Module 8: Quadrilaterals

Topic 5 Content: Applying Properties of Trapezoids and Isosceles Trapezoids Transcript

Hi, guys. Welcome to Geometry. In this topic, we're going to focus on how to apply the properties of trapezoids and isosceles trapezoids. Now, your knowledge of quadrilaterals is going to come in handy during this topic. You ready to get started? Let's go. Okay, so let's just review a little bit about the quadrilaterals that we've covered so far. Let's reveal these here, get these out of our way. We've discussed a parallelogram, right, and all the properties that a parallelogram has. We even spent some time setting up that Venn Diagram and seeing the relationship between difference types of parallelograms, right, one of which includes the rectangle, the rhombus, the square, and now what we have here are two additional quadrilaterals.

Now, what's different about these quadrilaterals is that they actually are not parallelograms, okay? One of them is the trapezoid, and the other the isosceles trapezoid. Let me show you that Venn diagram so you know what I mean about when I say that the trapezoid and the isosceles trapezoid are quadrilaterals, but they're not parallelograms, okay?

Let's take a look at this here. We discussed previously parallelograms. Those are a special case of four sided figures, right, quadrilaterals, and within our group of parallelograms, we have the rectangle, the rhombus, and the square. Now, in addition to the parallelogram, there's also another type of four sided figure, the trapezoid. Then, within our group of trapezoids, we have a special case, or special set, known as isosceles trapezoids, okay?

Here, trapezoids have their own set of properties that are not the same as the properties that parallelograms have, okay? A trapezoid is a quadrilateral because it is a four sided figure, but it's not a parallelogram. What I mean by that is that those properties of a parallelogram that were inherited by the rectangle, the rhombus, and the square are not inherited by the trapezoid. It's a four sided figure, but it's not a parallelogram, okay? Let's explore its properties a little bit. The trapezoid, the definition of a trapezoid is that it's a quadrilateral with only one pair of parallel sides.

If you look here, those parallel sides are known as the basis, so how a parallelogram had two pairs of parallel sides, a trapezoid only has one pair of parallel sides. Those parallel sides are known as the bases. The non-parallel sides are known as the legs, and another part of a trapezoid are the base angle pairs. Those lie right here on top. These are a pair of base angles, and right here on the bottom. These are another pair of base angles. Now, they're not always on the top and on the bottom depending on how your trapezoid is oriented, but the base angle pairs will always like along the same base.

These are a pair of base angles, and these are a pair of base angles, okay? Now, in addition to the trapezoid, we also have the isosceles trapezoid. Now, let me just reveal this so you can see exactly what it is. It is a trapezoid with congruent legs, okay? See here we still have the bases, that pair of parallel sides, but in an isosceles trapezoid, the legs are congruent. Now, another property of the isosceles trapezoid, lets reveal this here, is that it's base angle pairs are congruent, so what that means, let me get my pen, is my pair of base angles right here along this base, these angles are congruent, and my pair of base angles along this base, these angles are also congruent, okay?

That's one of the properties of the isosceles trapezoid is that its base angles are congruent. Another property of the isosceles trapezoid is that its diagonals are also congruent. If you have an isosceles trapezoid, you'll know that the diagonals of it will be congruent. Let's start applying some of these properties in the next couple of examples. Here, we have trapezoid GRMV. We're going to find the measure of angle M and the measure of angle R.

Okay, so let's see how we're going to dive into this one. Let me move a few things around here. I want to just focus on this trapezoid for a second. Okay. What we're going to use to figure out the measures of those two angles is the fact that we know that the bases are parallel. What we can do, because we know that, going to pull something out of here for you. I'm going to highlight this. I'm going to highlight this, and I'm going to highlight this. Do you recognize that set up?

We've dealt with it several times before. Going to pull it out here, see if it looks familiar, this situation right here. What we have here is a pair of parallel lines intersected by a transversal. Because that's what's going on in this trapezoid, we can recall those angle pairs that we learned that were associated with this figure to handle this trapezoid, okay? Take a look at angle M in relationship to angle V. I see they're in the interior of those parallel lines, and they're on the same side of the transversal.

That means they're consecutive interior angles, or you may call them same side interior angles, but I know the relationship to each other is that they're supplementary. That means that their measures are going to have to add up to equal 180. If angle V has a measure of 50 degrees, then 180 minus 50 is 130. Angle M, 130 degrees, okay? The same relationship is happening right up here. You have those same pairs of parallel lines, that same pair. Then, this is your transversal this time. Again, you have consecutive interior angles or same side interior angles. These are supplementary to one another.

