Module 8: Quadrilaterals

Topic 6 Content: Proving the Properties of Quadrilaterals Transcript

Hi, guys. Welcome to Geometry. In this topic, we're going to focus on how to prove the properties of quadrilaterals, so your knowledge of parallelograms, and rectangles, and rhombi, and squares, and trapezoids, and isosceles trapezoids is all going to come in handy for you during this topic. You ready to get started? Let's go. Okay, so let's take a look at this Venn diagram and just review the relationships between different quadrilaterals. We know that a quadrilateral is a four-sided figure, and a special case of the quadrilateral is the parallelogram. Within our group of parallelograms, we have rectangles, and rhombi, and squares. The rectangles, and the rhombi, and the squares all inherit the properties of the parallelogram because they do belong to that group, or that subset. In addition to parallelograms, we also have trapezoids. Trapezoids are quadrilaterals. They have four sided. They're a four-sided figure, but a trapezoid is not a parallelogram, so in the way that rectangles, and rhombi, and squares inherited properties of the parallelogram, that isn't the case with the trapezoid.

It doesn't inherit those parallelogram properties because it's not a parallelogram. It's just a different type of four-sided figure. With a group of trapezoids, we do have isosceles trapezoids, right? That's a special case of the trapezoid, and there are some other quadrilaterals, but these are the ones that we're going to focus on here, and I want you to keep this in mind, and all of those properties related to the sides, and the angles, and the diagonals. Keep all that in mind as we work our way through these following examples. Okay, so let's start with this two-column proof, okay? Here we go. Given parallelogram HJKL, prove that HJ is congruent to LK, and HL is congruent to JK. All right, so here we are told that this quadrilateral is a parallelogram, and we're trying to prove that HJ's congruent to LK, so these pair, and that LK is congruent to JK, so this pair. Really what we're trying to prove here is that the opposite sides of a parallelogram are parallel. I'm sorry, are congruent. Okay?

Let's start here, so HJKL is a parallelogram. I know that because it was given information. Remember, most of your two-column proofs are going to start with the given statement, okay? The reason that I know that is because it was given, and then I'm also told that HJ is parallel to LK. The reason that I know that HJ is parallel to LK is because I was told that HJKL is a parallelogram, so I can use the fact that that's part of what we'll call the definition, I'm going to abbreviate that, of a parallelogram. I know that the opposite sides are parallel. I know at this point, you may be thinking, "Well, then couldn't we just say that what we're trying to prove is true, because it's part of the definition of a parallelogram?" You could. It is true, but what we're trying to do here is use different facts and different justifications to prove that that property of the parallelogram is true, that its opposite sides are congruent, okay? Let's keep working our way through this, all right?

What I'm also going to do is I'm going to mark that HJ is parallel to LK. I'm also told here angle five is congruent to angle four, and angle three is congruent to angle six. I'm going to mark that, and then I want to see if you notice the relationship here, where it came from. Angle three is congruent to angle six, and angle five's congruent to angle four. Okay. Do you notice something here? I know this keeps popping up. We covered it way back near the beginning of the course, but it pops up all the way through it. That figure again. You have parallel lines intersected by a transversal. Let me draw it again right underneath. I have HJLK, which are parallel, and the transversal. Here's angle three and angle six. Kind of free handing it, so it doesn't match exactly, but I want you to just kind of zoom in on that relationship there, and then angle five and angle four.

You notice what we have here. We're in the interior of our parallel lines, and we're on alternating sides, or different sides of the transversal. Those angle pairs are alternate interior angles, and from what I know about parallel lines being intersected by a transversal is that the alternate interior angles are congruent. That's the relationship that justifies, or that's the statement that justifies this relationship. The reason these angle pairs are congruent is because I have parallel segments in this case, but parallel lines intersected by a transversal, so I know the alternate interior angles are congruent. I know that's a lot to write, so I'm going to abbreviate it, and I'm going to say, parallel lines. I'm going to write down here, parallel lines intersected by transversal and we'll say, alternate. We'll right that down. Interior, let's abbreviate angles. Congruent, okay?

