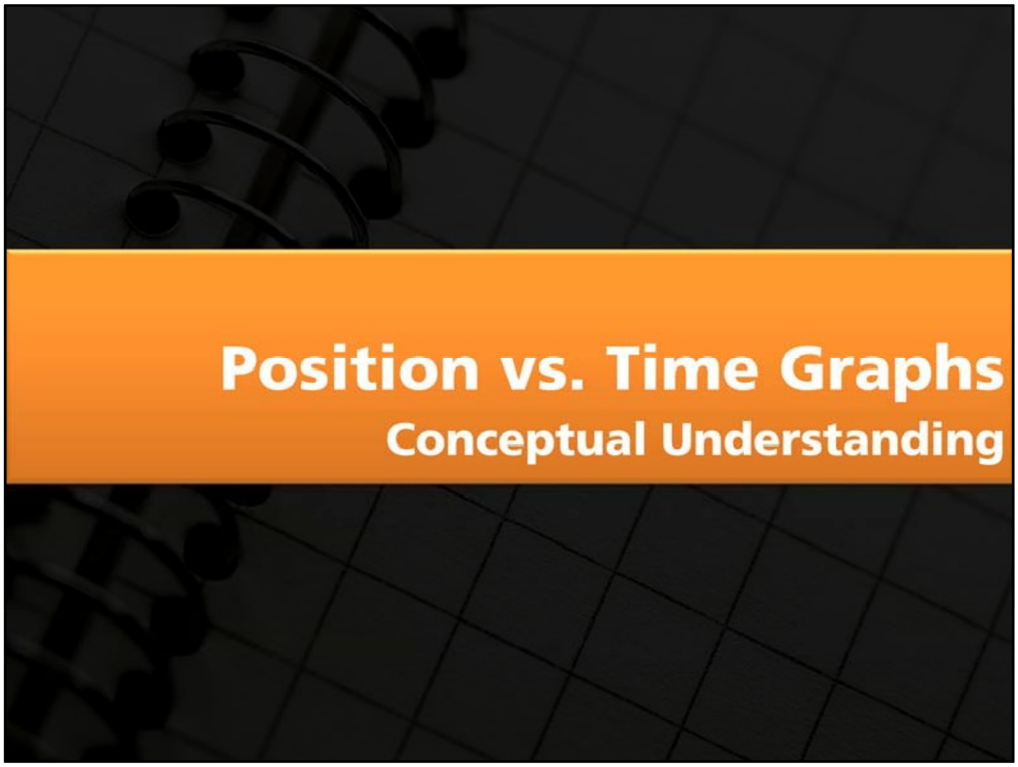
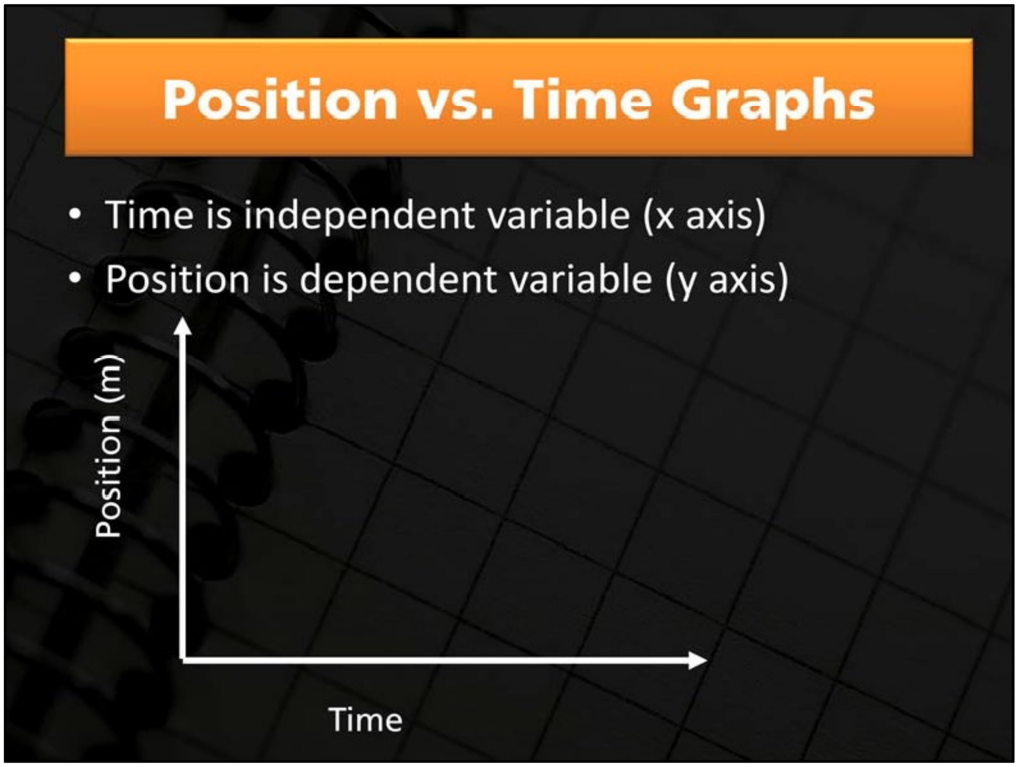
Module 1: Describing Motion
Topic Content: Position vs. Time Graphs

