Module 8: Quadrilaterals

Topic 4 Content: Applying the Properties of a Square Transcript

Hi, guys. Welcome to Geometry. In this topic, we're going to focus on how to apply the properties of a square. Now, your knowledge of parallelograms is going to come in handy during this topic. You ready to get started? Let's go.

Before we dive in to talking about the square, let's just review a little bit about the quadrilaterals we've discussed so far. By quadrilaterals, what I mean are the four-sided figures that we've discussed so far. Let's see. Let's uncover these. See what we have hiding behind here. We've discussed a parallelogram, and all the properties that a parallelogram has. Its opposite sides are parallel. Its opposite sides are congruent. Its consecutive angles are supplementary. Its opposite angles are congruent, and its diagonals bisect each other, right? Those are all the properties that a parallelogram has.

Now, we've also discussed a rectangle, which is a parallelogram with four right angles. A rectangle has all the properties that a parallelogram has, and then on top of that it has four right angles, and its own property that it has for itself is that its diagonals are congruent, okay? Along with those diagonals bisecting each other, because it's a parallelogram, in a rectangle, its diagonals are also congruent.

Then we've also discussed a rhombus. A rhombus is a parallelogram with four congruent sides. It has all of the properties that a parallelogram has, and then in addition to that, its diagonals are perpendicular, and its diagonals bisect at the opposite angles, right?

The new one for this topic is a square. Now, to show you the properties of the square, because the square is a parallelogram, so it is going to have the properties of a parallelogram, but in addition to that it also has a few more. I want to show it to you by way of this Venn diagram. Here, I'm going to zoom in first, focus first in on exactly where the square is. Notice a square or the squares are in the intersection of rectangles and rhombi, which remember, rhombi is just plural for rhombus. What that means is, a square is both a rectangle and a rhombus. It inherits all of the properties that a rectangle has, because a square is a rectangle, and it also inherits all the properties that a rhombus has, because a square is a rhombus, okay? It's the overlap of rectangles and rhombi, is where squares lie.

Now, if we look at this full picture and really look at what's going on here, let's zoom out and look at our big set here. Our big group is quadrilaterals, so those are all four-sided figures, right? Then a special case of quadrilaterals is parallelograms, so that's our subset of quadrilaterals, and then within parallelograms we have rectangles, rhombi, and squares. You can think about it as the smaller you go, or the further you go into those subsets, you're inheriting things from the larger subset. What I mean is, a parallelogram is a quadrilateral, so that means it's going to have four sides, right? Now a rectangle is a parallelogram, so it's going to inherit all the properties the parallelogram has, and then in addition it has that special property of its own. A rhombus is also a parallelogram, so it inherits all the properties of the parallelogram, and it has a couple of its own. Then a square is both a rectangle and a rhombus, so it inherits all of the properties of a rectangle, all of the properties of a rhombus, all the properties of a parallelogram, and then it's also a quadrilateral.

Module 8: Quadrilaterals
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While we work through these square problems, just keep in mind all of the properties that it has, which is basically everything that we've covered so far. Nothing's excluded, okay? Everything is fair game as we work through these examples.

Take a look at this first one. "Given the square below, find y." Let's kind of look at what's going on here. I notice that I have an expression for an angle measure down here in the corner. Now, because I know that a square is a rectangle, I know that it has four right angles, right? So I'm going to put that ... I'm just going to put one here and one here. I know there are four of them, but I just want to focus there.

There's a reason I'm going to show you right now. Because a square is also a rhombus, I know that its diagonals bisect those opposite angles. What that means is for these ... Here's the diagonal I'm going to focus on. For this right angle right here, this diagonal bisected it or cut it in half. What that means is I have 45 degrees right here, and then this angle right here also measures 45 degrees. In order for me to solve for y, all I need to do is set this expression equal to 45.

Let me get a little work space for us here. Let's slide the square over to the left, and let's get our equation set up and get ready to solve. Just group that together, get our pen back. Okay. Five y minus 10 equals 45, so I'm going to solve for y here. Add 10 to each side. Here we go. Let's cancel that out. We've got five y equals 55, divide by five. Get a little more work space here, and we end up with y equals 11.
There you go. We've solved for y.