When we look at that, we know this is what we're thinking, that here we had two pairs of alternate interior angles that were congruent. I know this is probably starting to get a little crowded for us, so let's just ... Now that we know that, we have that in mind, just going to erase these extra marks in the figure here. Let's get rid of that. Okay, and we'll leave that extra work at the bottom, just to jog our memory if we need it again, but we know ... Let me scoot it down a little bit. We know where that relationship came from. Okay, let me add those marks back in, though. I still need those, that these are parallel and that these angles are congruent. Let's get those back up there. Okay. That's our two-column proof. We're also told here, HK is congruent to KH, so HK congruent to KH. That's a side congruent to itself, right, and we know the property that always justifies that is the reflexive property, okay?

Now our next line in our proof here, triangle LHK is congruent to triangle JKH. Okay, so now we've jumped back to congruent triangles, so remember those five ways that you know of how to prove triangles congruent? Side side side, side angle side, angle side angle, angle angle side, and hypotenuse leg. Okay? I'm actually going to pause for a minute. I want you to think about that for a second, those five ways to prove triangles congruent. I want you to take a look at that parallelogram, and I want you to see if you can figure out which one of those five ways is it that proves that these two triangles are congruent, okay? Take a minute. Work your way through that, just that one line of that proof. Press play again when you're ready to check your answer. Okay. What'd you come up with? Was it angle side angle? That's what we needed here, because in each of our triangles, we have two pairs of corresponding angles congruent, and one pair of included sides congruent.

In this case we have the triangles congruent by angle side angle, okay? The last line of our proof. Let's look here. HJ is congruent to LK, and HL is congruent to JK. What happened here is now that I know that these triangles are congruent, now I can just refer to these parts of my congruent triangles. Remember that we could always take that from the statement, so I'll just use the statement right above it. I know that HJ is congruent to LK, and I know that HL is congruent is congruent to JK, because I know that the corresponding parts of my congruent triangles are congruent, right? The parts that match up, or the parts that correspond to each other in congruent triangles, have the same measure. Here, HJ is congruent to LK, and HL is congruent to JK, because the corresponding parts of congruent triangles are congruent. That statement is often abbreviated as CPCTC. The corresponding parts of congruent triangles are congruent, okay?

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Good job working through that proof. I know we've pulled in a lot of things that we've learned over the course here, but you worked your way through it. You persevered. We got it done. All right. Let's look at this next one here. What value of X will ensure that the quadrilateral is a rhombus? Okay, so let's think about what we know has to be true if this quadrilateral is a rhombus. What we're given here is we're given an angle measure in the form of an expression right at the intersection of our diagonals. Let's think about what we know about what's true about the diagonals of a rhombus. We know that where the diagonals intersect, we have four right angles, right? The diagonals of a rhombus are perpendicular, so if this quadrilateral is a rhombus ... Make sure I got my pen. Oh, I do. That means that this angle measure is going to have to equal 90 degrees. I need to figure out what value of X will guarantee that that angle measure is 90 degrees, okay?

Let's set up an equation. Let's get that part done. Okay, so 8X plus 10 equals 90. Get a little more workspace here. Subtract 10 from each side. 8X equals 80. Divide by X equals 10, so 10 is the value for X that would guarantee that this quadrilateral is a rhombus, because what would happen is if X equaled 10, then this expression would have a value of 90, so that angle measure right there would be 90 degrees. Okay? Okay, good job with those algebraic methods right here. Now let's get on the coordinate plane. All right, now let's go ahead and take a look at example three. Here you're asked to prove that the diagonals of the square are perpendicular. All right. When you see perpendicular, you can think a few different things, right? Perpendicular, we know that if we see that term, it tells us that we have two lines or two segments intersecting to form a right angle, right?