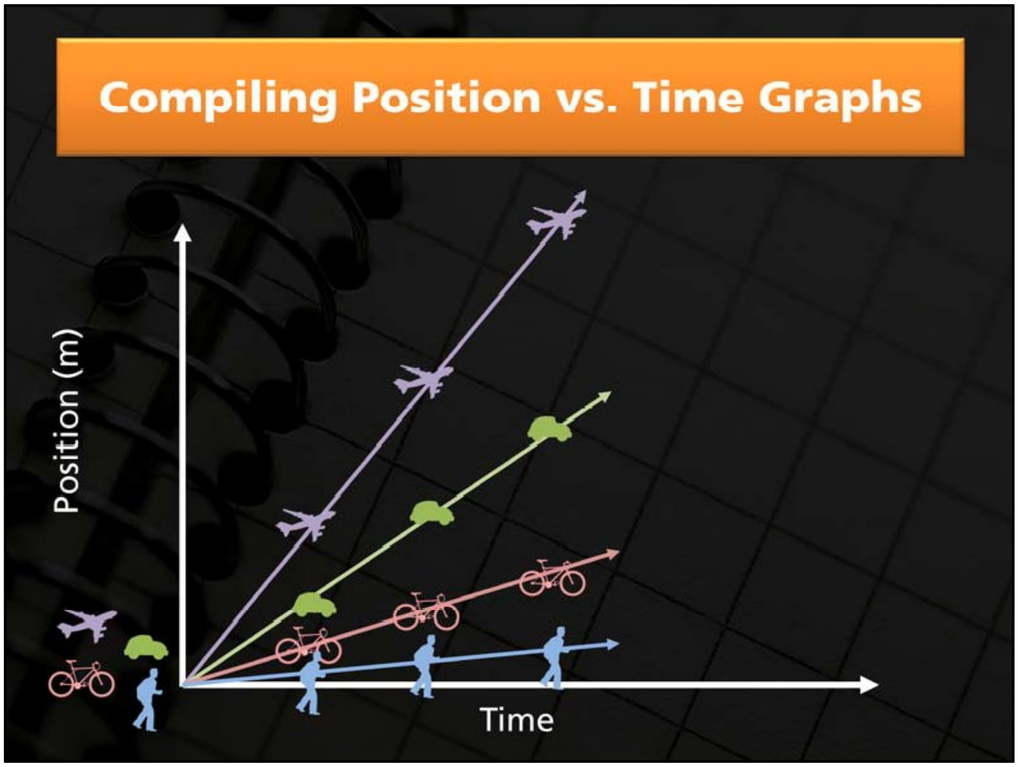


We are already able to calculate average speed and average velocity in several situations. We started simple, then got somewhat more complicated. But for a better understanding of motion, we will turn to graphs that not only represent the starting and ending points, but allow us some insight as to what is happening in between.

If we are able to capture the position at several points of time, we can construct a position versus time graph that will let us analyze changes in motion and give us a clearer view of how the motion changed over time.



