

Module 8: Quadrilaterals

Topic 1 Content: Applying the Properties of a Parallelogram Transcript

Hey guys, welcome to Geometry. This topic is going to focus on how to apply the properties of a parallelogram. Now some of what you learned in middle school about parallelograms is going to come in handy during this topic. You ready to get started? Let's go. Okay, now you probably learned in middle school, let me go ahead and remove this here, that a parallelogram is a quadrilateral with both pairs of opposite sides parallel.

Quadrilateral, that means it's a four-sided figure and then parallelogram its opposite sides are parallel, okay? That's the definition of a parallelogram. The properties of a parallelogram that we are going to be applying here, there are several. One is that its opposite sides are congruent. Okay and we see that here by our congruent marks. Its opposite angles are congruent and we see that here by those congruent arcs.

Its consecutive angles are supplementary, so what that means is that the angles that lie along one side of a parallelogram measure 180 degrees, the sum of them, okay? That means along every side, these angles here will add up to equal 180 degrees. These two angles would add up to equal 180 degrees and the same along this side, these two angles and then along these side as well.

Our consecutive angles measure 180 degrees, the sum of them. All right. Then also in a parallelogram its diagonals bisect each other. By bisect I mean they cut each other in half, so this diagonal was cut in half by this one and vice versa. That means along this diagonal, these two parts are congruent, their measures are equal, and then along this diagonal, these two parts are congruent, okay? All right. Now let's start using those properties to solve these problems.

Here we are given a parallelogram and asked to find X, Y, and Z. All right. When you are solving properties involving parallelograms it maybe that you just need to apply one property or more than one to get to your answer. It all depends on the information that you were given. In this case here I see I'm dealing with the angles in this parallelogram, so I'm going to consider my angle properties to solve this problem.

I'm given that this angle measures 120 degrees. Now I know from my properties that the consecutive angles of parallelogram are supplementary, so that means that the angles along this side have to measure 180 degrees. If this angle measure 120, that means that X is going to measure 180 minus 120, 60 degrees, so now that X, 60 degrees. Now I also know that my opposite angles are congruent, so if X is 60, that means Y is 60 because the opposite angles in my parallelogram are congruent and then if this angle is 120 degrees, this measure, that means that Z has to be the same thing because here my opposite angles have to be congruent.

I look and I've got all my measures. When I was asked to find X, Y, and Z, I could say that X is 60, Y is 60, and Z is 120. Okay? I've got it all. All right, so, you see how we apply that angle property, actually two of them to solve this problem. Let's take a look at this one. Here we are asked to find the measure of each angle on the parallelogram and our angle measures are represented by algebraic expressions. What that signals to me is that I'm going to have to set up an equation in order to get to the bottom of this one.

Module 8: Quadrilaterals

Topic 1 Content: Applying the Properties of a Parallelogram Transcript

Okay so here I was given angle D, its measure as an expression but I was given its measure, and angle G and they are opposite angles. I know that my opposite angles are congruent. I think I'm going to shrink this parallelogram a little bit, just so that we can kind of look at it and get the work down at the same time. Okay, that will help us a little bit. All right, so like I said, I know that my opposite angles, I'm going to abbreviate that, are congruent, so I can use that to setup an equation.

$10X$ plus five equals $5X$ plus 20, because I know that these measures are going to have to be the same so I will set my expressions equal to each other, so use my algebra skills, solve for X , subtract $5X$ from each side, that's going to cancel. We've got $5X$ plus five, equals 20. Let's get a little more space, all right. Subtract five from each side, cancel, we've got $5X$ equals 15 and then our last step, divide by five. Now we have that X equals three.

Now don't be tempted to stop right there because if you look back at the problem, we were asked to find the measure of each angle. Now that we know what X is, we are going to use that value to figure out the measure of each angle in this parallelogram. I'm going to write up here at the top that X equal three just so I remember, but I'm going to get rid of this work right here, just to get some more work space. Now on your notebook paper, you could keep it there; just write in a different area.

I'm just trying to keep us a little organized and a little less crowded up here. Okay, so now that I know that X is three, I'm going to just pick one of this angles and substitute X into it. Pick one of these expressions. I will pick the measure of angle D, because I know the measure of angle D equals $10X$, just write that a little better, that D looks a lot like a zero. Okay, the measure of angle D equals $10X$ plus five degrees. Now that I know that X is three, I'm going to go ahead and substitute that in there.

10 times three, plus five, 10 times three is 30, 30 plus five is 35. That means that the measure of angle D is 35 degrees. Now I know 35 degrees. Now if angle D measure 35 degrees, that means that angle G has also to measure 35 degrees because like we said before all opposite angles are congruent. I'll go ahead and write 35 degrees in there. Now, I can use what I know about the supplementary or the consecutive angles to figure out the measures of angle F and angle H. I know that in a parallelogram my consecutive angles are supplementary.

What that means, I think I will actually redraw this parallelogram so we can just focus on the angle measures and not get distracted by those expressions, okay, so 35 degrees, 35 degrees, so for angle F and angle H. I know that my consecutive angles are supplementary. The sums of their measures has to equal 180 degrees. If I find 180 minus 35, I'll figure out the measures of angle F and angle H. I believe that is 145. That means the measure of angle F, 145 degrees and because the measures of angle F and angle H have to be the same, also 145 degrees.