180 minus 120 is 60, so the measure of angle R, 60 degrees, and you're all done with that one, okay? If you want to write your final answer out, I could say that ... Turn this around here, the measure of angle R is 60 degrees, and the measure of angle M, 130 degrees. You're all done there, okay?

See how we pulled back to what we knew about the angle relationships when a transversal intersects parallel lines to help us solve for the angle relationships in this trapezoid, all right? Keep that in mind as we work through the next few ones here. Here, we have trapezoid ABCD, and we're asked to find the measures of angle B, angle C, and angle D. Now, even more specific than this being a trapezoid, I notice that its legs are congruent, so that means it's an isosceles trapezoid. What I know about isosceles trapezoids is that their base angles are congruent, so what that means, switch colors here.

I'll leave it black. That means that angle A is going to be congruent to angle D, and angle B is going to be congruent to angle C. Immediately, I already know the measure of angle D, because I know it's going to have to be the same as the measure of angle A. I'll write that in blue. 80 degrees. Now, as far as angle B and angle C are concerned, I can pull back to what I know about those consecutive interior angles, so let me slide this trapezoid over just a little bit. Scoot it over right here. Get that highlighter out. Okay. Here's my base. Here's my base. Those are my parallel sides, and here's my transversal. Here I have a pair of same side interior angles, or consecutive interior angles, so I know they're going to have to be supplementary.

If angle A is 80 degrees, then 180 minus 80, 100 degrees. Now, I know the measure of angle B, 100 degrees. I know that angle B and angle C are going to have the same measure, so angle C also 100 degrees. If you wanted to write that answer out formally, you could say the measure of angle B equals the measure of angle C, which is 100 degrees, and that the measure of angle D, 80 degrees. All right? Okay, good job on that one. Let's take a look at this next one.

It's your turn. Go ahead and press pause, take a few minutes, work your way through this example, press play when you're ready to check your answer.

All right. Let's see how you did on this one. Here you were told that the floor plan for the living room of a newly constructed home is modeled by the trapezoid below. Find the measure of angle L and the measure of angle S. Okay. Here, I notice that the legs are not marked congruent in this trapezoid, so it's not an isosceles trapezoid. It's just a trapezoid. What that means is in this case my base angle pairs are not going to be congruent. I'm going to go ahead and just use what I know about consecutive interior angles to figure out the measures of angle L and the measure of angle S.

Here are my bases. Parallel sides. Here's the first transversal I'll focus on. If angle T is 110 degrees, 180 minus ... Put the equal sign there, let's fix that. 180 minus 110, 70, so that means the measure of angle L, 70 degrees. Then, for angle S, same pair of parallel sides. This is the transversal I'll focus on, so here are my consecutive or same side interior angles, so 180 minus 160 is 20. Angle S, 20 degrees. Write it out formally, the measure of angle L, 70 degrees. The measure of angle S, 20 degrees. You're all done with that one. All right? Good job on that one.

Take a look at the next one. Here we have an isosceles trapezoid named EFGH. We're told that EG is 9x- 11 and FH is 4x+4, and we're asked to find x. Okay, so if I notice here, EG and FH are my diagonals, right, so once I see that, then I'll think back to the property of an isosceles trapezoid related to the diagonals which is that the diagonals are congruent. Since I know that, I can set up an equation where I set these expressions equal to each other and solve for x. Let's get that work going here.

Scroll a little bit. Get some space. We're going to have to scroll up and down for a second. 9x-11 equals 4x+4. I'm able to do this because I know the diagonals are congruent so I know that EG equals FH, okay? All right, so let's solve this equation here, get some more work space. Subtract 4x from each side. That's wiped out, so 5x-11 equals 4. Add 11 to each side there, some more space. We have 5x equals 15 and divide by 5. X equals 3. You're all done with that one, okay?

You see how we applied the property that the diagonals were congruent, we used that to set up our equation where we set those expressions equal. We solved for x, did all our algebraic work here, and we found out that x equals 3. All right, example four. Given the trapezoid below, find x. Now, this one's a little different, this problem, because it involved something called a mid-segment. I'm going to write that right down here. Now, the mid-segment is a segment that connects the midpoints of the legs. What I mean by that ... Here's our mid-segment here given by this length 4x+2. This end point is the midpoint of this leg, and this end point is the midpoint of this leg, okay?