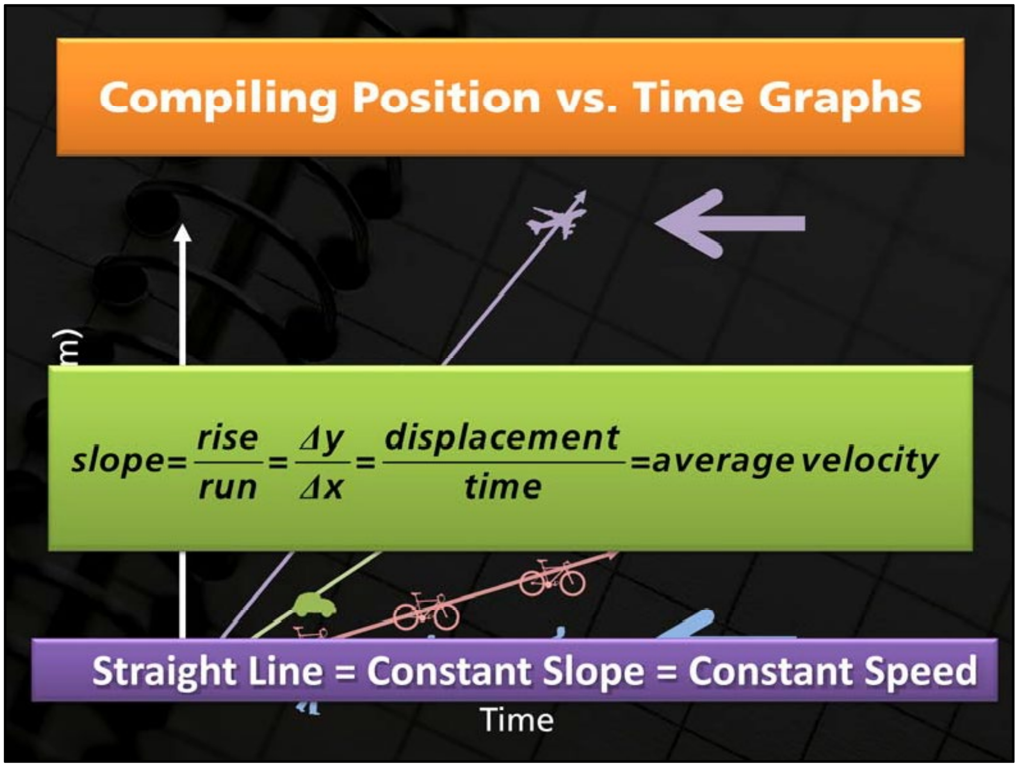
We will begin by setting up our position time graphs in a consistent fashion. We will always want to put time on the horizontal axis and position on the vertical axis. As we graph other measurements versus time, we will also put time on the horizontal axis, which will let us see certain relationships between the variables better.



Suppose that a walker, a bike rider, an automobile and an airplane all traveling in the same direction happen to cross the starting line at the same time. After a small amount of time, the walker would have traveled a small distance, the bike rider would have been further along, the automobile would have gone still further, and the airplane would have traveled the farthest. Marking these on the position-time graph would look something like this.

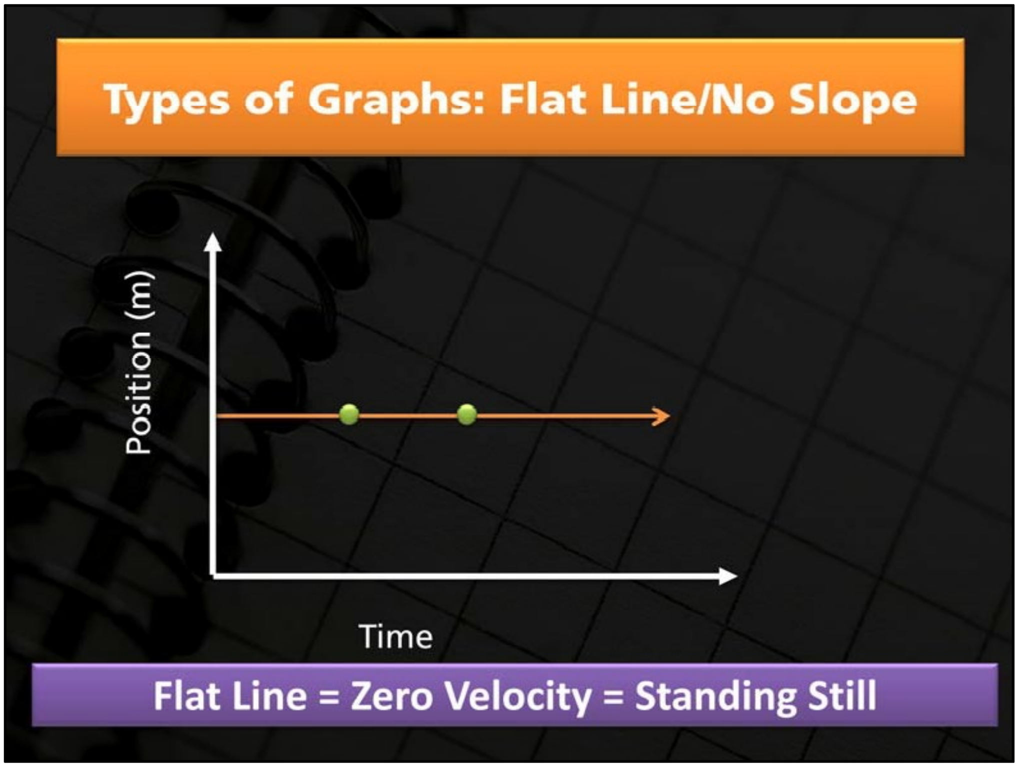
At a later time, each object would now be in this position. And still later, at this position. And by connecting the dots, we wind up with our first position time graph.

Remember that all four objects were heading in precisely the same direction. They did not go off at angles to each other. But, when we graph position over time, the line representing motion in a single direction may slope or even curve. Don’t let this confuse you. We are only dealing with motion in a single direction so far.

