Module 8: Quadrilaterals

Topic 3 Content: Applying the Properties of a Rhombus Transcript

Hi, guys. Welcome to Geometry. This topic's going to focus on how to apply the properties of a rhombus. Now your knowledge of a parallelogram and its properties is really going to come in handy for you during this lesson. You ready to get started? Let's go.

Why a parallelogram and its properties are important here, is because a rhombus ... Let's reveal this, is a parallelogram with four congruent sides. What that means is, because a rhombus is a parallelogram, it inherits all of its properties. Every property that a parallelogram has, a rhombus has also. Then what's special about a rhombus is that it has four congruent sides, so it actually brings some new properties into place.

Let's review the properties that the rhombus inherits, because it is a parallelogram. Okay, let's reveal some things here. Because a rhombus is a parallelogram, its opposite sides are parallel, its opposite sides are also congruent. Now, because a rhombus has the property that all of its sides are congruent, it just so happens that all of its sides are congruent, but because it is a parallelogram, you could also say that its opposite sides are congruent. Its opposite angles are congruent also, because it's a parallelogram.

Let's reveal this one. Its consecutive angles are supplementary. What that means is along each side of this rhombus, A plus B will equal 180. The measures of its consecutive angles have a sum of 180 degrees.

Then here, the last property it inherits from a parallelogram is that its diagonals bisect each other. These parts would be congruent and so would these parts. Their diagonals cut each other in half.

Now the new properties that it has, or the properties that are specific to a rhombus, one is this one, that its diagonals bisect opposite angles. What that means is, my diagonals, the opposite angles, that they intersect, they cut in half. For this rhombus, that would mean that the measure of angle one's congruent to the measure of angle two. The measure of angle five's congruent to the measure of angle six. The measure of angle eight's congruent to the measure of angle seven, and the measure of angle four is congruent to the measure of angle three.

Now because the opposite angles are congruent as well as being bisected by those diagonals, that also means that each of these angles, so angle one, angle two, angle five, and angle six, all of those measures would be congruent. Angle four and angle three, and angle eight and angle seven, all of those measures would also be congruent, okay? That's one property that's specific to a rhombus.

The next one here is that the diagonals are perpendicular. What that means, as the point at which those diagonals intersect, it forms a right angle, or they form right angles. I've represented that here by putting that right angle symbol right in the center. Just at one, but each of these angles here would measure 90 degrees. In a rhombus your diagonals are perpendicular, or in other words, they intersect to form right angles.

We're going to use all of these properties to get through these problems. Let's take a look at this first one. Here we're given a rhombus and we're asked to find X. What you want to do, look at what you're given, and use the properties that you know to get to the bottom of the problem here. I see I was given a measure of an angle, asked to find the measure of another angle. Now like we saw earlier, the diagonals bisect the opposite angles, so what that means, that if the measure of this angle's 40 degrees, so is the measure of this angle.

Now, also because my opposite angles are congruent, so the measure of this whole angle here is congruent to the measure of this angle here. That means that this is also, I'm going to just write that a little lower so I can squeeze 40 degrees in there. Oh, I'll write it on the other side. What that means is, because these angles are congruent and they're also bisected by this diagonal, is that angle's also 40 degrees, and so is this one.

Now, I can use what I know about triangles to figure out X her. If I focus my attention just on this triangle right here on the right side, and I'll even redraw it. Let's pull it out of this rhombus here. Just try to get a sketch that looks similar to it, kind of like that. It's a little over slanted, but you get the idea. 40 degrees, 40 degrees. I know that in a triangle, the measure of its angles, the sum, has to equal 180 degrees. So far, I have 80 degrees taken off from those two angles. 180 minus 80 will tell me what's remaining. 100 degrees. The measure, and I'll just put those degree symbols all around there. X in this case has to equal 100, because this angle has to have a measure of 100 degrees. X is 100. You're all done with that.

You see here how we used the properties of a rhombus, that its opposite angles were congruent, and that the diagonals bisected those opposite angles, and we used a little what we knew about triangles to find X in this case.