Notice what we did here. We used the properties that we knew, that belonged to a square, because it was both a rectangle and a rhombus, and used that to set up our equation and solve for y. Let's keep going. It's your turn, so go ahead and press pause. Take a few minutes. Work your way through this example. Press play when you're ready to check your answers.

All right. Let's see how you did here. Remember, a square is also a rhombus, right? So the property that I'm going to focus on to help me solve this one is, I know that the diagonals of a square are going to be perpendicular. A square inherits that property from the rhombus. What that means is right here in the center, I have four right angles, and I'm going to focus on that one to help me set up an equation and solve for X. Let me slide this out of my way one more time. Scoot that over to the left, and let's get that equation set up. Because I know that's a right angle, we're going to set up six X plus 60 equals 90, subtract 60 from each side. Six X equals 30, divide by six. X equals five, and you're all done. All right? Good job on that one.

Let's take a look at this one, example two. Marie's favorite quilt includes a pattern represented by the square below. If DC equals eight, what is the length of BE? Let me slide this square down just a little bit so I can include some measures around it. Actually, I might shrink it a little bit too. Let's make it a little smaller. What I'm told in this problem is this square represents a pattern on a quilt, and that DC equals eight. I'm going to go ahead and write that, get my pen back, go and write that right here. I'm asked to figure out the length of BE, right? So I'm going to call BE X, because right now it's unknown.

Module 8: Quadrilaterals

Topic 4 Content: Applying the Properties of a Square Transcript

Let's start thinking about the properties of a square, and start using some of those properties to get to this answer here. Now when you're working problems like this, you may not know exactly where to start, and that's okay. Just start with whatever you know, and eventually you'll start to see that the pieces will come together, you'll be able to get to what you're being asked to solve. Where I'm going to start here, is because I was given the length of one of the sides of the square, well I do know that a square has four congruent sides, right? So I'll just start there. I know DC is eight, so that means C is eight, so is AE, so is DA. Now what I'm asked to find here is the length of part of one of my diagonals in the square, so I'll start thinking about the properties of a diagonal and see what I run into to help me solve this problem.

Now, I know that because a square is a parallelogram, its diagonals bisect each other. I'm going to go ahead and put my congruent marks in here, and then before I do that, on top of that, what I also know is that because a square is a rectangle, its diagonals are also congruent. Because its diagonals are congruent, because it's a rectangle and its diagonals bisect each other, because it's a parallelogram, I'm going to go ahead and put my congruent marks in here to show that each of these individual segments are congruent. I'm just going to slash each of them one time, show that they're all equal.

Now, another thing I know since I'm working in the diagonals now, stay along those lines, is I know that because a square is a rhombus, its diagonals are perpendicular. That means they intersect to form right angles. What I'm noticing here is that I have a triangle right here that I can kind of zoom in on, and this is one of the sides of my triangle. I know that this is a right angle here because my diagonals are perpendicular. What I'm going to do is I'm going to focus my attention on this right triangle and I'm going to use it to solve for X. I'm going to pull it out of the square so we can really focus in on it when we start solving, and then we'll write our measure back in at the end. Let's group that together, slide it over. Okay. Get the pen back.

What I'm going to do is I'm going to pull this triangle out of that square. Let me just make that slash a little longer so we know that's another congruent side. I'm going to try to orient it the same way, just so you can really get the visual, and we'll have our eight, congruent mark, congruent mark, and our X. I'll put it on the outside this time.

Let's look at this closely here. What we have here is a right triangle. We know that, right? But also its legs are congruent. We have an isosceles right triangle. What that means is it's a special right triangle. Remember those? It's a 45, 45, 90 special right triangle. We can use one of those special right triangle formulas to help us solve for X here. For the 45, 45, 90, the formula was the hypotenuse equals the leg times the square root of two. That's what I'm going to use here to solve for X, and I'm going to just slide this triangle over just a bit. We're starting to get just a little crowded there. Let's get a little more workspace.

I'm going to label the sides of my triangle hypotenuse and leg, just to help me fill into my formula properly. I know across from my right angle, that's my hypotenuse. These are my legs here, and I'm focused on this one. I know those lengths are the same, so when I'm substituting into my formula here, I know that the hypotenuse equals the length of the leg times the square root of two. Let's see. Get the black ink back. Eight equals X times the square root of two. Let's go ahead and solve this equation for X, so we'll divide both sides by the square root of two. That's going to cancel out there. Need a little more space. I'm going to flip this around. I'm going to write, "X equals eight divided by the square root of two."

