Module 9: Polygons
Topic 1 Content: Interior Angles of a Polygon Transcript

Hi guys, welcome to Geometry. In this topic, we're going to focus on the interior angles of a polygon. Now your knowledge of different shapes and figures is going to come in handy during this topic. You ready to get started? Let's go.

Okay, so before we start talking about the interior angles of a polygon, I just want to review a little bit about what a polygon is. Now a polygon by definition, going to just scoot this out of our way here, it's a two-dimensional shape formed by three or more segments. What that means is that it's a shape that has a length and a width, and that it's formed by three or more segments. A three-sided figure, or a three- sided polygon is a triangle, a four-sided polygon, a quadrilateral, a five-sided polygon is a pentagon, and a six-sided polygon is a hexagon. The sides continue to increase, but I just wanted to focus on just a few, just to warm you up a little bit for this topic.

All right, now take a look here. In the case of polygons, you can have a convex polygon or a concave polygon. In a convex polygon, each of the interior angles is less than 180 degrees, and what I mean by interior is I'm talking about the angles inside of the polygon, the angles that open up towards the inside. In a convex polygon, each of those angle measures is less than 180 degrees.

In a concave polygon, you'll have at least one angle measure that's greater than 180 degrees. In this case, it's this angle right here. This is the angle in this polygon that's greater than 180 degrees. Another way to think about a concave polygon is you can look at it as, there's going to be a side in it or part of the polygon is going to appear inverted, or that it caves in in some spot. Okay, so convex, concave.

You also have regular and non-regular polygons. A regular polygon like this one is equilateral, so each of its sides are congruent, and it's equiangular, each of its angles are congruent. I've just showed one example of a regular polygon, it doesn't have to be a two, four, six, hexagon, doesn't have to be a six- sided polygon, but if you have a polygon that's both equilateral and equiangular, so all of its sides are congruent and all of its angles are congruent, that means that you have a regular polygon.

In a non-regular polygon, it's just the opposite. All of the sides are not going to be congruent, and all of its angles are not going to be congruent. That's regular versus non-regular.

Okay, so now let's start diving into those interior angles. Depending on the number of sides your polygon has, that's what determines the sum of its interior angles. You'll notice here I've titled this "The sum of the interior angles of an n-gon". We're going to let n represent the number of sides the polygon has. We know we could call these specific names, like I could call this a quadrilateral, which I know you're looking at it and you may think, "I think it's a rectangle," or I might want to call it that, but we want to name it by its most general name, because we don't know the properties that this has to prove that that is indeed a rectangle.

Module 9: Polygons

Topic 1 Content: Interior Angles of a Polygon Transcript

The most general thing I could call it is a quadrilateral, that's why I'm calling it that, just naming it by the number of sides it has. In this case, going to fill in some information here, and we're going to use it to help us derive the formula that we use to determine the sum of the interior angles of a polygon. Here, I have four sides, so we're going to say okay, n equals four. Now you see I have a blank space, and then I have triangles. We're going to determine the number of triangles we could create by drawing as many diagonals in these figures as we can, but we want all the diagonals to start at the same endpoint.

Let me show you what I mean, that one I think you really need to see. If I started at this endpoint, you could pick any one, that's going to be the only diagonal that I can make. In this case, I end up creating two triangles, I'm going to write that I created two. Now I'm going to determine the sum of the interior angles in this quadrilateral, based on the fact that I know it includes two triangles. Because I know that the sum of the interior angles of a triangle is always 180 degrees, and in this case, because I have two triangles, going to do a little work underneath here, going to say two times 180 degrees, that's 360 degrees. The sum of the interior angles in this quadrilateral, 360 degrees.

You've got to keep that in mind, we're going to figure out a pattern here. Okay, now in this quadrilateral, I have one, two, three, four, five, so n equals five, five sides. Let's see how many diagonals I can sketch, all beginning at the same endpoint. I'll start at this one, and I can go there, and I can go here. Okay, so I could draw two diagonals, and when I did that, I created three triangles, and each triangle measures 180 degrees, so 180, 180, 180. Now, the sum of the interior angles here, based on the number of triangles, three times 180 degrees, and I've actually done this math ahead of time, so I know that equals 540 degrees.

One more polygon. Okay, so this one has two, four, six sides, so n is six. Let's see how many triangles I can create by drawing diagonals. Okay, so one, two, I was able to get three diagonals, one, two, three, four triangles, and I know each of these triangles, 180 degrees. Okay, so let's do a little work underneath here. Four times 180, and I have done that work ahead of time too, I know that that's 720 degrees.