Okay, now take a look at this one. Go ahead and press pause, take a few minutes. Work your way through this one. Press play when you're ready to check your answer.

Let's see how you did here. Here again we're given a rhombus and asked to find X. I think I'm going to scoot it a little bit to the left. I know that the measure of this angle's 70 degrees. I'm just going to start using it and start working my way around this rhombus here. I know that the opposite angles in a rhombus are congruent, so if this angle measures 70 degrees, so does this one. I'll go ahead and write 70 degrees in there.

Now, what I also know is that this diagonal bisected the opposite angles. Right now I don't know what the measure of that angle is, but whatever it is, it's the same as this one and this one. All of these angles are congruent. Because I know that, now what I'll do is I'll focus on this triangle. That'll help me figure out what X is. Let's redraw that right off to the side. Let's put it in black. Let's get that right off to the side there. I'm just going to get those congruent marks for that. I'm going to leave X out for right now.
Just put in the congruent marks and that 70 degrees. I know that one measure of this triangle is 70 degrees. The remaining, because I know each, all the sums of all these angles has to equal up to 180, so I know the remaining degrees for this triangle is shared between those two angles.

Let's see what's left over. 180 minus 70, that's 110. 70 degrees are taken up in this triangle here, leaving 110 degrees that's split evenly between these two angles. 110, let's get those degrees symbols in there. 110. 110 degrees divided by 2, that's 55 degrees. That means that this angle has a measure of 55 degrees, and so does this one. Then now taking that triangle back to our rhombus, now we know what X is. X is 55. Good job on that one. See how you again, you apply your knowledge of those properties related to the angles of a rhombus, and a little bit about triangles, and you got to the bottom of this question.

Now take a look at this next example. We have a practical problem here. I'm going to go ahead and switch to the highlighter and I'm going to pull up that information that I need as I go. Okay, a kitchen table's in the shape of a rhombus. Each edge of the table measures five feet. One of it's diagonals measures eight feet. What is the length of the other diagonal?

Okay, so for this one I'm definitely, like all practical problems, want to get a picture drawn here. That's what I want to do first. I'm going to sketch a rhombus, which some people describe it as like a tall parallelogram, but you get the general sketch of what it looks like. Use your general idea. We see here each edge of the table measures five feet. I'm going to go ahead and just bring this down a little bit so I can write that measure here without running into the problem. Each edge measures five feet. I'm going to go ahead and write five all around. One of its diagonals measures eight feet, and I'm not going to write that eight in yet, but I'm just going to put that diagonal in red for right now is the reason for that. What is the length of the other diagonal? I'm going to put that one in black.

I'm given the length of the red diagonal, because it told me one of its diagonals measured eight feet, and I'm asked to find the length of the other diagonal. What I'm going to do here is instead of writing that this entire diagonal has a length of eight feet, I'm going to write the length of each of its halves. I know that diagonal's been bisected by the other one. If the whole diagonal's eight feet, that means that its halves are four feet, and that I'm asked to find the length of the other diagonal.

What I'm going to focus on first is just the length of half of that diagonal, and this is why ... Now I free handed this rhombus, like I said, so it's not perfect. It may not appear to be a right angle here, but because it's free handed, that's why. I know that one of the properties of a rhombus is that its diagonals are perpendicular, or they intersect to form right angles. I know that I have a right angle here, in the center of that rhombus.

What I'm going to do is I'm going to use the Pythagorean Theorem to figure out the length of the side of this right triangle. I think I'll highlight it just to see if you can ... Let me put an X there first. I want it to stand out for you so you can really see it in that rhombus, and I'm going to pull it out of the rhombus so we can solve it. We're going to focus on solving that right triangle. It'll give us half of that diagonal. Then we can use that information to figure out how long the entire diagonal is.

