Module 8: Quadrilaterals
Topic 2 Content: Applying the Properties of a Rectangle Transcript

Hi guys. Welcome to Geometry. This topic's going to focus on how to apply the properties of a rectangle. Now your knowledge of parallelograms is really going to come in handy during this topic. You ready to get started? Let's go.

Okay, so like I said, you're knowledge of parallelograms will help you in this topic and that's because the definition of a rectangle is that it is a parallelogram with four right angles, so what that means is that because a rectangle is also a parallelogram, it inherits all of the properties of a parallelogram. It does have one new one of its own but it inherits all of the properties that a parallelogram has. Let's review those.

Because a rectangle is a parallelogram, that means that it has opposite sides that are parallel, and that means it has opposite sides that are congruent. It's opposite angles are congruent and because the rectangle is a parallelogram with four right angles, it just so happens that all of its angles are congruent, but because it is a parallelogram, we can say that it's opposite angles are congruent. It just so happens that because it's a rectangle, those angles are all right angles, all 90 degrees. Okay? Also, because a rectangle is a parallelogram, it's consecutive angles are supplementary so along each side, the sums of the angles will measure 180 degrees. Now because each of those measures is 90 degrees, you can see very easily that all of the consecutive angles are supplementary. Let's get that out of the way but in a rectangle, your consecutive angles are going to be supplementary. All right?

Then, also because a rectangle is a parallelogram, it's diagonals bisect each other but what's specific about a rectangle, and this is the new property, is that the diagonals are congruent, so not only do my diagonals cut each other in half and create equal segments along each diagonal, but because my diagonals themselves are congruent, that means that I end up with four congruent parts, right in here. Let me say that again. Because a rectangle is a parallelogram, it's diagonals bisect each other, they cut each other in half but what's new, the new property of a rectangle is that it's diagonals are congruent. What ends up happening is we have four congruent parts, right in here. The four parts of the diagonal or the two parts of each diagonal, four in total. All right?

Let's use these properties that we know now, the ones that a rectangle inherits and the one that it has of its own and let's apply them to solve some problems. Okay, take a look at this example. We're going to find the measure of each angle in this rectangle shown here and we're given one angle measure, right there in the center, 40 degrees. Now, actually what helps us solve this problem is the property involving the diagonals of a rectangle. Okay, watch me do this one here. Now, because I know my diagonals are congruent, and they bisect each other, that means that I have four congruent pieces, right here in the center. Okay, so look at this as a whole, you see I have the parts of my rectangle, then I also want you to see these four individual triangles that were created in this rectangle. What I mean by that is this, we have one triangle here on the left. We have one triangle here on the right. Right here, these two facing each other and then we have the triangle here at the top and the triangle here at the bottom.

If you look here, right here, these angles in the center, these are a pair of vertical angles, so what I know is that my vertical angles are congruent, so if the measure of this angle is 40

degrees that means the measure of this angle is also 40 degrees. Now look at these two triangles. You actually have two congruent triangles, right? These triangles are congruent by side, angle, side. You have two pairs of corresponding sides that are congruent and a pair of included angles that are congruent, so what that means is, whenever I find the measures of the angles in this triangle, I'll immediately know the measures of the angles in this triangle, okay? I know we're bringing in a lot of old geometry that we learned, a little bit of new stuff, it's all going to kind of come together for you here working with these angles.

Okay, now focus on this triangle here on the left. I'm actually going to re-draw it so you can just kind of zoom in on it and just focus on that. Okay, so congruent marks and then we've got 40 degrees. Okay, so what I have here is an isosceles triangle. What I know about isosceles triangles is that the base angles are congruent, you I believe learned that back in middle school. You may have even touched on it a little bit in elementary school. Also, what we learned here in geometry is that because these two sides are congruent, that means the angles that are opposite those sides are congruent and I know in a triangle I have 180 degrees and 40 of those degrees are already taken up by this angle so that leaves me with 140 degrees that has to be split evenly between those two angles, so 140 divided by 2, that's 70. That means this angle, 70 degrees and so is this angle, 70 degrees.