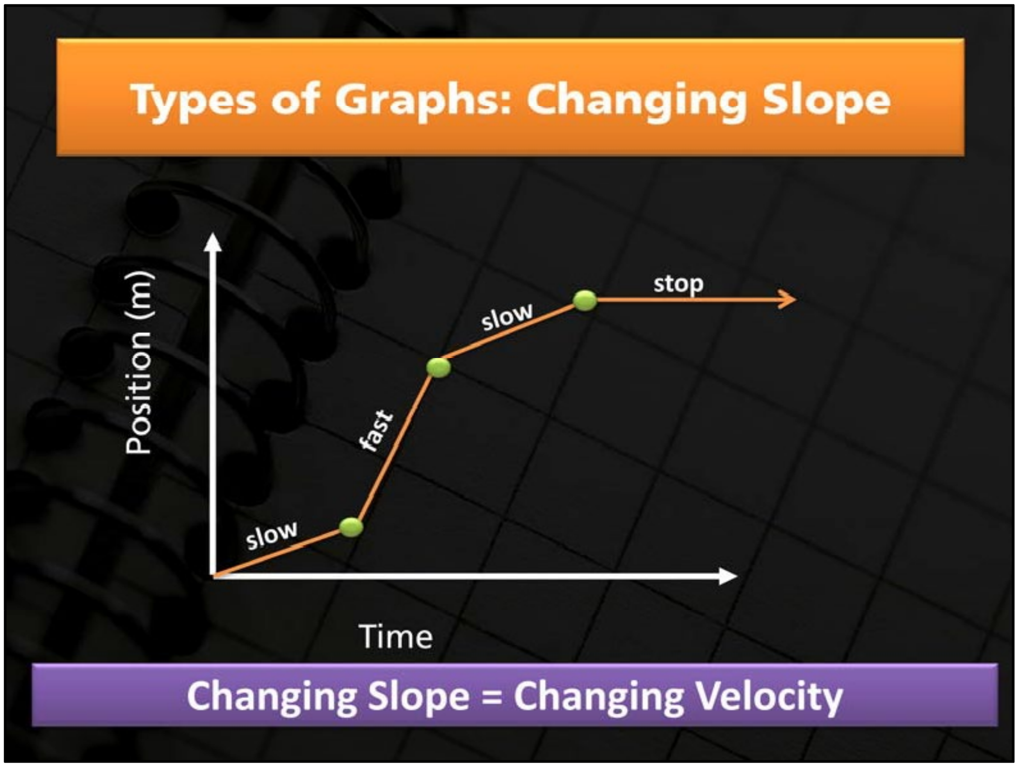


One of the first things you may notice is that the lines are not all equally steep. The slope varies from line to line. The least steep line is the one that tracks the motion of the walker, who was travelling the slowest, and the steepest line is the one tracking the airplane, which was moving the fastest. This is the first rule we see on our position time graph, that the steeper the line is, the quicker the object was moving.

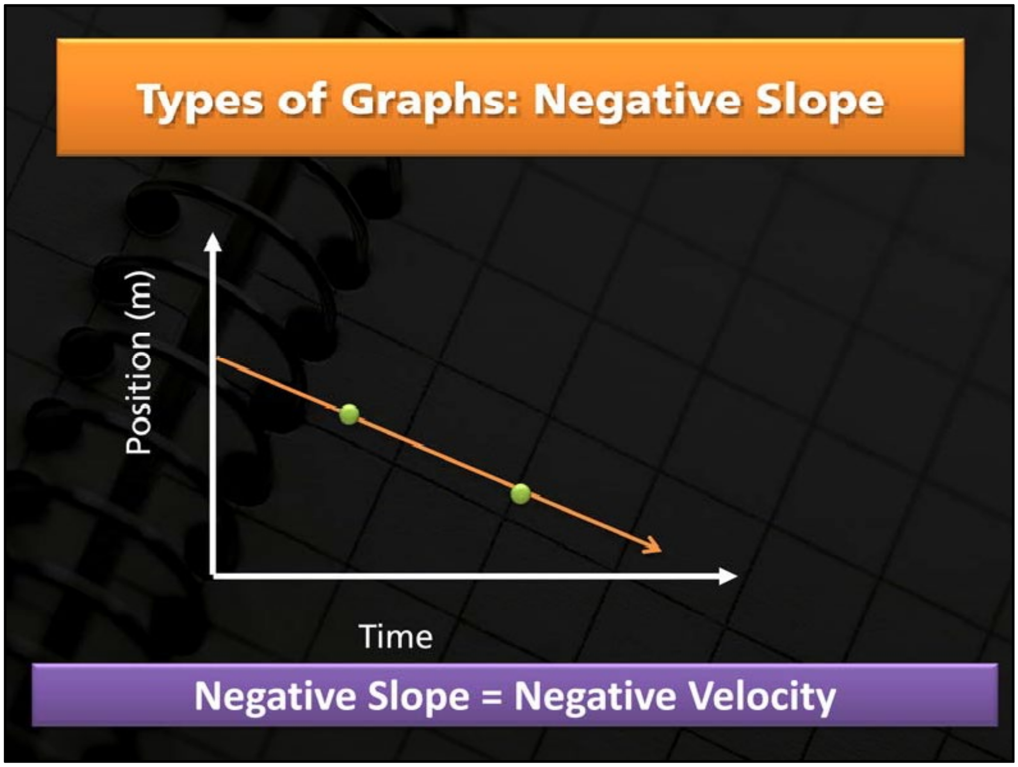
In fact, if you remember the equation for the slope of a line, it is rise over run, or change in the y-value over change in the x-value. In this case, it would be change in position, which is displacement, divided by change in time, which just happens to be our equation for average velocity. Since the slope is the velocity, any straight line will have a constant slope and therefore a constant velocity.