Remember when we learned that it's improper to leave an irrational number in the denominator? We're going to go ahead and have to rationalize this denominator. Remember how we did that is we multiply the numerator and the denominator by that square root of two, in this case. Let's multiply straight across, the black ink back. Eight times the square root of two is eight square roots of two. The square root of two times the square root of two. Let's do that off to the side. The square root of two times the square root of two, that's the square root of four, which is just two. In our denominator, we just have two. Now what we can do is divide just these whole number parts right here. Eight divided by two, that's four, so our final answer is four square roots of two. That's what X equals in this case.

Going back to that square, I'm going to scroll up a bit. What that means is the length of BE right here, the length of part of that diagonal, it's equal to four square roots of two, all right? I know that problem pulled in a lot of things for us. It pulled in some things we had learned previously about triangles, and then the properties of the square. What I want you to take away is to start with your given information. Just start using what you know about a square to get some measures in your figure, see what you've got to work with, and you may actually have to pull back to some old geometry you've learned before, okay? The final answer for this one, BE is four square roots of two.

Now take a look at the next one. I want you to give this one a try. Go ahead and press pause, take a few minutes, work your way through this problem. Press play when you're ready to check your answer.

All right. Let's see how you did with this one. Here you were told that the outline of a local community garden is modeled by the square below. If the perimeter of the square is 40 centimeters, find AB. Okay. What we have going on here is we're not actually told the length of any part of the square, but we are given a measure to help us determine the length of a part of the square. Notice in our problem, let me get my pen back here, that we're told the perimeter of the square is 40 centimeters. Remember, perimeter is just adding up the lengths of your edges in your figure. Here, if I know that the perimeter of the square is 40 centimeters, I know that the lengths of each of these sides when I add them together is 40.

Now, because I know that in a square I have four congruent sides, I can use this perimeter to figure out the length of each of these sides. That 40 centimeters has to be split up equally four ways, right? So 40 divided by four, that's 10. What that tells me is the length of each side in the square is 10 centimeters. I'm going to keep going here. I'm asked to find AB, so that's the segment right here. I'm just going to put an X there. See, now this is actually looking familiar. It's looking similar to the problem we just did. What I see here, I'm working with the diagonals again inside the square, so I know that in the square, the diagonals are congruent and they bisect each other, so that means that each of these parts are equal, and I also know that my diagonals are perpendicular. That means I have four right angles in the center.

This time what I'm going to do is I'm going to focus on this right triangle and use that to help me solve for X. I'm going to pull that triangle out of the square, let's get the pen back. All right, I'm going to try to orient it the same way that it's sitting in that square. We've got out 10, congruent marks, and our X. Again, I have an isosceles right triangle, so that means I have a 45, 45, 90, special right triangle. I'm going to use that same right triangle formula to solve for X here. Get a little more work space. Switch to blue ink for a second. Let's get that out of our way.

I know that the formula I need to use, here we go, is that the hypotenuse equals my length of my leg times the square root of two. In this right triangle, let's just bring it down a bit. It's starting to get a little crowded up here. Let's just focus on that triangle and that formula. I'm going to go ahead and label the hypotenuse and the leg in that triangle, so I know across from my right angle that's my hypotenuse.

These are my legs. I'm going to go ahead and substitute into my formula so the hypotenuse is 10, the leg is X, times the square root of two. Now to solve for X, divide both sides by the square root of two. Get a little more work space.

I'm going to flip this around. Going to write it as X equals 10 divided by the square root of two. Again, I need to rationalize this denominator, so let's multiply both the numerator and denominator by the square root of two. 10 times the square root of two, just 10 square roots of two. The square root of two times the square root of two is the square root of four, which is just two. Now what I'm going to do to simplify is just divide right here. 10 divided by two, which is five. My final answer is five square roots of two, okay? Do you see we kind of followed a similar process like we did in that previous example? Let's just take it back to the square here, remembering what we were initially asked to solve. We were asked to find AB, so AB equals five square roots of two, and you're all done with that one. Good job there.

All right guys, you've reached the conclusion of this topic on how to apply the properties of a square. I hope you saw how your knowledge of parallelograms, and rectangles, and rhombi, and even triangles all came in handy for you during this lesson. Bye.