Okay, so I'm going to jump back to our rectangle up here and let's get those measures in there. Okay so 70 degrees, 70 degrees. Now remember we said that these triangles were congruent triangles, so if these two angles measure 70 degrees, that means that these two angles also measure 70 degrees. Those are the angles that correspond to it. Okay? So far we've gotten some of the angles taken care of and we can use some more geometry to figure out the rest.

Now recall that a rectangle is a parallelogram with four right angles. That means that these angles right here, they're complimentary. The sum of those must add up to equal 90 degrees because I know they have to form a right angle here. If the measure of this angle is 70 degrees, that means the measure of this one has to be 20 degrees because 90 minus 70 will leave 20 degrees remaining, okay? That's going to be the same all around this rectangle, so another 70 degree angle, that has to be 20, this has to be 20, and this has to be 20. Now, I'm just left with having to find the measure of one angle in this triangle up top. Okay, so let's get rid of this down here.

It's kind of like we're putting together a puzzle with this. We're using what we have just to start finding some little pieces and eventually everything kind of starts to come together for you. Okay, so I'm going to re-draw this triangle just so we can focus in on it, right underneath this rectangle. Let me switch to the pen. All right and we have our congruent marks here and 20 degrees, 20 degrees. Okay, so I know that triangle has to add up to equal 180 degrees, it's measures, so 20 plus 20, that's 40. 180 minus 40, that's 140. That means the measure of this angle right here, 140 degrees. All right? I'm going to use that information, I'm going to take it back to my rectangle. Let's get that angle right in the center, 140 degrees. Now, if this angle is 140 degrees, so is this one because again, that's another pair of vertical angles or maybe you noticed that this triangle up at the top is congruent to this triangle at the bottom, also by side, angle, side. That means those included angles there; they're going to have to be congruent, all right?

I know we pulled together a lot of geometry, a lot of old angle relationships and triangle relationships that we knew of and some of the new relationships that you learned about a rectangle. Now there's more than one way to get to the answer to this one, so when you try the problem right after this, you may use a different strategy then I used, okay? I want you to just apply the geometry that you know to get to the bottom of this next problem, okay? Go ahead and press pause, take a few minutes, work your way through this one. Use everything you know to help you put together the pieces of this puzzle, and press play when you're ready to check your answer.

All right, let's see how you did here. The first thing I'm going to do is mark the congruent parts on that diagonal. I'm going to use that 50 degrees and just start working my way around this rectangle to figure out what I can. All right, well I know, if I focus in on this triangle on the left, and I'll go ahead and even re-draw it separately just to help you just focus on that part of this rectangle, that 50 degrees is one of the base angles of this isosceles triangle, so because these two sides of this triangle are congruent, these two angles, these base angles are going to have to be congruent also. If that angle's 50 degrees, then so is that one, so I'm going to go ahead and write that 50 degrees right in there. Now I'll go back to focusing on the triangle as a whole. I know that the sum of the measures of its angles is going to add up to equal 180 degrees, so right now I have 100 degrees taken up by those two angles in the triangle, so 180 minus 100, that's 80. This remaining angle in the triangle, 80 degrees, and because I know this triangle on the left is congruent to this triangle on the right, its corresponding angles are going to be congruent, so 80 degrees, 50 degrees, and 50 degrees.

Now, let me erase some of this work so we don't get too crowded. Then, I'm just going to step back and look at this rectangle as a whole and figure out what could I use to figure out some of these missing angle measures in here. Well, I think I'm going to go back to the fact that this rectangle has four right angles, right? That means that these two angles right in my corner here have to add up to equal 90 degrees, so if this angle is 50 degrees, this angle's going to have to be 40 degrees, so that I have a right angle right there in the corner and it's going to end up being the same all along the corners of this rectangle. Okay?

Then, I see I'm left with looking at this triangle up top and this triangle on the bottom. I think I'll focus on the bottom one this time. I'm just going to re-draw it outside of the rectangle and I've got 40 degrees, 40 degrees. Okay, so I know the measures of each of these angles, and the sum of the measures of these angles has to be 180 degrees because I have a triangle. I have 80 degrees taken up by those two angles there, which leaves me with 100 degrees, so the remaining angle in this triangle has to measure 100 degrees, okay? If this angle is 100 degrees, then so is this one because they're vertical angles. You're all done with finding the measures of each angle in this rectangle. Now, maybe you took a different route then I did but as long as you got to these measures in the end, using the appropriate properties then you're good to go. All right?