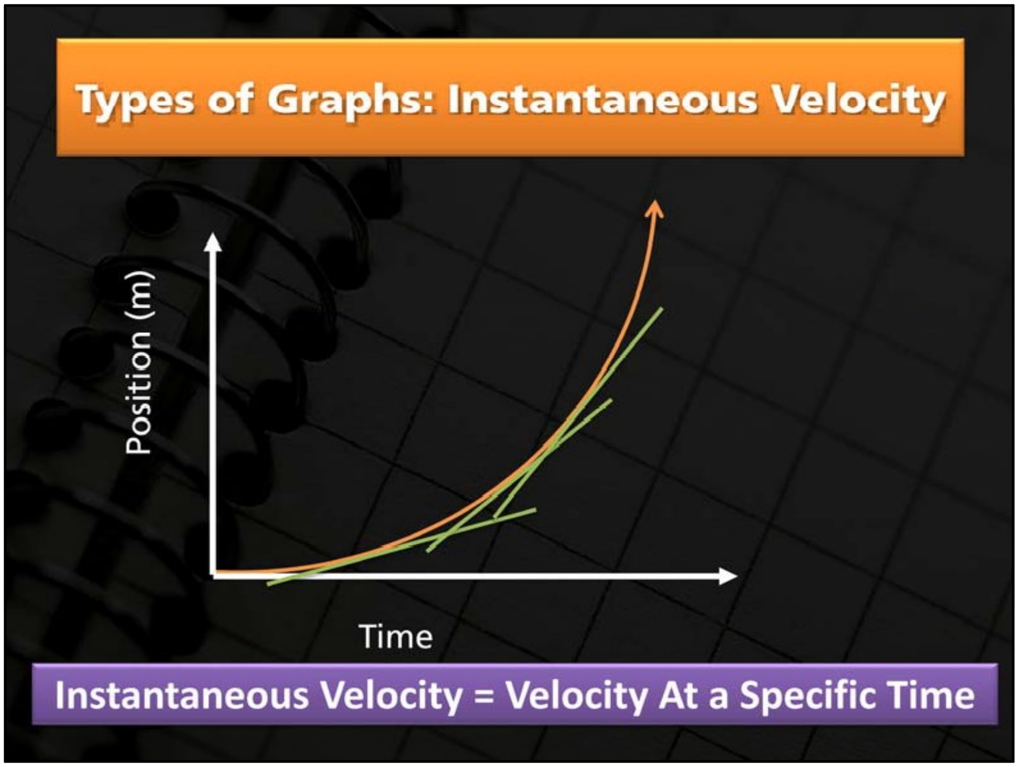
Now, we're going to take a look at a series of position time graphs so that you will begin to see the relationship between the shape of the graph and the motion of the object. First, let's take a look at a flat line on a position time graph. At one time, the object is at a certain position. At a later time, the object is still at that same position. It hasn't moved! An object that is not moving is standing still and has a velocity of zero. The slope of a flat line is zero, so the velocity is zero.



How would you describe a position time graph like this, with a crooked line? Remember that a straight line has a constant slope and therefore represents a constant speed. In this case, the slope changes. It starts out with a small slope, then gets steep then lessens again and finally flattens out. The greater the slope, the greater the velocity, and the lesser the slope, the lesser the velocity. In this case, the object begins to move slowly, then travels faster, slows down again, and finally comes to a stop.



