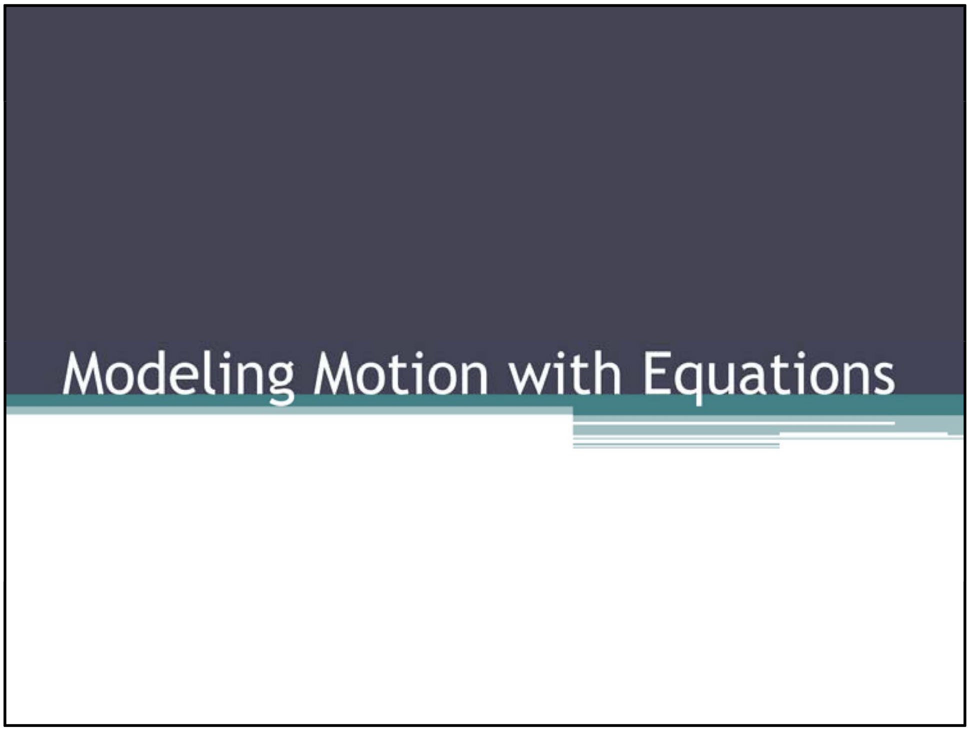
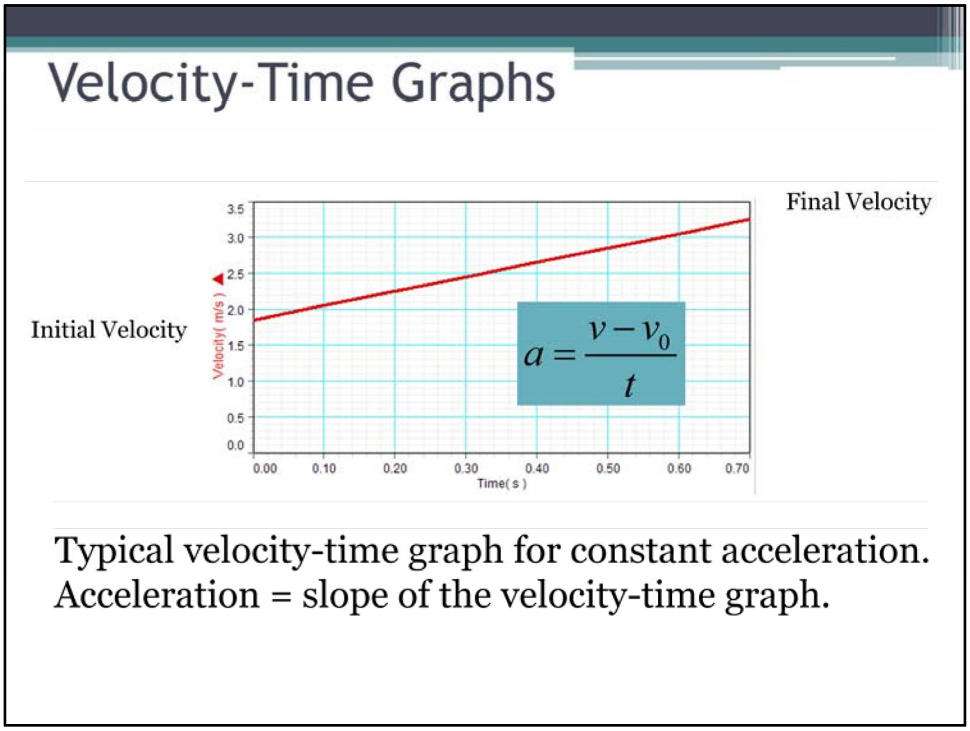


Modeling Motion with Equations



We know that constant acceleration means velocity changes at a steady rate. On a velocity-time graph, this steady change is represented by a steady slope. The slope of the velocity-time graph is the acceleration. Acceleration is equal to final velocity minus initial velocity divided by time.

Kinematics Equation #1
Finding v from v_0 , a, and t.

$a = \frac{v - v_0}{t}$	The definition of acceleration
$at = v - v_0$	Multiplying each side by t
$v = v_0 + at$	Rearranging

We want to be able to find velocity from initial velocity, acceleration and time.

First, we will do this algebraically, by rearranging the equation.

We start with the definition of acceleration using the slope of the velocity-time graph.