Let's take a look at the next one. Okay, we've got a practical problem here. A local park is in the shape of a rectangle and includes a 15 mile diagonal walking path. If one edge of the part is 12 miles long, what is the perimeter of the park? All right, so let's pull out what we need

here. We're going to get a rectangle drawn and then figure out which properties we need to use to solve this one. Okay, so we have a park in the shape of a rectangle and it has a 15 mile diagonal walking path. We're told that one edge of the park is 12 miles long. We're asked to find the perimeter of the park. Okay, so let's get our rectangle drawn.
We've highlighted the key information here. We've got our rectangle and we were told it includes a 15 mile diagonal walking path. I'm just going to sketch a diagonal of this rectangle, as perfect as I could make it. I didn't get it perfect but we've got a 15 mile long diagonal here. Okay, then we're also told that one edge of the park is 12 miles long. I think I'll just focus on this bottom edge.

It doesn't tell me specifically which edge so you could pick, either the bottom or the top or the left or the right, completely up to you. Then, we're asked to find the perimeter of the park. From your earlier geometry in middle school and probably elementary school also, you know the perimeter of a figure is the sum of the lengths of its edges. What I'm going to have to figure out here is the length of every edge in this rectangle and then I can use that information to get the perimeter. Okay, so what I'm going to focus on here is the fact that I have a right triangle in this rectangle. I know that it's a right triangle because I know that a rectangle is a parallelogram with four right angles, so here I've got a right triangle that I can focus on and I'm going to use the Pythagorean Theorem to figure out the length of this edge of my rectangle. I'm going to call that X because again, the length of that edge of my rectangle is also a leg of this right triangle, all right?

I'm going to pull that triangle out of the problem there and let's use the Pythagorean Theorem to figure out that unknown length. We've got X, we've got 12, we've got 15. Okay, so the Pythagorean Theorem, it's C squared equals A squared plus B squared. C is your hypotenuse, so let's get that labeled, the side opposite our right angle. Then, A and B are our legs. We can label those however we want. Now, let's go ahead and fill in to the Pythagorean Theorem over here. Switch back to black ink, okay so C squared, that'd be 15 squared equals X squared, plus 12 squared. Okay, so let's simplify this. 15 squared, that's 225, equals X squared plus 144, so I'm going to go ahead and subtract 144 from each side. Okay, let's get some more work space here. That's going to cancel, so 225 minus 144, that's actually 81, so 81 equals X squared and then to figure out what X is, I'll need to take the square root of each side.

The square root of 81 is 9 and the square root of X squared is X, so X equals 9. Okay, so let's go back to our rectangle up here. This is no longer X, it's no longer unknown. I know that this edge is 9, miles in this case. Now, don't be tempted to stop there because you were asked to find the perimeter of the park.

Now that you know what the edge is of the rectangle are, you can figure out the perimeter of the park. Okay, let's get some more work space here. Let's erase this. We know we used our right triangle and the Pythagorean Theorem to get to the bottom of that. Let's erase that, okay.

Let's re-sketch this rectangle just focusing on its edges. Let's scoot it down just a little bit. It's a little close to that one at the top. Okay, so I no longer need to know the length of that

diagonal. I just used it to figure out the edge. I know that this is 9 and this is 12 and because a parallelogram is a rectangle, I'm sorry, because a rectangle is a parallelogram, I know that it's opposite sides are congruent, so if this is 12 miles so is this. If this is 9 miles, so is this. Now I know the length of every edge of this rectangle and I can go ahead and find the perimeter. I have 12 plus 12, plus 9 plus 9. Another way that you could represent this, this perimeter, we'll say that P equals this, is we could say that P is 2 times 12 plus 2 times 9. You could also represent it that way, whichever way you're more comfortable with in finding the perimeter of a rectangle. Okay, so I'll use that just to help me get to the bottom of this here. 2 times 12, that's 24, plus 2 times 9 is 18. Let's use a little mental math here, I believe that's 42. Go ahead, we'll check our work just to be sure. 24 plus 18, okay so 24 plus 18 is 42. Okay, so let's go back full screen here. Bringing this back to the meaning of our word problem, what that means is the perimeter of the park is 42 miles. You're all done there. Okay?