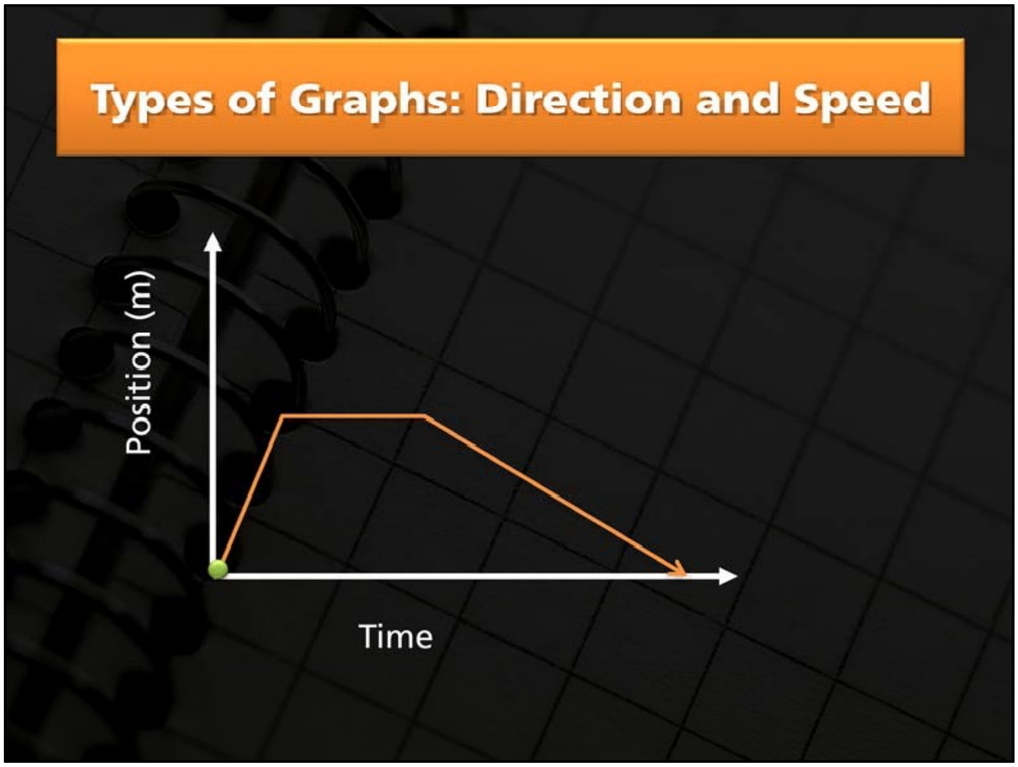
Here is a position time graph with a downward sloping line. Again, to get a good feel of what this graph represents, compare two different points in time and see what happens to position. At this time, the object is at a certain position away from the origin. At a later time, the object is closer to the origin than it had been. It gets closer to the origin as time passes resulting in a move toward the left, a negative direction (since you set right as positive). The slope of this line is negative, and in this case, the object has a negative velocity, otherwise known as a velocity in a negative direction.



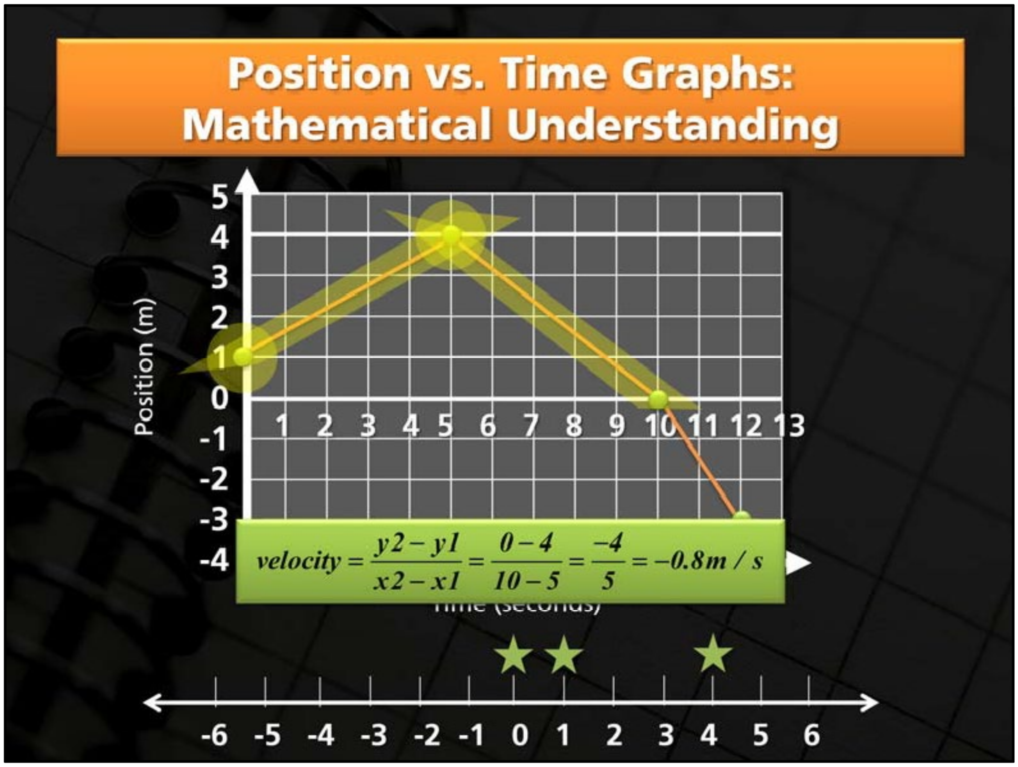
What could we say about this graph, a curved line?

