Module 7: Right Triangles

Topic 4 Content: Practical Problems Involving Right Triangles Transcript

Hi guys, welcome to Geometry. This topic is going to focus on practical problems involving trigonometry. Your knowledge of sine, cosine, and tangent is going to come in handy during this topic. You ready to get started? Let's go.

Okay, so before we start diving into those practical problems, let's just review our trig functions for a second. In our earlier topic we covered sine, cosine, and tangent, and we learned about a mnemonic device, SOH CAH TOA, that could help you memorize those trig functions. Sine—in this case we're referencing angle A—is the opposite leg over the hypotenuse; cosine, the adjacent leg over hypotenuse; and tangent, the opposite leg over the adjacent leg. SOH CAH TOA is what we use just to help us remember those functions: that sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent.

Something we didn't mention before that I want to mention now is a special relationship that exists between the sine and the cosine of complimentary angles. Let's take a look at this triangle—make sure I've got my pen here. We have a right triangle. In this case, angle C is our right angle, so it measures 90 degrees. We know that the measures or the sum of the measures of the angles of any triangle is always 180 degrees. If I have a right triangle, I know 90 degrees is taken up by one of my angles, and my remaining angles represent that other 90 degrees.

I could say that the remaining angles in my triangle—I'm going to call them my acute angles because I know that their measures—will be less than 90 degrees. One thing I also know is that the sum of the measures of those angles has to equal 90 degrees because I've got 90 degrees taken up here, another 90 degrees between these two angles, together that give me my 180 degrees.

Those acute angles in my triangle are also complimentary. When you bring in sine and cosine, the special relationship that exists is the sine of one of your acute angles equals the cosine of your other acute angle. Let me show you what I mean. In this case, I'm going to talk about the sine of angle A. I know that sine is opposite over hypotenuse, so the leg opposite of angle A is CB. My hypotenuse is AB. Now let's talk about the cosine of angle B. I know that cosine is the adjacent leg over the hypotenuse. My adjacent leg, to angle B is side CB, and my hypotenuse is AB. You see how those ratios were the same? That's going to be the case in any right triangle. The sine of one of your acute angles is equal to the cosine of that complimentary angle.

That's just a special relationship that exists in right triangles that we hadn't discussed before that I wanted to make sure you were aware of now, okay? All right, now let's talk about those inverse functions. We use these to determine the measures of unknown angles in a right triangle. We have inverse sine. If we wanted to know the measure of angle A, we could take the inverse sine of the opposite leg over the hypotenuse, or we could take the inverse cosine of the adjacent leg over the hypotenuse, or we could take the inverse tangent of the opposite leg over the adjacent leg. These were another set of functions, our inverse trig functions that we could use to find the measures of unknown angles in a right triangle. Again, we can use that same mnemonic device, that SOH CAH TOA, to help us memorize what these are here also.

All right, now let's keep all that in mind, and let's start diving into these practical problems, these real-world applications. Okay so let's take a look at our first example. A gymnastics wedge at a local recreation center is in the shape of a right triangle. The wedge rises at an angle of 40 degrees, and its base is seven feet long. What is the height of the wedge? Round to the nearest foot.

Okay, so like with any practical problem that doesn't have an image, we always want to draw some figure, some shape, some graphic to help us represent what we see in the problem. Let's start pulling out the important information that we need so that we can get a sketch of a right triangle with our measures in the right spots, and then we can go ahead and start solving and figuring out the answer to this one. Okay, so I'm going to switch to my highlighter and start picking out some things here. All right, so a gymnastics wedge at a local recreation center is in the shape of a right triangle; that's important to know. The wedge rises at an angle of 40 degrees, and its base is 7 feet long. What is the height of the wedge? Round to the nearest foot.

Okay, so I'm going to go ahead and get a right triangle sketched. I'm just going to sketch it sitting upright. My right angle in there. I was told that the wedge rises at an angle of 40 degrees. If I take a look at this right triangle, if I were focusing on this angle right here, this would be the angle of my incline would say. That's the angle where I'm rising, right there. It's this angle, right here, that's 40 degrees. We're also told that the base of the wedge is seven feet long, so when you hear base you can kind of think of the bottom. Generally that's what it's referring to. Not always, but because this is a practical problem, so I'm envisioning a gymnastics wedge in the shape of this right triangle, I can go ahead and assume that when they say *base*, they're talking about the bottom of the wedge. They're telling me here, that the base is 7 feet long. I'll go ahead and get a 7. What is the height of the wedge?