You see how you had to apply a lot of different properties and a lot of different things you know about geometry to get to the bottom of this problem here. You'll find that a lot when you're working with parallelograms and rectangles because a rectangle is a parallelogram, that sometimes you just use the properties that the rectangle has inherited from the parallelogram, maybe you're going to use the property that it has of its own and maybe you have to pull in the Pythagorean Theorem or another thing that you know about geometry to help you get to the bottom of a word problem, or the bottom of any problem really. All right? Okay.

Now, I want you to go ahead and try this one. Press pause, take a few minutes, work your way through this one. Press play when you're ready to check your answer. All right, let's see how you did here. We had a rectangle, it was called ABCD and I didn't mention before but you name a rectangle, and really just any polygon by just going around its advertises, either clockwise or counterclockwise, so I could tall this rectangle ABCD. I could all it rectangle DABC, as long as I just go around the figure when I name it, okay? This is rectangle ABCD. BE is 3x plus 4. I'm going to go ahead and write that in there, 3x plus 4 and AC equals 20. I'm going to actually just follow along that here so this is 20. We're asked to find X. Okay, so we're going to need to use the diagonal property of a rectangle in order to get to the bottom of this one. Now, what's a little different about this problem is that we're given the length of one whole diagonal and then the length of half of the other one. I know that it's half because I know as far as the properties of the rectangle go, the rectangle does, is that the diagonals are congruent and the diagonals bisect each other. I know that this has to be half of this whole diagonal.

I also know that this whole diagonal is equal to this whole diagonal, so I could approach this two different ways. Let me actually, I'm going to re-draw this underneath just so you can focus on the diagonal parts for a second and ignore the measures. Okay, so I know that the diagonals are congruent and that they bisect each other, so I could figure out what X is by setting one half of a diagonal equal to one half of another diagonal, or I could figure out what X is by setting one whole diagonal equal to one whole diagonal. This one also has two strategies. This is what I mean. Let's go ahead and just write that one here. Okay, so I know AC, which we're going to say is this one right here. Let's write the vertices in there. Okay, so I know AC is 20, right? That means that AE must be 10 and that EC must also be 10 because those would be the two halves of that diagonal. One thing that I could do is I could say, "Well,

okay," BE, which I know is 3X plus 4, so BE equals AE because that's one half of a diagonal equal to one half of another diagonal. I can use what I know about algebra to set up an equation and solve for X.

Let's see. We have 3X plus 4 equals 10. Okay, so we'll subtract 4 from each side, all right. That cancels so 3X equals 6, divide by 3. X equals 2. Okay, so that's one strategy that I could take in order to solve for X. Now maybe you didn't use that strategy because you also had another option here. I'm going to scoot that off to the side for a second. Let's go ahead and group it together and then let's just shrink it a little bit. Okay, so if you didn't use that strategy then you may have used this one. Let's switch back to red ink here. Okay, so I know that BE is 3X plus 4, so that also means that ED is 3X plus 4 because that's the other half of that diagonal, so I could also use the fact that I know that the diagonals are congruent to set up this equation. Okay, so because I know that BD equals AC, all right, so BD is 3X plus 4, plus 3X plus 4, or I could represent that as 2 times 3X plus 4, not quantity, equals AC, it's easy to represent, that's 10 plus 10, that's 20.

This is another equation that I could set up to solve for X. I'm still using my property of a rectangle except for this time I'm using the fact that I know my diagonals are congruent, so 2 times the quantity, 3X plus 4 equals 20. Again, I'll use my algebra skills to go ahead and solve for X. Okay, so the distributive property, distribute that too, so 6X plus 8 equals 20, subtract 8 from each side, 6X equals 12 and then last step, divide by 6. X is 2. Either one of these strategies would have been appropriate to take in order to solve for X, okay?

You can really kind of get creative in how you want to solve these problems because for a lot of them, like I said before, there's more than one way to get to the right answer. All right? You've reached the conclusion of this topic on how to apply the properties of a rectangle. I hope you saw how your knowledge of parallelograms as well as your knowledge of some earlier geometry helped you to get to the end of this lesson. Bye.