We've found the measure of each angle in our parallelogram and if you wanted to just write that out formally here is what we could do, we could say that the measure of angle F equals the measure of angle H, which equal 145 degrees, and we could also say that the

Module 8: Quadrilaterals

Topic 1 Content: Applying the Properties of a Parallelogram Transcript

measure of angle D equals the measure of angle G, which equals 35 degrees. All right and we are all done with that one.

All right, so you see how we again used the properties relating to the angles of a parallelogram to help us set up an equation, solve for X and then figure out the measures of those angles. All right, okay, now it's your turn. Go ahead and press pause, take a few minutes, work your way through this one, press play when you are ready to check your answer. All right, let's see how you did here. I see here I'm asked to find the measure of angle R in this parallelogram and I'm given a pair of consecutive angles.

I know from my properties that the consecutive angles, just going to abbreviate that, consecutive angles, are supplementary. That means that I can setup an equation, a little more space, $2X$ plus 20 plus $6X$ equals 180 because those measures of those angles have to add up to equal 180 degrees. Let's solve for X. We combine like terms $2X$ plus $6X$ is $8X$ plus 20 equals 180 . Subtract 20 from each side, scroll a little bit here, that's going to cancel, so $8X$ equals 160 , divide by eight, let's get a little more space here.

X equals 20 but remember we weren't asked just to find X, we were asked to find the measure of angle R. Let's see how we can use X to figure out the measure of angle R. We know that X is 20 and if I look angle R and angle T are opposite angles. If I can figure out the measure of angle T, I will immediately know the measure of angle R. Let's get a little more workspace. Let's erase some stuff up here because we don't it to get too crowded. Let's get rid of that work and get rid of that answer we wrote up at the top.

Okay, so the measure of angle T, let's find that. The measure of angle T is $6X$ degrees and now that we know that X is 20 , six times 20 , 120 . The measure of angle T is 120 . Right. If T, if that angle has a measure of 120 degrees, that means that angle R also has a measure of 120 degrees. These are a pair of opposite angles, so I they are congruent. As my final answer I can say, the measure of angle R is 120 . Now this was one strategy you could have used to solve this problem, maybe you did it this way, maybe you found the measure of angle T and used that to find the measure of angle R.

Another option is you could have found the measure of angle S and use that to find the measure of angle R and just in case you took that strategy into that route, let me show you how you would have gotten to that answer. Then also just because if you want to see what is the other way you could have solved this, because a lot of times in math there is more than one way to get to the right answer, so I showed you one way but here is an alternate strategy. Now let's say I erase this, pretend like I don't know this yet in here.

Let's say okay I know that X is 20 degrees and I want to use the measure of angle R, or the measure of angle S to figure out angle R, so the measure of angle S is $2X$ plus 20 degrees and I know that X is 20 , so I can substitute that in there. Two times 20 plus 20 , so two times 20 that's 40 , plus 20 , that equals 60 . I know that the measure of angle S is 60 degrees. Let's go back up to the parallelogram. If I knew the measure of angle S was 60 degrees, then I can use the fact that these consecutive angles have to be supplementary and 180 minus 60 is 120 .

Module 8: Quadrilaterals

Topic 1 Content: Applying the Properties of a Parallelogram Transcript

That's another way you could have used X , use that value, to figure out the measure of angle R . Maybe you used angle T like we did at first or maybe you used angle S to get to that answer. All right, a good job on that. Now let's take a look at this one. We've got a practical problem here. A wool blanket includes a decorative pattern in the shape of parallelogram below. Find the length of the diagonals. Okay, so I know in a parallelogram my diagonals bisect each other. In other words they cut each other in half.

If I focus on this diagonal first, I can see that if this part of it is eight centimeters, that means that this part of it also has to be eight centimeters, because if one half is eight centimeters, then so is the other half. If I focus on this diagonal, if this part of it is seven centimeters, then this means this part also has to be seven centimeters because that's my other half. Now when I want to find the length of the diagonals, I need to add the parts together to find the whole length of each diagonal, so along this diagonal eight plus eight, that's 16.

I'll just show that work there, so eight centimeters plus eight centimeters, that's 16 centimeters, so that's one of the diagonals. Then we have the other one here, so seven centimeters, plus seven centimeters, 14 centimeters. These are the lengths of our diagonals of the parallelogram and this wool blanket. Good job there applying that diagonal property and it your turn again, so go ahead and press pause, take a few minutes, work your way through this one. Press play when you ready to check your answer.

All right, let's see how you did here. Here we have the opposite sides of the parallelogram, they have their measures given, and we are asked to find X . What I know about the opposite sides of the parallelogram is that they are parallel, which won't help me get to the answer here but that is one of the properties but I do know that they are also congruent. The fact that the opposite sides of a parallelogram are congruent will allow me to setup an equation and solve for X .

Here, $11X$ plus three, equals $7X$ plus 23. All right. Let's scroll down here to get some work space. I'll subtract $7X$ from each side, that cancels. We have $4X$ plus three equals 23. Subtract three from each side, that's going to cancel. We have $4X$ equals 20 and then divide by four. We have that X equals five. In this problem since we were only asked to solve for X , we are all done here. All right, so you see how we used that property to setup an equation and figure out that in this case X equals five. All right, good job on that. All right guys, we've reach the conclusion of this lesson on how to apply the properties of a parallelogram. I hope you saw how some of your background knowledge and some new information you learned here helped you get to the end of this lesson. Bye.