If you're on the coordinate plane and you see perpendicular, think slope, okay? Because you know that the slopes of perpendicular lines are opposite reciprocals, or you could also say that their product is negative one. In this problem, where you're asked to prove that these diagonals are perpendicular, we want to show that their slopes are opposite reciprocals or that the product of their slopes is negative one. You can do either one, whichever one you find easier to do, okay? Let's go ahead and get started on that. Let's begin by getting the coordinates of the diagonals. I'm going to work with diagonal AC and then I'm going to leave you to work with diagonal BD, okay? I'm going to go ahead and get the coordinates of A and C. I'm going to go ahead and get my pen here. Okay, so A is at -2, 3, and C is at 1, 2, 3, -2.

I know I'm working with the slope formula. I'm going to go ahead and write it right here off to the side. We know we represent it by M. it equals Y sub 2 minus Y sub 1 over X sub 2 minus X sub 1. I'm going to go ahead and label my coordinates, and then substitute into that slope formula, and figure out the slope of AC. All right, so X sub 1, Y sub 1, X sub 2, Y sub 2. Get a little workspace down here. We're going to have to scroll up and down a bit on this one, so I'm going to get the slope of AC, that diagonal. We need first Y sub 2 minus Y sub 1, so that is -2 minus 3, so -2 minus 3. Over X sub 2 minus X sub 1, so that is 3 minus -2, so we're going to scroll back down here. 3 minus -2. The first thing jumping out at me is that double negative. I'm just going to go ahead and clean that up. -2 minus 3. That's -5. 2 plus 2. That's 5, and -5 divided by 5 is -1. Right now, I know that the slope of AC is -1.

What I want you to do. I'm going to scroll back up here. I'm going to erase the coordinates I have down so far, so we don't get too crowded. Okay, so now what I want you to do is I want you to figure out the slope of BD, okay? I want you to get that slope, and then I want you to compare it to the slope of AC and see if we do indeed have perpendicular segments, okay? Go ahead and press pause, take a few minutes, work through that. Press play when you're ready to check your work. All right. Let's see how you did.

Okay, so let's go ahead and get the coordinates for B, 1, 2, 3, 1, 2, 3. Let's get the pen, so 3, 3, and D, -2, -2. Okay? I'll label this one X1Y1, X2Y2, get some workspace down here. We're going to be scrolling. Okay, so now we're getting the slope of BD and there we go. Okay, so first thing we need is Y2 minus Y1, so that is 3 minus -2. Scroll back down.

Let me just scoot this down a little bit so we don't start to run into the work we already. Have there we go. Just get a little more space. Okay? That was 3 minus -2, over now we need X2 minus X1, so that is again 3 minus -2. Scroll back down. Okay, so 3 minus -2. We have two sets of double negatives here to clean up. Okay, so 3 plus 2. That's 5, we have 5 divided by 5, which is 1, okay? Now let's compare our slopes. I think that what you can see probably quickest on this one is that the product of these slopes is -1. -1 times 1 is negative one. You could also discuss that in the way that these slopes are opposite reciprocals, because that is also true, but I'm just going to go ahead and run with the fact that the product of these slopes is -1, okay? All right, now because we know that, what that proves to us is that these diagonals are indeed perpendicular, because the product of the slopes is -1.

Let's go ahead and write that out formally, just so you can see that, because a lot of times in geometry, and I may have said this before, your teacher will want to see the work. They want to see all the math that you did, all the calculations, but they also want to see a statement where you've pulled it all together, and analyzed your work, and you know what it really represents, what it means, what it stands for. Okay? All right, so let's just get a little statement written down here. It doesn't have to be perfect, just a statement to show you know what this means. Okay, so our diagonals here were AC and BD. Okay, so we can say the product of the slopes of AC and BD equals -1. Therefore, AC is perpendicular to BD. All right, and you're all done with that one. Good job on that one.

Okay, so let's take a look at this one. Here we're asked to prove that the diagonals of the parallelogram bisect each other. In this case, when you want to prove that your diagonals bisect each other, you want to show that the midpoints of your diagonals is the same. We haven't used the midpoint formula in a little while, so let me write it down here. Our midpoint formula, midpoint formula is X1 plus X2 divided by 2. It's not actually an equal sign. Let me fix that. Our answer is an ordered pair. So used to writing equations. Y1 plus Y2 divided by 2. We'll use this formula to determine the midpoint of a segment. In this case, because we're trying to determine that if our diagonals bisect each other, we're trying to prove that, we want to show that PR has the same midpoint as QS, okay? Let's just draw those diagonals there. You want to show that PR has the same midpoint as QS.