Let's get a little bit more workspace, and I'm going to sketch just that right triangle. Going to try to orient it in the same way you see it there, just so you can really see the relationship. That's what we're focused on. We're going to solve this right triangle to figure out the length of that part of the diagonal. Let's use, I'm going to use the Pythagorean theorem. See if we can just scoot that over there. There we go, and let's figure out the missing side in this triangle.

My Pythagorean Theorem, C squared equals A squared to B squared. Just label the sides of this right triangle. Five's going to be the length of the hypotenuse. Let's go ahead and just write C in there. Okay, so that's going to be our C. A and B are the legs. Let's substitute into the Pythagorean theorem. That'd be five squared equals X squared plus four squared. Five squared is 25, equals X squared, plus 16. Let's get a little more space. Subtract 16 from each side. 25 minus 18, that's nine, equals X squared and then to solve for X, take the square root of both sides. Square root of nine is three. Square root of X squared is X. X equals three.

Now let's take that back to our rhombus up here and let's go ahead, let's erase X, because now we know what it is. We know that it's three. I'm going to circle it just so it stands out a little bit. Now don't be tempted to stop there, because that's just the length of half of the diagonal. We were asked to find the length of the whole diagonal. If this half of it is three, so is this half. The length of the entire diagonal, three plus three, it would be six. What's our units here? Units is feet. It is six feet. Then we're all done with that one.

You see we brought a lot of things into play in that problem in order to get to the answer there, but it all starts with knowing the properties of your rhombus. We use that given information in the problem. We highlighted what we needed. We filled out what we had. We applied those properties to figure out some of the measures in our diagonal. That let us use the Pythagorean theorem. We solve for X. We got the length of part of it. Then we used that value to get the length of the whole diagonal.

Like we've seen in a few of our earlier topics, when you're working with parallelograms, which include rectangles, and rhombi, that's the plural name for a rhombus, a lot of pieces come together sometimes. There's a lot of parts to it. Just persist through those problems, apply those properties, and get to the bottom of what you're being asked to solve.

Keep all that in mind and go ahead and try this one. Press pause, take a few minutes. Work your way through this problem. Press play when you're ready to check your answer.

Let's see how you did here. We were given a rhombus. We were given QT. Its length was eight, and PR had a length of 12. We were asked to find PS. Let me put that in red for a second, just so stands out. QT is eight. PR is 12. What I'm going to do, is instead of writing 12 in there, I'm going to go ahead and bisect the diagonal. I'm going to put the length of its halves. Six and then six. Then we were asked to find PS. I'm going to call that X. Let's use what we know about a rhombus, about its properties to figure out how we could find out the length of PS.

Let's go ahead and fill out, go ahead and complete the length of this diagonal. If I know this is eight, then that means TS is also eight, because the diagonals bisect each other. Now I also know that my diagonals are perpendicular. That means I have a right angle right there. What I can do in this one is I can focus my attention on this right triangle. Didn't get it perfect, but you know what I mean, along PS. I'm going to use the Pythagorean Theorem to figure out the length of PS and help me get to the bottom of this problem and figuring out X.

Let me sketch. I'm going to pull that triangle out of that rhombus. See I have black ink here. Here's the triangle that I'm focusing on here. Here's X. Here's my right angle. There's six. There's eight. Let's go ahead and get our parts labeled. I'm going to scoot this over. Then let's use the Pythagorean theorem to find X. Pythagorean theorem, C squared equals A squared plus B squared. C is my hypotenuse, which in this case would be X. Legs are A and B. X squared equals six squared plus eight squared. X squared equals 36 plus 64. 36 plus 64 is 100, so X squared equals 100. Take the square root of each side. A little more space. X equals 10.

Now let's go back to the problem and see what x represented. X represented PS. When we found out that X is 10, we actually had gotten to the end of this problem. That means that PS equals 10 units. Good job on that one.

All right, guys. We've reached the conclusion of this topic on how to apply the properties of a rhombus. I hope you saw how your knowledge of parallelograms, as well as some other geometries you've learned about the Pythagorean theorem and triangles and angle relationships all came in handy for you in this topic. Bye.