Okay, in practical problems, height is always straight up and down, perpendicular to the ground, right? If someone asked you how tall you are, you always stand straight up tall, and you measure your height, right? Straight up and down. The height of this wedge is what we're begin asked to find, so I'll call that *x*. That's my unknown. All right, so I'm going to take a look, I've got this right triangle. I know the measure of an angle, and one of my sides. I'm going to have to use my trig functions to figure out the length of this side. I'm going to go ahead and write SOH CAH TOA just to help me remember—once I get this all labeled—which trig function I need to use. Let's go ahead and start labeling, I'm going to switch colors.

Okay I'm going to label my hypotenuse first so that I don't get it confused with my legs. Know that's the side across from the right angle. In reference to this 40-degree angle, that's my opposite leg. I'll call that *opp*. Let's get a little more workspace for a little bit here. Now in reference to this angle, this is my adjacent leg. I'll label that *adjacent*. Now let's take a look at what we have here. I know my opposite leg is what I'm solving for, that's labeled with *x*. The adjacent leg is 7. The hypotenuse doesn't have a measure, so I'm not going to be working with that side at all. I'm just going to go ahead and erase it so it doesn't distract me.

If you're working on notebook paper, you could just cross it out; you could also erase it. Just anything to signal to you that it's not going to play a part in your math that you're going to use to solve this. Okay, so I see I'm working with the opposite leg and the adjacent leg, or O and A. Let's look at our mnemonic device: O and A are together with T, so I'm going to use tangent to solve this right triangle. Let me go ahead and just scoot this to the left a little. I'm going to do my work right to the right side. Let's group that together. There we go. Get the pen, okay? I'm going to take the tan of 40 degrees, and it's going to equal my opposite leg over the adjacent leg. Just going to write it like that for now, then I'll substitute the values in. Okay, scroll down some more get some more space.

The tangent of 40 equals the opposite leg—in this case is *x*—and the adjacent leg is 7, and I'm going to go ahead now and put that tan 40 over 1 because I'm going to treat this like a proportion to cross multiply. All right, get some more space here. I'm going to go ahead and start cross multiplying. 1 times *x*, that's *x*, equals 7 times the tan of 40 degrees. Now I'm going to use my calculator to go ahead and determine the 7 times the tan of 40 degrees. All right, so let's switch to our graphing calculator. We're going to clear the memory. I've actually already cleared mine, but just so you're familiar, I'll go ahead and show you how I did that. I'm going to press the *2nd* button, the blue button right here in the upper left corner. The addition sign, right above *ENTER*. Then I'm going to select *RESET*, which for me is number 7, *ALL RAM*, and then *RESET*.

Okay. My memory is clear. Now remember: When we're working with trig functions, we always want to change our mode to *degree*, because the measures of our angles are going to be in degrees. Let's go ahead and press *MODE*, which is right beside that blue second key. If you clear the memory like I did, mostly everything to the left, on the left side, will be highlighted. You'll want to go down to the fourth row. You'll notice *DEGREE*, *RADIAN* is highlighted because right now the calculator is set to measure angles in radians, but we want to change it to degree, so go ahead and press your right arrow key so your cursor is blinking on *DEGREE*, and then hit *ENTER*.

Now *DEGREE* should be highlighted, the cursor should be blinking on *DEGREE*. Go ahead and press *CLEAR* just to get back to your home screen. Now I'm going to switch back to my work for a second so I can remember what I needed to put in the calculator. I need 7 times the tan of 40 degrees. I'm going to be rounding to the nearest foot, so back to the calculator. 7 times tan 40. Going to go ahead and close my parentheses, and rounded to the nearest foot, it would be approximately 6 feet. Let's go back to our work, go full screen here. All right, so *x* is approximately—get a little more space—6. Bringing it back to the units in our word problem, that means that our wedge is approximately 6 feet tall. Okay?

You see how we were given this word problem; I'm going to scroll back up to it. You were given the word problem, we highlighted that key information that we needed. We used it to sketch the image of our triangle, and from there we went ahead and used our skills of solving trig problems to get to the solution to this one.

Now take a look at this one. I want you to go ahead and try one. Press pause, go ahead and take a few minutes, work your way through this problem. Remember, highlight that key information, get your triangles sketched, and then use what you know about trig functions to solve the problem. In this case, maybe you're going to be using right triangle trig, or inverse right triangle trig. It all depends on what you're given there. Press pause and take your time, work your way through it. Press play when you're ready to check your answer.