I'm going to show PR. I'm going to show you how to get the midpoint of this segment. Then I'm going to leave you to get the midpoint for this one, okay? All right, so let me get the coordinates for P. P is at 1, 2, 3, 4, 5, 6, so -6, 1, 2, 3, 4, so -6, 4, and R is at 0, 0. Okay, so let's come down here, and let's get the midpoint of PR, so midpoint. I'm going to abbreviate that, of PR. That segment. Okay, so X1 plus X2, so let's go back up here. X1 label, X1Y2, X2Y2. X1 plus X2, -6 plus 0. Okay, so -6, and that slid on me a little bit. That's let get that up a tad. Okay, so -6 plus 0, divided 2, and then Y1 plus Y2, so let's get that. That's 4 plus 0. Scroll back down, so 4 plus 0 divided by 2. Okay, so -6 plus 0, that's just -6, so here I just have -6 divided by 2. 4 plus 0, that's just 4, so 4 divided by 2. Okay, -6 divided by 2 is -3. 4 divided by 2 is 2, so the midpoint of PR is -3, 2.

Now what I want you to determine is I want you to get the midpoint of our other diagonal, which in this case is SQ. If SQ has the same midpoint as PR, then that means our diagonals did indeed bisect each other. Okay, so I'm going to write the midpoint formula right up here, just so you have it while you're looking at the coordinate plane, so X1 plus X2 divided by 2, Y1 plus Y2 divided by 2, and just so things don't get too crowded for you, let me erase my old coordinates and I'll draw diagonals back in there for you. Okay, so again, I want you to determine the midpoint off SQ, and if it's the same as PR, then we know our diagonals did bisect each other. Take some time, press pause, work your way through that.

Press play when you're ready to check your work. All right. Let's see how you did on this one. Let's go ahead and get the coordinates of S and Q.

S is at -1, 2, 3, 4, 5, 0, and Q is at -1, 1, 2, 3, 4. Okay, so I'll label this X1Y1, and this one, getting crowded, X2Y2. Let's come down here and do some work. I know the first thing that I need is -5 plus -1, so just go ahead and keep that in mind. All right. Let's get some workspace here, so we're getting the midpoint of SQ, so -5 plus -1 divided by 2. Let's get the other ones there. Okay, and we have 0 plus 4. Scrolling back down, so 0 plus 4 divided by 2. Okay, so -5 plus -1, so that's -6 divided by 2, and 0 plus 4. That's 4 divided by 2. Okay, keep going here, so -6 divided by 2, that's -3, and 4 divided by 2 is 2, so let's look at what we have here. The midpoint of SQ, of that segment, is -3, 2, and then back up to our other work, the midpoint of PR is also -3, 2. The midpoints of those diagonals are the same, so what that means for our parallelogram is that our diagonals do bisect each other because they share the same midpoint, in this case -3, 2.

Let's go down to the bottom and just write that out formally, get some empty space up here. Okay, so let's, all right, get the black ink here. Okay, so let's say, let me get the names of our diagonals here. SQ and PR. PR and SQ bisect each other because they share the same midpoint. That's just one way you could explain it, or you could say because PR and SQ share the same midpoint, they bisect each other. Any way that you could explain that you understand that if those diagonals share the same midpoint, that they bisect each other. All right? Okay, now take a look at the next one here. Going to have you do this one all on our own. Here you're asked to prove that the diagonals of this isosceles trapezoid are congruent. Before you start on this one, I want you to think about what formula do you need to use in order to determine lengths on a coordinate plane?

