Module 7: Right Triangles

Topic 3 Content: Inverse Trigonometry Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on inverse trigonometry. Your knowledge of the trig ratios are going to come in handy during this topic. You ready to get started? Let's go.

Okay, so before we dive into inverse trigonometry, just want to review a little bit about the ways that we know of how to solve a right triangle. I'm going to reveal some right triangles hidden behind these circles. Okay, so given this right triangle, we know that we could use the Pythagorean Theorem to solve for the unknown side. We could use $C^2 = A^2 + B^2$, and figure out what x is in this case, right? Okay, let's take a look at this one in the middle. Okay, so here I noticed that 45-degree angle. It's shooting out at me right now, so I know that I have a 45°-45°-90° special right triangle. I could use my formula, the hypotenuse equals my leg times the square root of 2, to figure out what x is in this case.

Okay, let's take a look at the next one. Okay, now how about this one? I see here I have a 60-degree angle in this right triangle. I could use my special right triangle formulas for the 30°-60°-90° special right triangle to solve for x. We know that those are the hypotenuse equals 2 times, or short leg, and the long leg equals the short leg times the square root of 3, right? I could use that, either of those formulas, depending on exactly what my given information is here, to solve for x. Okay, let's scroll and look at the one we have next. Okay, so here I have a right triangle that also includes at 74-degree angle. In this case, it's not a special right triangle. I'd have to use my trig ratios to solve for x here. That sine, cosine, or tangent, one of those would get me to the value of x in this triangle. Now take a look at this next triangle, this last one here.

Now this one's a little different. We haven't faced a right triangle set up in this way. Here we're give two links in the triangle, and we're asked to actually solve for one of the missing angle measures in this right triangle. Now when you're faced with a triangle that's set up like this, you're going to use inverse trigonometry to figure out what x is, okay? All right. Let's take a look at those inverse trig ratios. Just half a right triangle trig, we had sine, cosine, and tangent. For inverse trigonometry, or inverse trig for short, we have inverse sine, inverse cosine, and inverse tan, and we use our inverse formulas, or inverse ratios when we're solving for an unknown angle measure in a right triangle, okay? Just look at inverse sine, and we're referencing angle A in this case, so the measure of angle A would be given by the inverse sine of the length of the leg opposite angle A, over the length of the hypotenuse, okay? In this case, let me actually slide this over a bit just to write what that would be. Yeah, we can put it in black ink.

The length of the leg opposite angle A, that would be a here, this lowercase a, and the length of the hypotenuse is c. The inverse sine of a over c would give us the measure of angle A.

Okay now, inverse cosine, let's take a look at that. Okay, so inverse cosine, the measure of angle A will be the inverse cosine of the length of the leg adjacent to angle A, over the length of the hypotenuse. In this case, that would be the length of the leg adjacent to angle A, so that'd be b, and the length of the hypotenuse, that's c. The inverse cosine of that ratio would give us the measure of angle A. Then finally, inverse tangent. All right, let's scoot that over a little bit. The measure of angle A is also given by the inverse tangent of the length of the leg opposite angle A, so that'd be a. Let's get the pen. Over the length of the leg adjacent to angle A, so that'd be b. We're going to make sure I got the right variable written down there. The inverse tan of a over b would also give us the measure of angle A, okay.

These are our inverse trig ratios that we can use. Okay, and then just for short—to give you a little shortcut way to remember this, we can use that same mnemonic device, that SOH CAH TOA, to keep straight what these ratios are also, or what these trig functions are going to be. The inverse sine, and the inverse is denoted by that little negative one exponent. When you see that, you're referencing an inverse. The inverse sine—and we'll see it abbreviated like this a lot—is opposite over hypotenuse. A lot of times you'll see it written like that, and that's how we're going to pretty much write it when we work our way through these examples in a minute. Okay, and then the inverse cosine, or inverse cos, you'll see written like this. cos, adjacent leg, over the hypotenuse. Notice that negative 1 exponent right there. Then inverse tan, a lot of times you'll see written like this: tan, with a negative 1 exponent, opposite leg, over adjacent leg.

That same mnemonic device, that SOH CAH TOA, can help you remember these formulas as well. Sine, inverse sines, opposite over hypotenuse, inverse cosine, adjacent over hypotenuse, inverse tan, opposite over adjacent. They're just so you can keep straight what those function are. Okay, so now let's apply this knowledge and work our way through the next couple of examples.

Here we're asked to solve for x, and we're going to round to the nearest degree. I'm going to start out by writing SOH CAH TOA, just to keep in mind what my trig functions are here, my inverse trig functions are, and I'm going to label the hypotenuse first. It's just a good rule of thumb so that you don't get it mixed up with your legs. Again, hypotenuse is right across from the right angle. Then x, this is my reference angle in this case. The leg opposite from it, that's right here. Then we have the leg adjacent to it.

Now, stepping back and looking at exactly what I have here, I'm given a length for the hypotenuse and the adjacent leg but nothing for the opposite leg. Let's just erase that, we're not going to use that at all when we're solving this triangle. We're going to be focusing on the opposite leg and the hypotenuse, I'm sorry, the adjacent leg and the hypotenuse. If I use my mnemonic device here, adjacent and hypotenuse, A and H, those are together with cos, okay? To solve for this angle measure here, I'm going to use inverse cosine of the adjacent leg over the hypotenuse. Then to fill in my values here, and that's going to tell me what *x* is, so go ahead and put that in there, too. *x* is going to equal inverse cos of 17 over 29. Once I know what that value is, I'm going to know what this angle measure is in my right triangle.