All right let's see how you did here. Going to go ahead and switch to my highlighter. While I'm reading it, I'm going to go ahead and highlight the key information. A 25-foot ladder—that's important—is leaning against the side of a house. The bottom of the ladder is 8 feet away from the bottom of the house. What is the measure of the angle formed by the bottom of the ladder and the ground? Round to the nearest degree. Okay. I pulled out of this problem what I need to help me get my triangle. Let's start using that information to get our picture sketched.

All right, so first I'm going to go ahead and sketch it right here, and then I'll pull that triangle out of the problem. We have a 25-foot ladder, so 25, and it's leaning against the house. The bottom of the ladder is 8 feet away from the bottom of the house. Bottom of the ladder, bottom of the house, this distance right here, that's 8 feet. Then I need the measure of the angle formed by the bottom of the ladder and the ground. That's that angle right here.

I'm going to pull this right triangle out of this image, then we're going to use what I know about trig, or inverse trig, going to dig deep here, and see which one I need to solve. I'm going to use that to get to the bottom of this one. All right, let's get that triangle right underneath. Okay get it here. My right angle, 25, 8, and then *x* degrees. Okay.

Here I notice I'm trying to find the measure of an unknown angle, okay. In this case, I'm going to use inverse trig to figure out the answer to this one. I can still use SOH CAH TOA to help me figure out which formula I need, so I'll go ahead and write that down. Let's actually slide this triangle a little to the left. Okay. Get the pen back. Okay, let's get everything labeled. My hypotenuse, that's 25. Label that. Now I'd reference to my angle, this is the opposite leg. This is the adjacent leg. In this case, I have measures for the adjacent leg and the hypotenuse but not for the opposite leg. I'll go ahead and erase that; it's not going to play a role in this problem.

All right, caught a little bit of my triangle, let's get that back. Okay. Adjacent leg and hypotenuse, A and H. A and H are together with C. I'm going to use cosine, or in this case inverse cosine, to figure out that unknown angle measure. *x* will equal the inverse cosine of the adjacent leg over the hypotenuse. Go ahead and fill in those values. Inverse cos, adjacent leg is 8, hypotenuse is 25. In my calculator I need the inverse cosine of 8 over 25. Let's get our calculator up. We need to press the *2nd* key, and then press *COS*, so that we could pull up that inverse cos function. It's stamped right above *COS*. 8 divided by 25. Close the parentheses, let's press *ENTER*, and I believe we were asked to round to the nearest degree, so in this case that would be 71.

Back to our word problem, let's go full screen here. That means that *x* is approximately 71 degrees, which means in reference to our problem—let me get back up to the picture—that the measure of this angle formed right here by the bottom of the ladder and the ground is 71 degrees. Okay? All right, good job on that.

Now before we leave practical problems involving trigonometry, I want to talk a little bit about angles of elevation and angles of depression. For a second, we're going to work with this graphic here. You see we have an airplane flying in the air, and we have a duck flying in the air. Notice that, they're facing each other—the airplane and the duck—and that the duck is flying a little lower than the airplane. Okay? Okay, so consider that. Bare with me here, we're going to keep diving into this. Now envision two horizontal lines, right? One at the duck, going straight from the, like, the duck's eye just straight across, a horizontal line, and then one right here from the cockpit window of that airplane going straight across: two horizontal lines.

Now remember from what we've studied before that all horizontal lines are parallel, so in this case that means that these two horizontal lines that I'm envisioning in the sky are parallel lines. All right, still with me? Let's keep going. Now consider this. Consider this line of sight, this solid line in between these parallel lines. I'm going to go ahead and throw those parallel symbols right, touching and put it in the center. We're going to write some things in here. On my dotted lines, those horizontal lines, so parallel. Now consider this line of sight, and what I mean by that. From where the duck is, envision, imagine that the duck is looking up at the airplane right up at that cockpit. This is the line of sight from the duck to the airplane's cockpit. The angle formed right here, between the line of sight for the duck, from the duck's eye to the airplane cockpit, and this horizontal line, we call that an angle of elevation.