We know that the slope of each line segment represents the velocity. At this point, we introduce the concept of instantaneous velocity, or the velocity at a specific instant of time. Since we can't find the slope of a curvy line using our normal methods of rise over run, we will look at a particular instant of time. In order to determine the slope of the line at this instant, we will have to draw a line that is tangent to the curve and determine the slope of that line.

Near the origin, a tangent line would be relatively flat, indicating a small initial velocity. At a later time, however, the tangent line is steeper, indicating a greater velocity. Still later, the tangent line is increasingly steep, indicating an even higher velocity. This curve is an example of a continually changing velocity, which you might already recognize as acceleration. We'll be studying this in a later lesson. Remember that the object here is moving in a straight line at ever increasing velocity. It is not moving along a curved path. The curve of the graph shows the relative change in speed over time.



Now, let's combine what you know to describe the motion represented by some other graphs. Take a moment to describe the motion represented in this graph. Be sure to indicate direction and relative speed for each section of the graph. The object appears to start at the origin and is moving relatively quickly in a positive direction, as indicated by the steep, upward sloping line. Then, the object stops for a period of time, as indicated by the flat line. Finally, the object moves back towards the origin at a relatively lower speed than when it was moving away, as you can see from the downward sloping, not so steep line. The object ends its motion at the origin, according to the graph, as it is back at position zero.



We have learned to interpret motion from a position time graph. We will look at these graphs in more detail to mathematically determine the motion. Let's see what we can determine about the motion of the object represented in the position time graph here.

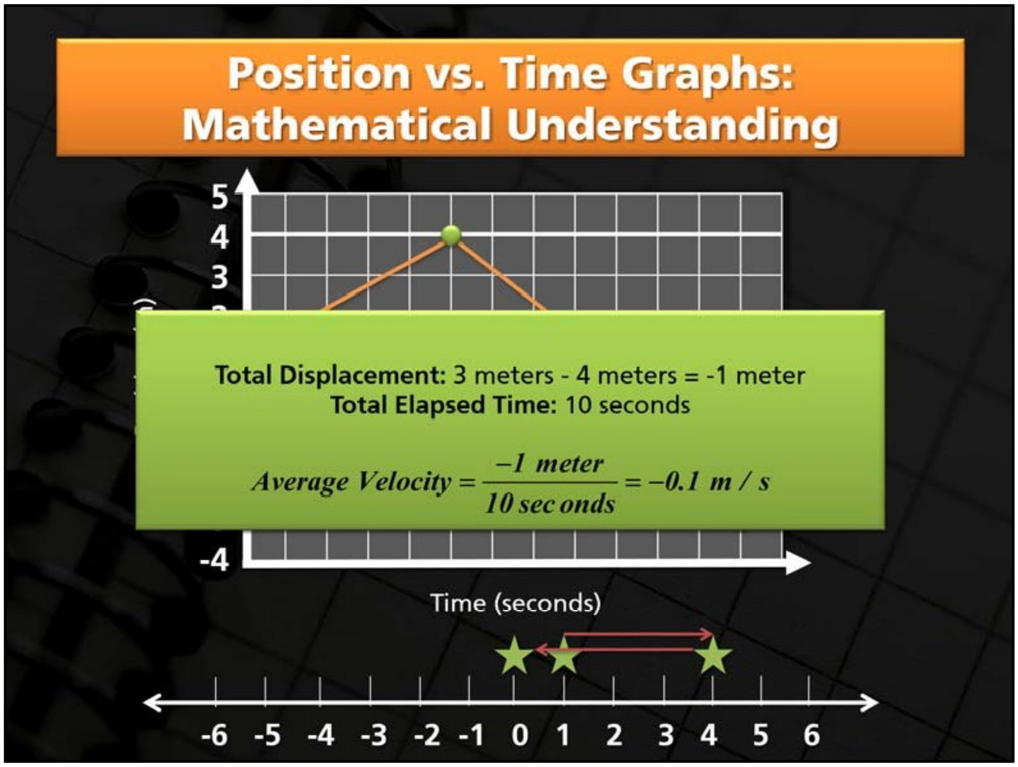
The first thing we can see is the starting position, which is the position when the time equals zero. In this case, the object starts at a position of +1 meter. If we think about the number line and positive to the right, the object starts 1 meter to the right of the origin.

Now, we look at the first line segment. After 5 seconds, where is the object? Look at time equals 5 seconds and follow it up to the line, then look to the y-axis to see that the position is now 4 meters. You should recognize that the object has a positive velocity because the line slopes up. But now, we've got numbers to work with.