Now, another property of the mid-segment is that it's parallel to the bases and you'll notice that by our parallel marks here. Another property of the mid-segment, and this is the one that's going to let us solve for x, is that the length of the mid-segment is equal to half the sum of the lengths of the bases. Okay? I'm going to say that again. The length of the mid-segment is equal to half the sum of the lengths of the bases, okay?

We're going to use that fact to set up an equation and solve for x here. All right. Let's get going with that. Make sure I've got my pen. Okay, so here's our mid-segment, so that length, 4x+2, equals one half x+3 plus 3x+5. Okay, so here's our mid-segment, and it's equal to half of the sum of the lengths of the bases. Now, notice the equation may look a little intimidating right now because it has a lot of terms, but what we're going to do to simplify it is we're going to begin by just handling this right side.

Let's just start out by simplifying this right side, and we'll get something a little more manageable to work with. Then, we'll go ahead and we'll solve for x and get to the bottom of this one, okay? I'm going to get a little more work space and come off to the side, do a little scratch work. Let's just get that right side down, that one half of x+3 plus 3x+5. All right, so let's simplify that sum first. X+3x, that's 4x, so one half of 4x plus 3+5, that's 8. All right, so we have one half of the quantity 4x+8, so let's go ahead and distribute that one half. One half times 4x, or you could think about it as one half of 4x.

That's 2x. One half times 8. We could think about that as one half of 8. That's 4, so the right side of that equation broke right down to 2x+4. Let's take that, and let's drop it back into our original equation.

Okay. Scrolling a little too far there. All right, so we'll have 4x+2 equals 2x+4. Let's just go ahead and get rid of the scratch work, because we got what we needed out of that. We don't want it to start to crowd our work solving this equation. Back up here, we've gotten that right side simplified, so let's solve 4x+2 equals 2x+4. Subtract 2x from each side. Scroll down here.

We have 2x+2 equals 4. Subtract 2 from each side. Get a little more work space, so we have 2x equals 2. Last step divide each side by 2, and we're left with x equals 1, and we're all done with that one, okay?

Let me just bring it back up to the top just to review what we did. We realized in this problem that we had a mid-segment, right? We learned that the length of the mid-segment is equal to half the sum of the lengths of the bases. We used that fact to set up an equation. Came off to the side and we did a little scratch work just to simplify that right side. Then, we went ahead. We solved for x. We landed at x equals 1. All right. Okay, good job on that one. Now take a few minutes and go ahead and try this one. Press pause, take some time, work your way through this one. Press play when you're ready to check your answers.

All right, let's see how you did on this one. Here, you had an isosceles trapezoid, and you were told that ZW is 4x+13. YV is 11x-1, and you were asked to find ZW. All right. In this problem, you'll notice you're dealing with the diagonals of an isosceles trapezoid. What you know about those is that they're congruent, right? That means that I know that ZW equals ... Let's write the other name there. Copying that work down. Getting ahead of ourselves. ZW equals YV, because I know the diagonals of an isosceles trapezoid are congruent.

I'll use that fact to set up an equation, so 4x+13 equals 11x-1, and after I've solved for x, I'll be able to figure out the length of ZW. Let's subtract 11x from each side. Got that canceled out. Let's get some more work space here. We have 4x-11x, so that's negative 7x+13 equals negative 1. Subtract 13 from each side, a little more work space. Negative 7x equals negative 14. Last step here, divide each side by negative 7, so x equals 2. Now, don't be tempted to stop there, because now that we have x, we need to substitute it into the expression that represents the length of ZW so we can find out exactly how long that diagonal is, okay?

ZW is 4x+13. Let's get a little work space here. Just going to move some things out of our way a bit. I can squeeze it right in there. I'll just change colors. Okay, so now that we know that x is 2, we're going to go ahead and substitute 2 right in here and figure out the length of ZW. Let's come right off to the side and put that in black ink now. ZW, and we can get it right underneath here. ZW equals 4 times 2 plus 13. 4 times 2 is 8, so we have 8+13, and 8+13 is 21. Now we know the length of ZW is 21. We're all done with that one, all right?

Just review just to make sure we can pull it all together. Scroll back up to the top here. Okay ,so we knew the diagonals of our isosceles trapezoid were congruent. We used that, we set up an equation, we solved for x. We figured out that x equals 2. Now that we knew that, we could go ahead and we could plug it in to ZW, right, so we used that value of x being 2. We substituted that in there, and we ended up that ZW equaled 21, okay?

All right, good job there. All right, guys, you've reached the conclusion of this topic on isosceles trapezoids and trapezoids in general. I hope you saw how your knowledge of quadrilaterals and different angle relationships came in handy during this topic. Bye.