I'm going to need to switch to my calculator; let's get that up here. I'm going to clear the memory just to be sure that everything's wiped out, so *2nd* and then the plus sign, *RESET*, *ALL RAM*, and then *RESET*. Then because we're working with trig ratios here, and in this case specifically the inverse ratios, we're going to need to change the mode. Anytime you're working with those trig ratios, sin, cos, and tan, or the inverse ratios, inverse sin, inverse cos, inverse tan, you're going to want to make sure that your calculator is set to *degree* mode because in the case of our triangles, our angles are measured in degrees, not in radians. *RADIAN* is the fourth row down, and right beside it is *DEGREE*. Arrow down until your cursor's blinking on *RADIAN*, arrow to the right so that it's blinking on *DEGREE*, and hit *ENTER*. Now degree should be highlighted and the cursor should be blinking on *DEGREE*.

Go ahead and clear just to get back to the home screen. Now we're ready to put in what we need the calculator to figure out for us. Going to look back at the problem, we need the inverse cosine of 17 over 29. Okay, so if you look at the *SIN*, *COS*, and *TAN* buttons, those are the second row above 7, 8, and 9. Right above those buttons written in blue you see *INVERSE SIN*, *INVERSE COS*, *INVERSE TAN*. We want to call up that inverse cos function. We're going to need to press the *2nd* button here on the upper left corner and then press *COS*, and there's our inverse cos function. Then I'm going to need to look back at that ratio one more time, so we need 17 over 29. 17 over 29, go ahead and close the parentheses, press *ENTER*. We were asked to round to the nearest degree, so this is about 54 degrees.

Let's go ahead and get that written down. Then in the case of this triangle, our angle—in this case it was *x* we're asked to solve for—is approximately 54 degrees. You're all done.

You see how we labeled our triangle, similar to how we would label it—actually, the same way that we would label it when we're working with our trig ratios—we labeled our triangle, we figured out which inverse trig function we needed, and then we solved, worked our way through this problem to solve for *x*. All right. Let's try another one together. Here, solving for *x*, and we're asked again to round to the nearest degree. I'm going to write my mnemonic device, that's SOH CAH TOA. Again, it's just a way to help you remember the formulas. You don't have to write it; it's just a little shortcut, kind of tricky, kind of way to remember what your functions are. Okay, so got that written down. I'm going to label the triangle starting with the hypotenuse, so that's 12. Then I'm going to go to my reference angle, so opposite the reference angle, that's the opposite leg, put it O for opposite. Then right next to it, is the adjacent leg.

Now, looking back, I'm given that the opposite leg's nine units, the hypotenuse is 12 units, the adjacent leg is blank. That means I'm not going to be working with it at all when I solve this triangle, so I'm just going to erase it, don't even have to worry about it. Here, I'm working with the opposite leg and the hypotenuse, or O and H. I look through my SOH CAH TOA here, O and H are together with S, so I'm going to be using inverse sin to figure out what x is in this case. Okay, so get the black ink there. x is going to equal inverse sin of the opposite leg, over the hypotenuse. Inverse sin, opposite over hypotenuse. Let's fill in the values. Opposite leg is 9, hypotenuse is 12. In my calculator, I need the inverse sin of 9 over 12. Okay, so let's get that. Second sin, 9 over 12. Let's close those parentheses, and if we round to the nearest degree, this will be 49 degrees, right?

Let's go back to our work. This means that x is approximately 49 degrees, and we're all done there. Okay, so you solve for x in this right triangle. Keep this work that we've done with these previous examples in mind, and go ahead and work your way through this one. Press pause and take a few minutes and do all the work you need to, to solve for x in this case. When you're ready to check your work, go ahead and press play.

All right. Let's see how you did here. I'm going to start by writing SOH CAH TOA, okay. Now I'm going to label my triangle, so opposite the right angle, *hypotenuse*. Have my reference angle here, it's x. 21's the opposite leg. 14's the adjacent leg. Looking back at my triangle, it's the hypotenuse that isn't given a length, isn't given a measure. Let's go ahead and erase the hypotenuse. I'm not going to be working with it at all. What I'm going to be working with is the opposite leg and the adjacent leg, or O and A. Check my mnemonic device here, O and A are together with T, so this is an inverse tan problem.

x is going to equal inverse tan of the opposite leg, over the adjacent leg. Okay, so x equals inverse tan, opposite leg's 21, adjacent leg is 14. In our calculator, we need inverse tan of 21, over 14. All right. Let's get that calculator up. Inverse tan, so second tan, 21 divided by 14. Press *ENTER*, round it to the nearest degree; that'd be 56 degrees. Let's go back to our work, and this means that x is approximately 56 degrees, and you're all done. Okay, so in this triangle, that missing angle is about 56 degrees. Okay, good job there.

All right guys, you've reached the conclusion of this topic on inverse trigonometry. Hope you saw how your knowledge of those trig ratios, sine, cosine, and tan, helped you get an understanding of the inverse trig functions that you needed to figure out here: the inverse sin, inverse cos, and inverse tan. Bye.