Let's interpret this a little bit and see if we notice a pattern. Focus first on the number of sides in relation to the number of triangles. In our first polygon, we had four sides, two triangles, five sides, three triangles, six sides, four triangles. You notice the pattern there? If I had another polygon that had seven sides, by the pattern we seem to have established here, we probably would have five triangles if we kept up with that pattern. Because it appears that for however many sides I have, I have two fewer triangles, so whatever n is, I could represent my number of triangles by n minus two, because the number of triangles is always two fewer than the number of sides. I have four sides, two triangles, five sides, three triangles, six sides, four triangles.

Let's scroll down a little here and I'm going to write a little bit right above this that we're going to represent the number of triangles by n minus two, so keep that in mind that n minus two represents our number of triangles. Then notice when we were determining the sum, we took the number of triangles and multiplied it by 180. Here we had two triangles, so two times 180, three times 180 for our three triangles, and four times 180

whro EDUCATION

for our four triangles, right? Because n minus two represents the number of triangles, I could say that the sum is represented by n minus two, times 180 degrees. That is actually the formula to find the sum of the interior angles of an n-gon, which is just a way to say, of a polygon of however many sides you're working with.

N minus two, times 180 degrees that will give you the sum of the interior angles of your polygon. Now while we're working some of the examples, you may notice me write it like this, 180 times n minus two, and you can also write it like this just to have a working example, but either way is fine, as long as you know that n minus two represents that n stands for your number of sides, so n minus two, that represents those number of triangles that we created, and then 180 degrees for each of those triangles.

To really get a handle on this, let's start practicing. Let's put this example to work. Here, example one. The top of the gift box is in the shape of an octagon. What is the sum of its interior angles? To handle something like this, we know that we're going to be using that 180 times n minus two, and you remember that n is the number of sides. Either you could take the number of sides from the problem itself, that it says you have an octagon, or you could count the number of sides on the top of the gift box. One, two, three, four, five, six, seven, eight, whichever way you want it, but you know that n equals eight for this problem.

To determine the sum of those interior angles, just substitute eight right in there for n, so 180 times that quantity, eight minus two, which is 180 times six. I know that's going to be a pretty large number, so I'm going to switch to my calculator. All right, and I've already got it clear, so 180 times six. That's 1,080, so let's go back to that problem, and what that means for us is that the sum of the interior angles of this octagon is 1,080 degrees. Now that is a pretty large number, but it's true for this octagon.

You see how we applied that formula in order to get to this practical problem in this case. All right, now take a look at the next one here, I want you to try this one. What is the sum of the interior angles of a 15-gon? I know you've noticed 15-gon instead of a name like hexagon or octagon or pentagon. When you see something like this, it's just another way to tell you how many sides your polygon has. When the numbers of sides start getting kind of large, you'll notice that sometimes the polygons are named just like that, like by the number of sides like 15-gon or 25-gon or 108-gon, but that just tells you the number of sides.

Go ahead and use that formula that we derived, take a few minutes and work your way through this one. Press play when you're ready to check your work. All right, let's see how you did on this one. Here, like we said earlier, n is 15, so I'm going to use my 180 times n minus two, that quantity, we're going to use that formula. 180 times 15 minus two, scroll down a little bit here, so that's 180 times 13, which is going to be a really large number, so let's go to our calculator. Okay, so 180 times 13, that is 2,340 degrees. Let's get back up here, all right so 2,340 degrees. That would be the sum of the interior angles of a 15-gon, or of a 15-sided polygon, really large number on that one.

Module 9: Polygons

Topic 1 Content: Interior Angles of a Polygon Transcript

All right, let's take a look at the next example. Now we're going to use some algebra skills to work through this one. Here we're asked to find x, and notice that your angle measures here are representing your interior angles. I'm going to need to set up an equation to help me solve for x, and it's going to have to relate to the interior angles of this polygon. The first thing that I want to figure out is how many sides this polygon has, so let's count. One, two, three, four, five, six, and I didn't say, but if you don't want to count the sides, or sometimes the sides lay almost completely flat and it's difficult to tell if that's one side or two sides, a polygon will always have as many angles as it does sides.

You could also count the angles to figure out the number of sides, so one, two, three, four, five, six, that'd work also to help you figure out the number of sides. Okay, so n is six, and then we know that the formula, 180 times n minus two, so we're going to use that to figure out what all of these angle measures should add up to equal, and that'll help us set up that equation. 180 times six minus two, that's 180 times four, and let's go ahead and get that just so you can see in the calculator, so 180 times four, that should be 720. Let's go back to our work there. What that tells us is that now we'll know that those angle measures should add up to 720 degrees.

