Module 9: Polygons
Topic 3 Content: Tessellations Transcript

Hi guys. Welcome to Geometry. In this topic, you're going to focus on tessellations. Your knowledge of interior and exterior angles of polygons are going to come in handy for you during this topic. You ready to get started? Let's go.

Okay. What exactly is a tessellation? A tessellation is a tiling or a linking together of polygons so that you leave no gaps or spaces in between. I've taken a little snippet, a little piece of a tessellation. Notice how these blue and green triangles link together, just fit right together like puzzle pieces. They don't leave any weird gaps or spaces in between. That means that those figures will tessellate the plane. That's what we call it. Both regular and non-regular polygons will tessellate the plane.

Another thing to know about tessellations is that the sum of the angles around a point of tessellation is always 360 degrees. What do I mean by that? If I circle right here, if I focus in on these angles right here with the little dots on them, those are the angles around this point of tessellation. What that means is the sum of each of these angles or the sum of those angles is 360 degrees.

Just a few things just for background knowledge that you can know about tessellations ... and I always want you to know that tessellations appear in random spaces. Look at this pineapple for example. You have tessellations in nature. The pattern on the skin of a pineapple, that's a tessellation. Right? Those shapes fit together, link together like pieces of a puzzle. Even the back of a turtle shell, that's also a tessellation. Notice those scales. Right? They just link right together, fit together like puzzle pieces. You can find tessellations even out in the everyday world, in nature.

Take a look at this one. We have tessellations in art also. Here we have four sided figures. It looks like they're outlined in white, pink and red. They link together just like puzzle pieces. They have this art piece on the wall. It's an example of a tessellation. Then also we have tessellation in construction. Here you see the tiles on this bathroom wall and even on the bathroom floor. Those are also examples of tessellations or tiling of the plane. Each of those pieces of tile links right together like a puzzle piece. That's an example of a tessellation in construction.

Now that you have a little background knowledge on tessellations, what they mean as far as geometry is concerned and just examples that you find out in the real world, let's start working a few examples. Will a regular hexagon tessellate the plane? When you're asked that question, what you're being asked to determine is can you link together hexagons like puzzle pieces? Will they just snap together without any gaps or spaces in between? What you need to know to be able to answer this question is that for a regular polygon, if the measure of its interior angles is a factor of 360 then that means that the polygon will tessellate the plane.

Let me show you how to figure that out. The first thing you need to know is what's the sum of the measures of the interior angles and then we can use that information to figure out the measure of each interior angle in this regular hexagon. I'm going to use my formula, my 180 times N-2. I have a hexagon. That means that N is 6. I'm going to substitute that 6 in there for N. So 180 times 6-2. That's 180 times 4. I've done this calculation a few times so I know this is 720. I know that the sum, I'm going to write that this represents the sum of the measures of the interior angles of this regular hexagon, is 720 degrees. Now to figure out the measure of each interior angle, I need to take that 720 and divide it by 6 because that hexagon has 6 angles, right? So 720 divided by 6, that's 120 degrees. Each measure of the interior angles of my regular hexagon is 120 degrees.

Now what I need to figure out, I'm going to write each right here, is I need to determine if 120 is a factor of 360. The quickest way you can do that is to do 360 divided by 120. If you get a whole number for your answer, then that means 120 is a factor. I'm going to get a little bit more work space here. 360 divided by 120 is 3 and 3 is a whole number. What this tells me is that 120 is a factor of 360. Will a regular hexagon tessellate the plane? Yes it will. So the answer to this question is yes.

Just to review what we did, to take you through that, scroll back up to the top. You were asked to determine if a regular hexagon would tessellate the plane. The first thing you needed to do was figure out the sum of the interior angles of our hexagon. Put a little 1 by that because that's the first thing that we did. We figured out the sum of the interior angles. It was 720 degrees. Then we found the measure of each interior angle. That was the second thing that we did. Each interior angle of that regular hexagon, 120 degrees. Then the third thing we did was figure out was 120 a factor of 360. Remember if you get a whole number for your answer when you do 360 divided by that 120 then you know yes, that is a factor. That's just the quickest way to figure that out. So 360 divided by 120 was 3. That's a whole number so we know that now 120 is a factor of 360. Then to finally answer the question, we know that yes, a regular hexagon will tessellate the plane.

Let me show you what that really means. What that means to us is that regular hexagons will snap together or link together just like puzzle pieces. They'll fit together without leaving any gaps or spaces in between. I just wanted to show you a visual of what that meant just to go along with the work. Now I want you to go ahead and try one. Go ahead and press pause. Take a few minutes. Work your way through this one. Press play when you're ready to check your work.

All right. Let's see how you did here. Will a regular pentagon tessellate the plane? I know the first thing I need to do is figure out the sum of those interior angle measures. Right? I have a pentagon. That means I have a five sided polygon. So 180 times 5-2, that's 180 times 3. I've done this calculation a few times so I know that it's 540. That's the sum of the interior angles of the pentagon, 540 degrees.

Now I need to figure out the measure of each interior angle of the pentagon. So 540, take that 540 degrees and divide it by 5. I'm going to the calculator for that one. Let's pull the calculator up, 540 divided by 5. That's 108. Back to our work. That is 108 so each interior angle of this regular pentagon measures 108 degrees. Now I need to figure is 108 a factor of 360. To do that, this is when we figure out is it a factor, I'm going to take 360 and divide it by 108. If my answer is a whole number, then that means that 108 is a factor. Let's go back to the calculator, 360 divided by 108. You see your answer is not a whole number. You have 3.3 repeating. I'm going to go back to the work. Let's get that down. The answer was 3.3 repeating. Is 108 a factor of 360? No, it's not.

