as that angle. Let's think about what those measures have to be.

We know that a triangle, the sum of its angles—so the sum of the measures of its angles—is always 180 degrees. We have 90 degrees taken up by the right angle. 180 minus 90 leaves 90 degrees for the angles in the triangle. If we know that those last two angles are congruent, then that 90 degrees has to be split evenly between those two angles. 90 divided 2, that's 45. That's how we know that each angle, or each of these remaining angles, has a measure of 45 degrees. That's how we come about the name of a 45°-45°-90° special right triangle.

Now, what's special about it is that there's a shortcut to solving for the lengths of the sides in a 45°-45°-90° right triangle. You can actually use this formula to determine your side lengths. The hypotenuse is always equal, the length of the hypotenuse is always equal to the length of your leg times the square root of 2. That relationship is always true for any 45°-45°-90° special right triangle. You don't actually have to use the Pythagorean Theorem to solve for any of your side lengths. You can use the formula that the hypotenuse equals the leg times the square root of 2. You notice the square root of 2 is left in that form. It's not actually approximated to a decimal value. That's because we often use the formula for the 45°-45°-90° when we want to come up with an exact answer, with an answer that doesn't include a decimal value, but that's exact.

Let's take a look at a couple of examples, and see how we can apply this formula. Okay, here we go. Starting with a practical problem. A landscaper plans to plant rose bushes along the diagonal of a square-shaped garden. If the length of each side of the garden is 7 feet, what is the exact length of the diagonal? With practical problems, especially in geometry, if you're not given a picture or graphic, some kind of image, you're going to want to sketch one, because really, how you get to the problem is oftentimes tied into some image. It's easier to solve the problem if you have the image. Sometimes it's given to you, but sometimes it's not. You're going to have to come up with it on your own. Let's take out the important information in this triangle.

We're told that the landscaper is planting rosebushes along the diagonal of a square-shaped garden. Let's go ahead and sketch a square. You know from your prior knowledge that a square has 4 congruent sides. Let's mark those 4 congruent sides. You also know that a square has 4 right angles. You covered some of that stuff even as early as elementary school. I'm going to draw two symbols for my right angles in here. Now the second part. Let's get the red ink back. If the length of each side of the garden is 7 feet, so each side, 7 feet, what is the exact length of the diagonal? Okay. Let's get that information on our sketch. We're told that the length of each side is 7 feet. I'm going to put 7 here. I'm just going to label it on one side. I know that it's the same for all of them. We're asked to find the exact length of the diagonal. Let's get a diagonal on here. That is what we're solving for, the length of the diagonal, so I'm going to put a red *x* right there.

Now, notice that the word *exact* was in italics and underlined. That's a hint that you're going to use a special right triangle formula to solve the problem. In this case, it's the formula for the 45°-45°-90°. It's just a little hint that they're not asking you to round, that they want an exact answer. If you end up with a length that has a square root in it, and it's not a perfect square, leave it as it is. If we look here, we actually have two right triangles trapped inside this square. I'm going to pull this one out, this one on the left side, where I've kind of written the most information. Let's see here. I've got a right angle and I'm going to take those congruent marks, going to take the 7, and I'm going to take the *x*.

If you look at the triangle standing on its own, you see it's an isosceles right triangle. We can use that 45°-45°-90° formula to solve for the missing side length here. In this case, we're being asked to solve for the hypotenuse. When you're working with special right triangle formulas—let's shift a few things around here. You are still going to want to label the sides of your triangle, but you're not going to label them as *a*, *b*, and *c*, when you were using the Pythagorean Theorem. You're going to label them as the hypotenuse and the leg because those are the parts of this formula. I'm going to label—actually, I'll do it in red—that *x*, this represents my hypotenuse. I'm going to abbreviate it. These are my legs, these other sides.

The 45°-45°-90° formula is one that you're going to have to memorize. It's a good idea to just write it every time you have to use it. The hypotenuse—I've abbreviated it—equals the leg times the square root of 2. In this case, my hypotenuse is *x*. Let's get a little more work space here. *x* equals my leg, which I'm told is 7, times the square root of 2. You're actually all done because if I take this any further, I'm going to end up with a value that I have to round, and I was asked for an exact length. Exactly *x* is 7 square roots of 2. Let's take it back to our problem to get the units. It's in feet. The exact length of the diagonal that these rose bushes are going to be planted along is 7 square roots of 2 feet long. You're all done with that one. You see how we use the right triangle, we label the parts, we use the formula, and then we got our answer at the end.

Keep that in mind, and go ahead and try this one. You're given a practical problem, so make sure you sketch that image first. Take a few minutes and work through this one. Press play when you're ready to check your answer.

All right, let's see how you did here. You were told that a wooden counter top is in the shape of an isosceles right triangle with legs 6 feet in length. What is the exact length of the longest side of the counter top? Let's pull out that important information here. We're told that the counter top's in the shape of an isosceles right triangle with legs that are 6 feet. We're asked to find the longest side of the counter top. Let's go ahead and sketch an isosceles right triangle. I'm going to mark those legs congruent. We were told that the legs were 6 feet. I'll go ahead and write my 6. We're asked to find the longest length, or the length of the longest side of the counter top. I know on a right triangle, my longest side's the hypotenuse. I'm solving for *x* here.

