Module 7: Right Triangles

Topic 3 Content: Right Triangle Trigonometry Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on right triangle trigonometry. Your knowledge of right triangles and the relationships between the sides and the angles of triangles is going to come in handy during this topic. You ready to get started? Let's go.

Okay, so before we start talking about right triangle trigonometry specifically, I want to just review a bit about the different ways we know about how to solve right triangles. Let me just reveal the triangles that are hidden behind these circles here. If you were given this triangle—you don't know any angle measures, but you do know the lengths of two of the sides, and it's just one unknown length—you could use the Pythagorean Theorem to solve this triangle. You could use the formula $C^2 = A^2 + B^2$. You would eventually work your way to the answer to this one. Let's go ahead and circle that just so it stands out a bit.

Let's take a look at the right triangle beside this one. Here I'm only given the length of one of the sides, but I do know it's a right triangle and it also has an angle that's 45 degrees. Here I could use one of my special right triangle formulas to figure this one out. It's a 45°-45°-90°, so I use my hypotenuse equals the leg times the square root of 2. I would eventually work my way to the answer to this one. Let's keep going through our right triangles here. Let's scroll down.

All right, let's look at the triangle hidden behind this circle. Here, again I'm only given one side length but I do know that one of the angle measures in this right triangle is also 60 degrees, besides that right angle. This is another special right triangle. It's a 30°-60°-90° special right triangle. There are two formulas that I could use, or that are available to me, to figure out the lengths of the sides of this triangle. That the hypotenuse equals 2 times my short leg and that my long leg equals—let's fix that one there, getting ahead of myself—that my long leg equals the short leg times the square root of 3. Those formulas are available to me when I'm working with 30°-60°-90° special right triangles.

What happens in this case? Let's look at this last triangle here. I have a triangle. I only know the length of one side. I am given an angle measure, but that angle measure is 74 degrees. It's not a 45°-45°-90° special right triangle, so I can't use the formula for that. It's not a 30°-60°-90° special right triangle, so I can't use these formulas either. How do I actually figure out what x is in the case of this triangle?

For this case, this is what you do: You're going to use right triangle trigonometry to figure out the answer to this one. There are three trigonometric ratios, or trig ratios for short, that are available to you when you're solving a right triangle that's not a special right triangle. Let's look at the formulas here.

The ratios are sine, cosine, and tangent, and there's some ratios behind there that I'm going to show, but just so you're familiar, when I say, "sine," "cosine," or "tangent"—sometimes I'll say, "sine," "cos," and "tan" for short—that you know what I'm referring to.

Here you see we have a right triangle with angle measures *A*, *B*, and *C*. Those are in capital letters, and then we're representing our lengths with lower case letters *a*, *b*, and *c*. We're going to use our formulas; we're going to reference angle *A* in our formulas. That's going to be what we call our reference angle, or the angle around which we describe these ratios. For example, the sine of angle *A* is equal to the length of the leg opposite angle *A*. Let's get the pen here. The leg opposite angle *A*, that's lower case *a*, over the length of the hypotenuse, which in this case is *c*. The sine of angle *A* is equal to the length of the leg opposite angle *A* over the length of the hypotenuse.

Now let's look at the cosine of *A*. Let's get that out of the way. I got to switch to my pointer tool. The cosine of angle *A* is equal to the length of the leg adjacent to angle *A*. By *adjacent*, I mean "next to angle *A*." If *a* was the leg opposite angle *A*, *b* is the leg adjacent to angle *A*. Then it's the length of that leg over the length of the hypotenuse, which is *c*. Let's get that pointer tool again.

Now the tangent of angle *A*. The tangent of angle *A*—let's get the pen back—is equal to the length of the leg opposite angle *A*. That's *a* over the length of the leg adjacent to angle *A*. That's *b*. I know this seems like a lot right now and it's really because we haven't attached it or haven't applied it to a problem yet, but I want to get you familiar with it at first, and I promise as we keep going it's going to start making a lot more sense.

Let's move on through this. I want to show you some abbreviations—and I'm going to flip back and forth for a minute—of sine, cosine, and tangent. A way that you can remember what these ratios actually are. We recall sine of angle *A* is the length of the leg opposite angle *A* over the length of the hypotenuse. The cosine of angle *A* is the length of the leg adjacent to angle *A* over the length of the hypotenuse. The tangent of angle *A* is the length of the leg opposite angle *A* over the length of the leg adjacent to angle *A*.