What that means is, going back to our question; will a regular pentagon tessellate the plane? It won't. What that means is that if you try to link together pentagons, they will not link together without leaving gaps or spaces. Let me show you what I mean. I've got a visual for that one, too. In the way that our hexagons link together without leaving any gaps or spaces, pentagons will not do that. There's no way that they'll tessellate the plane without leaving gaps like we have this one here. They're going to leave gaps and spaces. You won't be able to tile the plane ... and I keep saying plane. I just mean a flat surface when I say that. You can't tile the plane with regular pentagons. All right? Good job on that one.

Now take a look at this one. We've got a practical problem here. A carpenter is designing a countertop modeled by the composite figure consisting of a regular hexagon and a regular octagon. What is the measure of angle DFG? There was probably a lot going on in that one. Let me break a few things down here. First of all, we ran into the term composite or composite figure. What that means is when you have a composite figure, it's a figure that's made up of more than one type of figure. For example, this is a composite figure because it's one whole figure but its parts consist of a regular hexagon and a regular octagon. Two different things. When you see the term composite figure, know that you have more than one figure. In this case, you have a regular hexagon, a regular octagon, and you're asked to find the measure of angle DFG. That's this angle here opening up right on the outside.

What's we're going to do to answer this one is we're going to use our knowledge of exterior angles to figure out the measure of angle DFG. What I'm going to do is I'm going to start out by extending this side right here where these two figures are adjacent, where they connect. I'm going to extend it. What I'm going to do is I'm going to ignore the octagon for a second and I'm going to figure out the measure of this exterior angle of this regular hexagon. Then I'm going to ignore the hexagon and figure out the measure of this exterior angle of this regular octagon. Then I'm going to add those measures together and get the measure of that entire angle on the outside. Okay? Let me show you what I mean.

Recall for your polygons the sum of your exterior angle measures is always 360 degrees. Because I was told that this hexagon is regular, I know that that 360 is split up evenly between those exterior angles because I know all of my sides are congruent and all of my angles are congruent. To figure out the measure of the exterior angle for the hexagon, 360 divided by 6 because I know a hexagon has 6 sides. So 360 divided by 6. Let's go to our calculator. 360 divided by 6 is 60. I know that each measure of those exterior angles, each one of them, measures 60 degrees. I'm going to write that 60, I'm going to switch colors, right here. Remember, I'm ignoring the octagon and this is the measure of this angle right here, just part of angle DFG. Now I'm going to figure out the measure of the exterior angle for the octagon. Let me write that 60 in there and I'll just mark this was for our hexagon.

Now for our octagon, we know an octagon has 8 sides, so 360 divided by 8. Let's go to our calculator for that one. 360 divided by 8, that's 45. That means that the measure of each exterior angle for that regular octagon is 45 degrees. I'll write octagon beside this one so we can keep them straight and I'll write in here 45 degrees.

Now I need to determine the measure of angle DFG. Angle DFG is just the sum of those two exterior angles. I'll just add those together and then I'll know the measure of angle DFG. Right underneath here, get a little more work space. I'll put that in blue. Let me just make sure I can see it. The measure of angle DFG equals 60 degrees plus 45 degrees. That is 105 degrees. Then you're all done for that one.

You see the things that came into play for this problem. You had to first see that you had a composite figure. You knew that you had a regular hexagon here, the measure of each of its exterior angles. We used our math to figure it out, 60 degrees. Then we also had a regular octagon. I didn't mark those sides congruent, but we also know that that octagon is regular so all of its sides are congruent. The measure of the exterior angle here, 45 degrees. We did our math right to the left to figure that out. Then once we had those exterior angle measures, we found the sum and that gave us the measure of angle DFG.

Keep all that in mind and I want you to try this one. Go ahead and press pause. Take a few minutes. Work your way through this one. Press play when you're ready to check your work. All right. Let's see how you did on this one. The play area at a local park is modeled by the composite of a regular hexagon and a regular pentagon. What is the measure of angle ABC. Let's use the same strategy that we used on that previous example to get to the bottom of this one. I'm going to extend that adjacent side, that side they share. I'll focus first on the pentagon. I'm told it's regular, so I know the sum of its exterior angles, the measure of those, is 360. For the pentagon, let's go ahead and label it. I'll take that 360 divided by 5. Let's go to the calculator for that. So 360 divided by 5, 72. Back to our work, 72 degrees. That's the measure of this angle right here.

Now for our hexagon, we actually did that work on the previous example but just to show you again, just to reiterate ... For our hexagon, it's also regular so 360 divided by 6, remember that was 60. That means the exterior angle for that hexagon, 60 degrees.

Now to figure out the measure of angle ABC, we'll just find the sum of those two angles. Let's get some space right down here. Okay. The measure of angle ABC equals 72 degrees plus 60 degrees. If we add those together we'll get 132 degrees. That's the measure of angle ABC. Good job on that one.

All right, guys. You've reached the conclusion of this topic on tessellations. I hope you saw how your knowledge of interior and exterior angles came in handy for you during this lesson. Bye.