In order to determine the velocity of the object during the first five seconds, we need to determine the slope. The slope of a line equals the rise over the run or y_2 minus y_1 over x_2 minus x_1. Here, the position went from 1 to 4 meters while the time went from zero to 5 seconds. 4 minus 1 over 5 minus zero equals 3/5 or 0.6 meters per second. So, we can say that for the first 5 seconds, the object moved an average velocity of 0.6 meters per second.

The next line segment is sloping down, so you should recognize that it will have a negative velocity, with the object moving to the left on our number line. This motion also happens to take 5 seconds, in which the object moves from +4 meters to zero meters.

The displacement from its last position is zero minus 4 or -4 meters. The elapsed time is 5 seconds. So, the velocity is -4 over 5 or -0.8 meters per second. Recall, however, that although the displacement is negative for meters for this segment of motion, the distance traveled here is simply four meters.

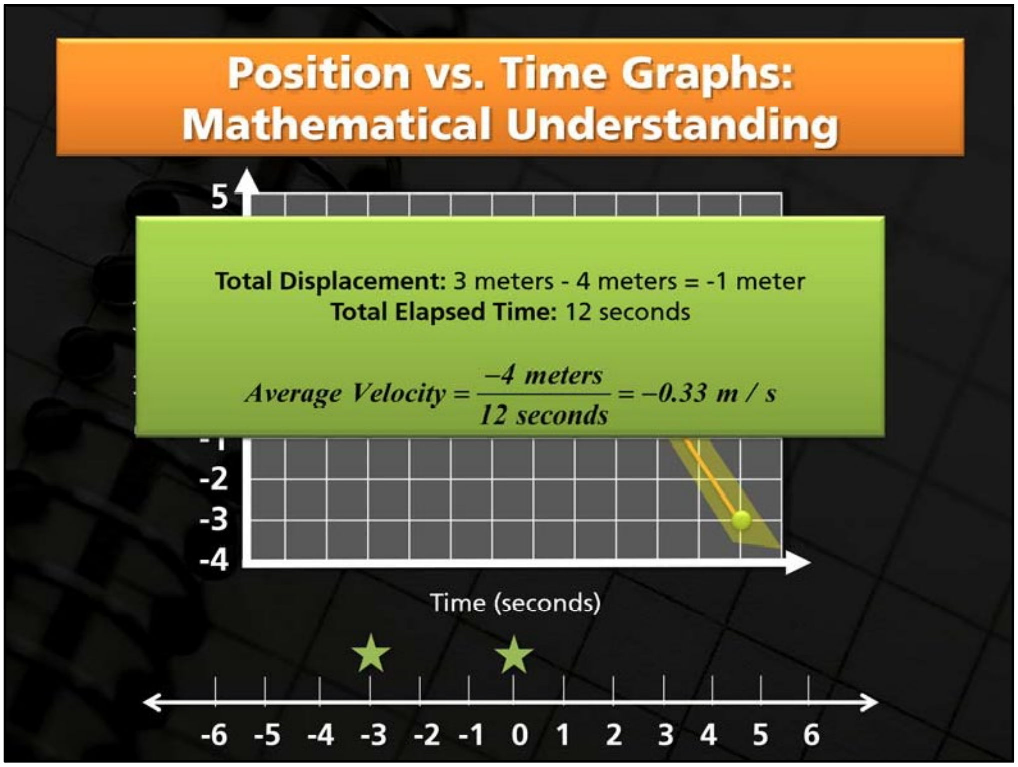


Now, let's compare the average speed for the first 10 seconds of the trip with the average velocity over this same time period.

To calculate the average speed, we need the total distance traveled and the total elapsed time. The total distance traveled is going to be 3 plus 4 equals 7 meters. The object moved a total of 7 meters in 10 seconds, so the average speed is 7 divided by 10 or 0.7 meters per second.

To calculate the average velocity, we need the total displacement over the elapsed time. With the displacement, the sign is important, so our total displacement is +3 minus 4 equals -1 meter. You can also see this, since the object started at 1 meter and is now at zero meters, which is a net change of -1 meter.

-1 meter divided by 10 seconds gives -0.1 meter per second for our average velocity. Interestingly, this is also the slope of the line connecting the starting point with the ending point that we are evaluating.



We have a third segment of motion to analyze. What happens in the final 2 seconds? The object moves from zero to -3 meters. The distance traveled is 3 meters. The displacement is -3 meters. The velocity for this segment, again, is the slope of this line, which is -3 over 2, or -1.5 meters per second.

What is the average speed for the entire trip? The object moved a total distance of 3 plus 4 plus 3, or 10 meters, in a total time of twelve seconds, which comes to 0.83 meters per second. The average velocity for the entire trip is the total displacement over the total time, which is -4 meters over 12 seconds, which is -0.33 meters per second. Again, this is also the slope of the line connecting the starting point of the graph to the ending point.