Now I'm going to go ahead and label my right triangle. I know that *x* is my hypotenuse, and these other sides are my legs. Go ahead and get that formula down for the 45°-45°-90°. The hypotenuse equals leg times the square root of 2. My hypotenuse is *x* equals the length of my leg is 6 square roots of 2. Go back to my problem for the units, we were in feet. It's 6 square roots of 2 feet. You're all done with that one. All right? Good job on that.

Before we leave special right triangles, I want to show you how you solve for one of the legs in the right triangle. It actually takes a little bit of an extra step to get the length of a leg versus the length of the hypotenuse. I want to show you what that's all about before we leave this topic. Here, we're asked to solve for *x*. I'm going to go ahead and write my formula at the top, that the hypotenuse equals a leg times the square root of 2. I'm going to label my triangle. Switch to red here. My hypotenuse is 20. I know my leg is *x* here. Leg. It's not labeled down here, but I go ahead and put "leg" also. I'm going to fill in the information that I know. I'm going to substitute into this formula. The hypotenuse is 20, equals, the leg is *x*. Then times the square root of 2. Again, that's multiplication. I didn't write the symbol; I just put them next to each other, but we know we're multiplying.

In order to isolate *x* here, we're going to need to divide both sides by the square root of 2. Divide by the square root of 2, divide by the square root of 2. This cancels out over here. I'm going to use the law of symmetry and flip this around. I'm going to write that *x* equals 20 divided by the square root of 2. Right here is where it gets interesting. In math, it's improper to leave an irrational number in the denominator of a fraction. What I mean by that is, the square root of 2, it's not perfect. If I take the square root of 2, it's a non-repeating non-terminating decimal. It goes on and on and on and on for infinity and never repeats itself. There's no pattern. It's improper to leave that value in the denominator. To get rid of it, what we have to do is we have to rationalize the denominator. What that means is that we have to manipulate this fraction in a way that doesn't change its value, but that gets that irrational number out of the denominator.

There's several steps, there's several different ways you can do it, depending on the fraction you've been given. For us, in geometry, it's pretty much the same kind of step you're going to do every time. You're going to multiply your numerator and your denominator by whatever is in your denominator. In this case, we have the square root of 2 in our denominator. We're going to multiply by the square root of 2 over the square root of 2. That doesn't change the value that we have here, because essentially, all we've done is multiply by 1, because the square root of 2 divided by the square root of 2 is just 1. What this allows us to do is change the way this fraction appears, so that the denominator is now rationalized.

Let's keep going here. We're going to multiply straight across. 20 times the square root of 2. We're going to represent that as 20 square roots of 2. I see I'm going to start running out of space here. I'm going to shift this a little over to the left. We have 20 square roots of 2 on top. On the bottom, the square root of 2 times the square root of 2, that's the square root of 4, which is just 2. Do you know what I mean by that? Write the scratch work. The square root of 2 times the square root of 2 is the square root of 4, which is just 2. In the denominator, I'm left with just 2. We're almost done.

There's one more step to take here. We're going to simplifying this whole number portion. We're going to do 20 divided by 2, and you get that value. 20 divided by 2 is 10. My final answer, I'm going to have to scoot this to the left a little more—not the triangle, let's leave that. Just the work. Let's scoot that over just a little bit more. We lost that. There we go. 20 divided by 2 is 10. This is 10 square roots of 2, and we're all done. Those are the steps that you're going to have to take in order to rationalize your denominator. The final answer is 10 square roots of 2. You can't actually leave it as 20 divided by the square root of 2 because of that denominator. You're going to represent it as 10 square roots of 2.

Go ahead and keep that in mind, that whole process in mind, and go ahead and try this one. Press pause, take a few minutes, work through it, press play when you're ready to check your answer.

All right, let's see how you did with this one. Let's label, we have the hypotenuse. Then we have our legs. I'm going to go ahead and write my formula up here. The hypotenuse equals a leg times the square root of 2. Our hypotenuse, 16, our leg is *x*, times the square root of 2. To solve for *x*, we're going to divide both sides by the square root of 2. Let's get a little more space here. Because I remember how much I had to move things last time, let's go ahead and get some space. We're going to write this as *x* equals 16 divided by the square root of 2. You got to the point where it got interesting, and you had to rationalize that denominator.

Let's multiply the numerator and the denominator by the square root of 2. 16 times the square root of 2. Let's switch to black ink here. That's 16 square roots of 2. The square root of 2 times the square root of 2 is the square root of 4, which is just 2. Then we can divide right here. 16 divided by 2—I'm going to write it underneath this time. That's 8. Our final answer, 8 square roots of 2. You're all done with that one. All right. Good job on that. All right, guys, we've reached the conclusion of this topic on the 45°-45°-90° special right triangle. Hope you saw how your knowledge of right triangles and isosceles triangles and some algebra skills all came in handy during this topic. Bye!