We're going to set up an equation where we add up these angle measures, set them equal to 720, and solve for x. All right, I'm going to scoot this out of the way here. Okay, so let's see here, we have 3x plus 12x plus 5x plus 160 plus 150 plus 130 equals 720. We've got a lot of terms to combine here, let's clean this up. 3x plus 12x, so now that's 15x, 15x plus 5x, that's 20x. My x terms combine to 20x. Let's see how much mental math we can do here. Okay, so 160 plus 150, that is 310, and 310 plus 130, that is 440, but just to double check, make sure we got it right, so 160 plus 150 plus 130, 160 plus 150 plus 130, because our numbers get pretty large when we start dealing with our polygons. It is 440.

Okay, let's go back to our work here. That equals 720, so let's solve this equation. Subtract 440 from each side, so we have 20x on the left, 720 minus 440, back to our calculator for that one since those numbers are kind of large. 720 minus 440 equals 280, back to our work. We've got to do a lot of flipping back and forth on this one, so that's 280, now divide each side by 20, so x equals. Now let's get 280 divided by 20, so 280, we've already got that in there, divided by 20, that is 14. Here we have that x equals 14.

This one had a good decent amount of steps, but really what it all broke down to is that we first needed to figure out what the sum of these interior angles should be, so we used our formula to figure that out, that it should be 720, then we set up our equation, solved for x, and we figured out that x equaled 14, all right? Okay, good job on that.

Let's go on to our next one, and it is your turn, so go ahead and press pause, take a few minutes, work your way through this one. Press play when you're ready to check your answer. All right, let's see how you did here. On this one, n is four. We have four sides, we have four angles, so let's use our formula to figure out what the sum of the interior angles should be. 180 times four minus two, that's 180 times two, that's 360. I know that the sum of the measures of these angles should be 360, so now I'll set up my equation so I can solve for x. 5x plus 40 plus 120 plus 7x plus 80 equals 360, let's just double check, make sure we got them all.

Let's combine like terms, go ahead and solve this, so 5x plus 7x, that's 12x, then we have 40 plus 120 plus 80, let's go to the calculator for that one. 40 plus 120 plus 80, that's 240, okay so back to our work. 12x plus 240 equals 360, subtract 240 from each side, get a little more space. Okay, so 12x equals, that's 120, divide by 12, so x equals 10, and you're all done with that one. You see how that one was similar to that previous example. You had to start out with, if we scroll back up to the top, start out with figuring out what your n is for this case. How many sides does this polygon have? Then figure out the sum of your interior angles, use that to set up your equation, and then use your algebra skills to go ahead and solve for x, all right? Okay, good job on that one.

Now I want you to take a look at this one. What is the measure of each interior angle of the regular hexagon shown below? Okay, now what's different about this one, you might have noticed it in the question there, I'll just scoot this over a little bit, is that here I'm being asked to figure out the measure of each interior angle. Not the sum of the interior angles, but the measure of each individual angle, and then I was also told here that this was a regular hexagon. Remember what we learned at the beginning of this topic, that regular means that each of those angles is congruent, and each of side lengths. In this case, what I want to focus on is those angle measures.

How I can use that to help me answer this question, is once I know the sum of the interior angles, I just need to split that sum up equally between each of my individual angles. Let me show you what I mean. Okay, so this is a hexagon, so that means it has six sides. 180 times six minus two, I'm using my formula, which is 180 times four, 720. That means that the sum of the interior angles is 720. In order to figure out the measure of each individual interior angle, I need to take that sum and divide it by six, because that 720 is split up evenly in this regular hexagon. Okay so 720 divided by six, and that's 120 degrees. What that means is that in this hexagon, each of these interior angles measures 120 degrees, because it's regular.

All right, so you see how that one was a little different. You did still have to use the sum, but you had to divide that sum by the number of angles you had to figure out the measure of each interior angle in your regular hexagon.

Okay, take a look at the next one here. I want you to press pause, take a few minutes, work your way through it, because this one's for you. Press play when you're ready to check your work. All right, let's see how you did on this one. Here you were told that your stop sign's in the shape of a regular octagon. What is the measure of each interior angle? For this one, I'm going to get some work space right underneath. I know that I have eight sides if this is an octagon, so n is 8. I'm going to use my formula for the sum to figure out the sum of those interior angles, so 180 times eight minus two, that's 180 times six, and let's go to our calculator and get that. 180 times six, that is 1,080. Okay, let's go back to our work; get a little more room here, so that is 1,080.

That is the sum of the interior angles of this octagon, but I'm asked to find the measure of each interior angle, so I need to take that sum and divide it by eight, because that'll tell me what each interior angle equals. Take that 1,080 and divide it by eight, and let's go to our calculator. 1,080 divided by eight, 135. All right, back to our work, so that means that the measure of each interior angle of this regular octagon is 135 degrees. All right? Okay, good job on that one.

All right guys, you've reached the conclusion of this topic on the interior angles of a polygon. I hope you saw how your knowledge of algebra and of figures and polygons in general really came in handy for you in this topic. Bye.