It's not the slope formula. It's not the midpoint formula, but it's the distance formula. Just in case you don't have that formula handy or you may not have memorized it yet, I'm going to write it for you right here, right underneath this trapezoid. Hopefully you can see them both at the same time. Okay, so your distance formula, the square root of X sub 2 minus X sub 1, that quantity squared, plus Y sub 2 minus Y sub 1, that quantity squared, okay? Go ahead and press pause, take a few minutes, work your way through what you need to do to prove the diagonals of this isosceles trapezoid are congruent. Press play when you're ready to check your work. All right? Let's see how you did here, so I need to first determine the coordinates of the end points of my diagonals, so let me just scoot the distance formula down a little bit so I can write the coordinates up there.

All righty. Okay, so Q is at -1, 2, 1, 2, 3, 4, so -2, 4. V is at 1, 2, 1, 2, 3, 4, so 2, 4. W's at 1, 0, and Z's at -1, 0. Okay, so I need to get the lengths of the diagonals, so I'm going to get QW, then I'll get ZV. Okay, so let's focus first on Q and W, so I'll label that X1Y1, and X2Y2. Let's start substituting into our distance formula. Going to have to scroll up and down for a little bit to get everything plugged in. All right, so this is going to be for QW. Let's see, we can start there. All righty. Maybe we can just scoot that up just a tad. Okay. Get our pen back here. Here we go, so X sub 2 minus X sub 1 for QW, that's going to be 1 minus -2, so 1 minus -2 squared. Y sub 2 minus Y sub 1 is going to be 0 minus 4 squared, so 0 minus 4 squared, and what's jumping out at me is I have a double negative, so let's clean that up and keep going.

1 plus 2 is 3, so this is 3 squared, and 0 minus 4 is -4, so that's -4 squared. Okay. 3 squared is 9. -4 squared is 16. 9 plus 16 is 25, and the square root of 25 is 5. It's a perfect square, okay? I know the length of QW is 5. If I can prove that the length of, what's my other diagonal here, of ZV is also 5, then I would've shown that my diagonals are congruent. Okay, so I'm going to erase this pair, just so I know I've already handled that one, and now I'm going to get the length of ZV. Now I'll label this X1Y1, X2Y2. Just to get us some room here so we don't have to scroll too far, I'm going to go ahead and write down that I know that QW is 5, okay? Let's erase this work here, and we'll put our new work right in the same space. All right. Almost got all of that gone. There we go.

Okay, so let's get the length of our other diagonal, ZV. Okay, so ZV. X2 minus X1, let's see what's that going to be. That's going to be 2 minus -1, so 2 minus -1 squared. Y2 minus Y1, that's going to be 4 minus 0. Okay, so 4 minus 0, that quantity squared. Again, I notice there's double negatives. Let's clean that up, keep simplifying. Okay, so 2 plus 1, that's 3, so 3 squared. 4 minus 0, that's 4 squared. Okay, so 3 squared, that's 9. 4 squared is 16. Again, I have 9 plus 16, which is 25, and the square root of 25 is 5. I've shown again with my distance formula that the length of the diagonal, in this case ZV, is 5. I'm going to go ahead and write that at the top, that ZV also equals 5. Here we've shown it. We've used our distance formula to show that the length of QW is 5 and the length of ZV is 5. Our diagonals are congruent.

To write that out formally, what we could say ... Let's get some empty space here, scrolling one more time. Okay, and so let's get the names of our diagonals one more time, QW and ZV. QW equals 5, and ZV equals 5, so therefore, the diagonals ... I'll have to run that together there. The diagonals, switch colors, get that back to black. There we go. The diagonals, just to keep it consistent, are congruent. Okay, so now you've written it out formally, so a lot of times when you're working your geometry problems, you'll have the work, but your teacher will want to see a statement where you've pulled it all together so that they will know that you're able to interpret that work, and you know what it means. You know that because QW and ZV have the same length, that your diagonals are congruent, okay?

All right. Good job on that. I know that one took a lot of algebraic skills, and some coordinate skills as well. All right, guys. You've reached the conclusion of this topic on how to prove properties of quadrilaterals. I hope you saw how your knowledge of direct proofs and algebraic and coordinate methods came in handy for you doing this topic. Bye.