You really could think about it simply; it's the angle that the duck is looking up at the airplane at. We call this angle an angle of elevation. Now also consider this angle right here. This is the line of sight from the cockpit down to the duck. We call this angle, formed from the airplane's line of sight down to the duck, and its horizontal line, we call that an angle of depression. You can think about that as the angle formed by—say the pilot, he's sitting in there, right? Or she's sitting in there, from the pilot looking down at the duck. The airplane, the duck is looking up to the airplane; that's our angle of elevation. And the pilot's looking down at the duck; that's the angle of depression.

Now because these are two parallel lines—actually what we have here are two alternate interior angles that are congruent. Your angle of depression will always be congruent to your angle of elevation. That's what we're going to discuss here. We're going to solve some problems involving angles of elevation, so one object or a person or a duck looking up at something else, the measure of that angle, and the angle of depression. If something higher looking down at something lower, okay? We actually use trig to solve problems like this. Take a look here at this first example.

All right, still working with that duck. A duck is flying at a horizontal path at a height of approximately 500 feet. The angle of depression from the duck to a lake's edge measures 34 degrees. What is the distance between the duck and the lake's edge? Round to the nearest foot. Okay, let's take what we need out of this problem, get a picture going, then we'll go ahead and solve it.

Okay so we have a duck flying in a horizontal path at a height of approximately 500 feet. The angle of depression from the duck to the lake's edge measures 34 degrees. What's the distance between the duck and the lake's edge? And we're going to round to the nearest foot. Okay, let's get some things going. Think I'll switch to red this time. We were told the duck is flying in a horizontal path. The duck is flying this direction right? We're also told that it's approximately 500 feet in the air. The distance from the ground to the duck, this distance right here, actually I'll put it up a little higher. Okay, from the ground to the duck, I'll make a dotted line. This is approximately 500 feet we're told. The measure of the angle of depression is 34 degrees. That's taking us from the duck to the lake's edge. We'll get that going just like that.

Here's our angle of depression. That's 34 degrees. We're asked to find the distance between the duck and the lake's edge. That would be this distance right here, that solid line. That's our x. Now so we can make this look familiar to us, I'm going to flash back for a second. Remember we talked about two horizontal lines and an angle of depression, that angle formed by looking down at something, at an angle of elevation, the angle formed by looking up at something. Because we started out with two horizontal lines, I'm going to get our two horizontal lines in our next picture. I know this one's represented by the duck's path, and then envision another horizontal line right here.

We picture that—here's our angle of depression, and here's our angle of elevation—because we know these angles are congruent. I know that if our angle of depression is 34 degrees, so is our angle of elevation. It's also 34 degrees. Now what I can do is I can focus on this right triangle to solve this problem. That's exactly what I'm going to do. I'm going to pull that triangle out of the picture, and use it to figure out the distance between the duck and the lake's edge. All right, let's get that down here, let's extend this page a little. All right, got our right triangle. Here's our right angle because we know that a vertical line and a horizontal line are always perpendicular. I know here's that height of the duck: 500. Here's the measure of that angle that we're going to use, and then there's x, what we're solving for, okay?

Let's use what we know about trig to get to the bottom of this one. I know I'm solving for the length of a side, so I'm going to be using my trig functions and not inverse trig. Let me actually go ahead and scoot this over a little bit so I can write some things to the right. Do want that to stay. Let's get this. There we go. Slide that over, just make sure that's all still lined up okay. This picture, all right there we go. Okay let me get my pen back, go ahead and get SOH CAH TOA down here to help me figure out which trig function I need to use. All right, let's go ahead and start labeling this triangle. I know across from my right angle, that's my hypotenuse, and in reference to this angle, that's my opposite leg, so that'd make this one my adjacent leg. If I look at what's labeled here, I have a measure for the opposite leg. I'm solving for the hypotenuse, so it's the adjacent leg that I'm not going to be using. Let's just erase that. All right, okay.

Opposite and hypotenuse, or O and H. That's together with sine. I'm going to solve for the sine, 34 degrees, and that equals my opposite leg over the hypotenuse. Okay now let's go ahead and substitute those values in there, the rest of those values. Sine of 34 degrees, our opposite leg is 500. The hypotenuse is x, and so that I can solve this like a prop. I'm going to go ahead and put it over 1. Let's get some more workspace here so we can cross multiply. Okay so x times the sine of 34 equals 1 times 500, which is 500. Right now I do not have x isolated. I'm going to have to divide both sides by the sine of 34. Sine of 34 degrees, sine of 34 degrees. On the left side that's going to cancel, and I'm just left with x. Let me just scroll back up here to the top, and make sure we were rounding to, yep, the nearest foot.