Keep that in mind. How we said that sine—I'm actually going to… let's uncover this. It's the same ratios from the previous slide, just without the triangle. The sine of angle *A*—we can abbreviate that as this, and this is how you'll see it written a lot. The sine of *A* is equal to opposite over hypotenuse. That's how you can keep it straight: a little shorthand way to keep straight that the sine is the leg opposite of angle *A* over the length of the hypotenuse.

The sign for cosine a lot of times you'll see that written as "cos." Matter of fact, most times you'll see it written as "cos." The cos of A is equal to the adjacent leg over the hypotenuse. Finally *tangent*, we'll call that "tan" for short. Tan of A is equal to the opposite leg over the adjacent leg.

A little mnemonic device that you can use to remember the formulas is SOH CAH TOA. I have it written right up here at top, and that helps you keep straight that—switch to red—that sine is opposite over hypotenuse. That's your S-O-H. Then that cosine is the adjacent over the hypotenuse, so that's your C-A-H. Then that tan is the opposite leg over the adjacent leg, so that's your T-O-A.

Sometimes what you'll see people write down when they're solving a problem where they have to use trig ratios is they'll write SOH CAH TOA at the top of their paper and that helps them keep straight that your formula for sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tan is opposite over adjacent. Just a little mnemonic device, a little pattern to help you remember these.

I know it's probably not crystal clear yet what happens with these but let's start applying them and I promise it'll get better. Our first example. Solve for x, round to the nearest tenth if necessary. Switch to black ink. The first thing I'm going to write is SOH CAH TOA. That's just going to help remind me of what my trig ratios actually are. Remember with our trig ratios, they're all based around a reference angle, and when you're given a problem, the reference angle is the angle that you're given besides the right angle measure.

I know it's a right triangle, so I know this is 90 degrees, but besides that right angle, I know that this is a 57-degree angle. We call this our *reference angle*. Good rule of thumb is to label your hypotenuse first, so I know my hypotenuse is opposite the right angle. Go ahead and call it h. In reference to this angle, x is the leg that's opposite from this angle. I'm going to label that with an O. This 11, this measure is for the length of the side that's adjacent or right next to this angle, so I'm going to label it with an A.

I've labeled my triangle with opposite leg, adjacent leg, and hypotenuse. Now what you want to do is look back and figure out: What are the side lengths that you're actually working with here, or which sides of your triangle should you focus on? You were asked to solve for x, which is the opposite leg. You were given the adjacent leg. The hypotenuse is a side that's completely blank. For the case of this triangle, just completely ignore the hypotenuse. It's not going to help you solve this problem in any way.

Let's get back here. We're asked to solve for the opposite leg, we're given the length of the adjacent leg, so we're working with the opposite leg and the adjacent leg or O and A. Look at your little mnemonic device here. Look for where the O and the A are together. In this case, they're together with T. Remember that was our tan formula, or tangent formula. To solve this problem, we're going to use the tan trig ratio.

The tan of 57 degrees, that's our angle, so tan equals opposite over adjacent. That would be x over 11. All right, so now that we have that we're going to actually solve this problem, I'm going to change this. I'm going to put that tan of 57 over 1 so that I can look at this like a proportion and then use what I know about solving proportions to cross multiply and figure out what my value is here for x.

Here we go. Let's start cross multiplying. 1 times x is x, and then I have 11 times the tan of 57. Now this is where you're going to need your calculator. The tan of 57 is actually some numerical value, and we're going to use the calculator to figure out what that value is. I'm going to switch to my calculator, and if you haven't already, go ahead and clear the memory in your calculator. I've already cleared my memory here. It's very, very, very important when you're solving your trig problems in geometry that you change the mode of your calculator. Right beside the second button, that button in the upper left corner, you'll see *MODE*. Go ahead and press *MODE*. When you go down to the fourth line, right now everything to the left is most likely highlighted for you if you reset your memory, if you cleared your memory.

You'll notice *RADIAN*, the fourth one down, *RADIAN* is highlighted and right next to *RADIAN* is *DEGREE*. We're going to want to highlight *DEGREE*. Arrow down and then over until your cursor is blinking on *DEGREE* and then press *ENTER*. Basically what happened here—a radian is another way that you can measure an angle but for our triangles we're working with degree measures for our angles. We're just telling the calculator that we're working with degrees, give us our answer appropriately.