In the calculator, we're going to put in 500 divided by the sine of 34 degrees, and we're going to round to the nearest foot, or the nearest whole number. Okay so 500 divided by the sine of 34 degrees. 500 divided by sine 34, go ahead and close my parentheses. Round to the nearest whole number, this would be approximately 894. Let's go back to our world problem, go full screen here. x is approximately 894, which in our case, we're going to go ahead and assign our units to it and give it meaning. That means that the distance between the duck and the lake's edge is approximately 894 feet. Okay? That's about how far away the duck is from the edge of the lake, 894 feet. All right, now I know that was a lot to put together. You're working with those angles, you used your angles to get your right triangle set up, and then you applied trig to go ahead and solve the problem.

Keep all that in mind, and I want you to go ahead and try one. Okay so press pause, take a few minutes, work your way through this one. Get those images, get the measures, get everything in your picture, and use that to help you figure out how to solve this one. Press pause, take your time, and when you're ready to check your work, press play.

All right, let's see how you did with this one. I'm going to switch to my highlighter. At a certain time of day, a 12-foot tall tree casts a shadow that is 17 feet long. What is the angle of elevation of the sun? Round to the nearest degree.

All right, let's see here. We know that we have a tree that's 12 feet tall, and we know that it's casting a shadow 17 feet long. We're asked to find the angle of elevation to the sun, or of the sun. Here what I'm going to do: I'm going to make my horizontal here. Actually, let me dot that one. I want to show you something that's a little special about this one. Now it might have thrown you a little bit that it said the angle of elevation of the sun, so I'm going to dot this horizontal here, and I'll dot this horizontal here, and then let's dot the one at the ground.

Okay, so we're told we have our shadow at 17 feet long, right? So we know this distance here. Now when it asked us for the angle of elevation of the sun, going to just get this line going just like this. Imagine that's perfect, if it's by hand or I couldn't get it perfectly straight, but just assume I got that perfectly straight there. When we're asked to find the angle of elevation of the sun, I know my angle of elevation means that I'm from somewhere low looking somewhere up, right? We're actually finding the angle of elevation which is this angle right here. That's our unknown angle. Even though it says the angle of elevation to the sun, if I'm here and I'm looking up to the tree and then again looking up to the sun, I'm still actually measuring that same angle. I didn't want that to throw you, that it said the angle of elevation of the sun. You're still trying to find that angle right there at the bottom, and what you probably see here is this right triangle, okay?

I'm going to close this off, and this is the right triangle that we're focusing on. There's our right angle because the tree is standing straight up and down, we're assuming on the ground. Let's pull that right triangle out of there. Right angle, 12, 17, and then here's x that's what I'm solving for. Because I'm solving for an unknown angle measure, I'm going to be using inverse trig to get to the bottom of this one. I'll still write down SOH CAH TOA. Let's get our triangle labeled. Across from our right angle, there's our hypotenuse. Then in reference to x, that's our opposite leg, and this is our adjacent leg. I notice the opposite and the adjacent legs, both have measures given to me. It's the hypotenuse that I'm not going to be working with here. Let's just go ahead and get that erased. Get that out of the way. Thought I might catch some of that triangle, let's just get that back in there.

Opposite and adjacent, or O and A, those are together with T. I'm going to be using inverse tangent to figure out this angle measure. x equals the inverse tan of the opposite leg over the adjacent leg. Let's go ahead and fill those values in. All right, so x equals inverse tan, the opposite leg is 12, the adjacent leg is 17. Then let me just go back up here to make sure we are rounding to the nearest degree, okay. In our calculator we're going to get the inverse tan of 12 divided by 17, and round to the nearest degree.

All right, so *2nd* and then *TAN*. Uh oh, our calculator is doing some thinking on us here. There we go, let's get that clear. *2nd*, and then *TAN*. 12 divided by 17. Close those parentheses, okay. To the nearest degree, this would be about 35 degrees. All right, so back to our problem, let's get that written down. All right, so x is approximately 35 degrees. That angle of elevation of the sun, which is the angle formed right here, right at the ground, that is approximately 35 degrees. All right? Good job on that.

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All right guys we've reached the conclusion of this lesson on how to use trigonometry to solve practical problems. I hope you saw how your knowledge of sine, cosine, and tangent and inverse sine, inverse cosine, and inverse tangent helped you solve all these real-world problems. Bye.