Once the cursor, you've hit *ENTER*, and the cursor is blinking on *DEGREE*, go ahead and clear and get back to this screen. I'm going to flip back to my problem here. What I needed to calculate was 11 times the tan of 57. I'm going to my calculator. 11 times. You see 7, 8, 9 on the calculator. Two rows above that. There's your sine, your cos, and your tan button. Go ahead and press *TAN* and then 57. Close the parentheses right around that 57. The calculator's going to tell us what 11 times the tan of 57 is. Press *ENTER*. We're rounding to the nearest tenth here, so it's approximately 16.9.

Let's switch back to our work and this means that *x* is approximately 16.9, and you're all done there. You see how we used our trig ratios to set up a proportion and then we used our calculator to actually figure out that unknown value.

I know trig, it's really a skill that you have to practice to get the hang of. We're going to do a couple more examples before we let you go and try one. Actually, we're going to do one more example before you try one on your own. All right. Let's look at this one. Again, we're solving for *x*. We're rounding to the nearest tenth, and we have our right triangle here.

The first thing I'm going to do is write my mnemonic device here, SOH CAH TOA. I'm going to label my right triangle. I know opposite the right angle there's the hypotenuse and the 23 degrees, that's my reference angle, and opposite the reference angle I have the opposite leg. That means this is my adjacent leg right here. Looking back here I was given that the opposite leg is 8. *x* is when I'm solving for my hypotenuse. My adjacent leg, or the leg that's adjacent to 23, is completely blank so it's not going to play a role in solving this triangle. I'm just going to completely erase it because I don't need to worry about it at all.

If I look here, I'm actually working with the opposite leg and the hypotenuse, or O and H. Look at your mnemonic device and find where O and H are together, and they're right here with sine. This time I'm going to actually write out that ratio for you to see. The sine of our angle, in this case 23 degrees, equals the opposite leg over the hypotenuse. This is what we're substituting those values in; just in case you didn't really see that on the last one.

The sine of 23 equals the opposite leg which is 8, over the hypotenuse which is *x*. Then what I'm going to do is first I'm going to scoot that down a little bit, just to get a little more space. I'm going to put that sign of 23 over 1 just so I can look at it like a proportion that I'm cross multiplying. Let's start solving this here. When I cross multiply, I like to start with the part of the proportion that's going to give me *x* on the left side, but it doesn't matter what side you start with. It's completely up to you.

I'm going to start with *x* times the sine of 23 equals 1 times 8, which is 8. In this case, I don't have *x* all by itself yet. It's not isolated. I'm actually going to need to divide both sides by the sine of 23 to get it isolated. Divide by the sine of 23, that's going to cancel on the left side. To figure this out, I need to go back to my calculator and figure out 8 divided by the sine of 23. Let's look at our calculator. 8 divided by the sine of 23. Round to the nearest tenth, that's about 20.5.

Back to our problem. *x* is approximately 20.5, and you're all done with that one. You see how we used the sine trig ratio to solve that right triangle. We've done a couple of examples together, so it's your turn now to go ahead and try one. Let me get you started off. I'm going to go ahead and write that mnemonic device up here. That's SOH CAH TOA, just so you can be mindful of what your trig ratios are. Go ahead and press pause and take a few minutes and work through this one. Press play when you're ready to check your work.

All right, let's see how you did here. Going to go ahead and label the sides of my triangle. Opposite the right angle, that's my hypotenuse. Opposite from my reference angle, go ahead and put an O there. Then x is the leg adjacent to my reference angle. Okay, so x is my adjacent leg. 19 is the hypotenuse, the opposite leg is completely blank, so it's not going to play a role here. I'm going to erase it.

This right triangle, I'm going to focus on the adjacent leg and the hypotenuse, or A and H. Let's find where A and H are together right here with cosine. I'm going to write out that trig ratio. The cos, 74 degrees, it equals the adjacent leg over the hypotenuse. Now let's fill in those values. Cos 74 equals x over 19. I'm going to go ahead and put the cos of 74 over 1, so I can look at it as a proportion and cross multiply. Let's see, 1 times x is x equals 19 times the cos of 74. Now I'm going to switch to the calculator and figure out what 19 times the cos of 74 is.

19 times the cos of 74, and we have—round to the nearest tenth, it's approximately 5.2. That means, let's go full screen again, that x is approximately 5.2, and you're all done with that one. Good job guys.

We have reached the conclusion of this topic on right triangle trigonometry, and I hope you saw how your knowledge of the relationships between the sides and the angles of your triangle helps you get through this topic. Bye.