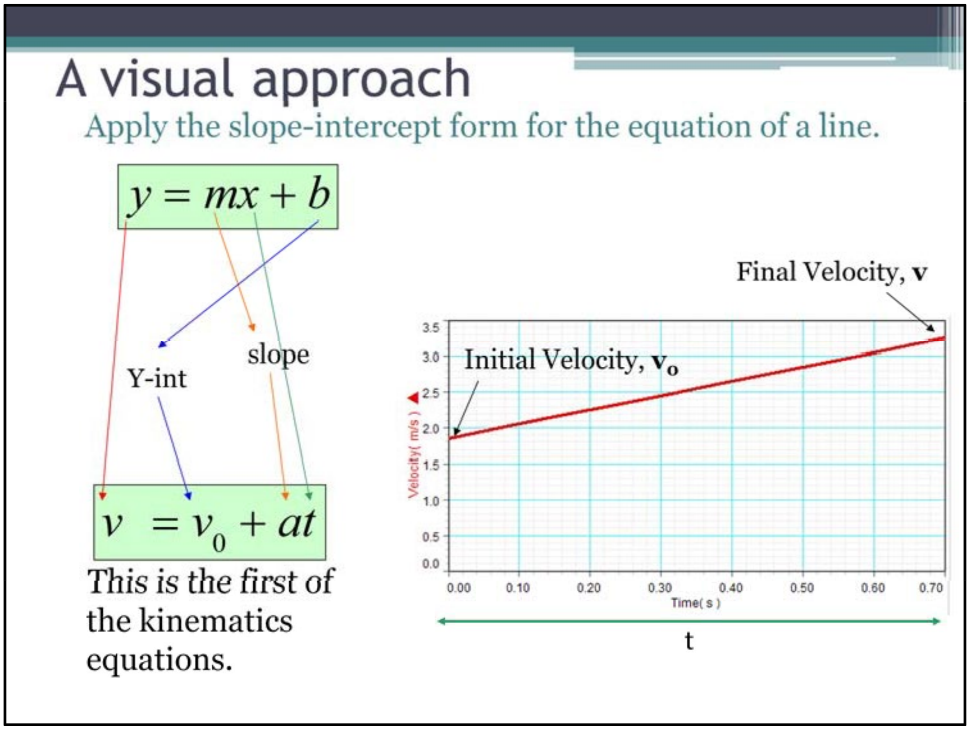
Slope is rise over run. The rise is the change in velocity, v minus v zero. The run is the time that passes, t.

Therefore, Acceleration is final velocity minus initial velocity divided by time.

We will put this equation in a more convenient form if we solve it for the velocity. To do this, we multiply each side by time, t.

Then we solve the equation for velocity by adding v zero to each side.

Our result is final velocity equals initial velocity plus acceleration times time. We will call this Kinematics Equation Number One.



We can also look at this equation visually.

First, we see that the graph is linear. We will use the slope-intercept form of the equation of a line to create our model.

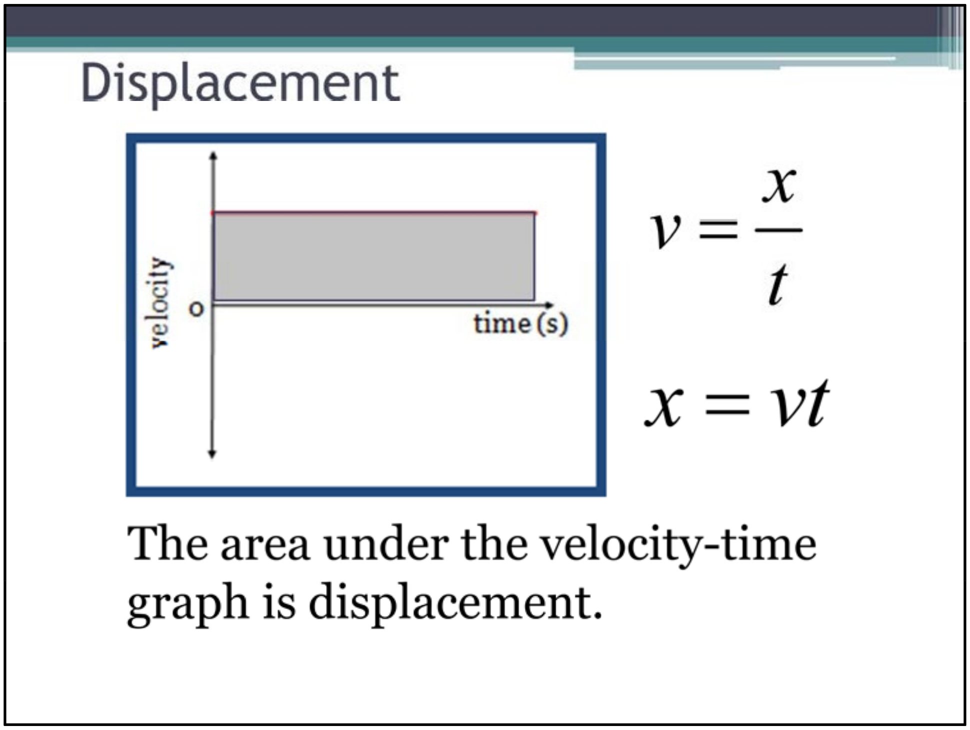
The slope-intercept form of a line is y equals m times x plus b.

Let's start at the beginning of this equation. The "y" is the y-axis variable, which is velocity.

The "m" is the slope. For this graph, slope is v minus v zero divided by t, which is acceleration, by definition.

The slope is multiplied by the x-axis variable, which is time.

B is the y-intercept, which on our graph, is the initial velocity, or the velocity when time is zero.



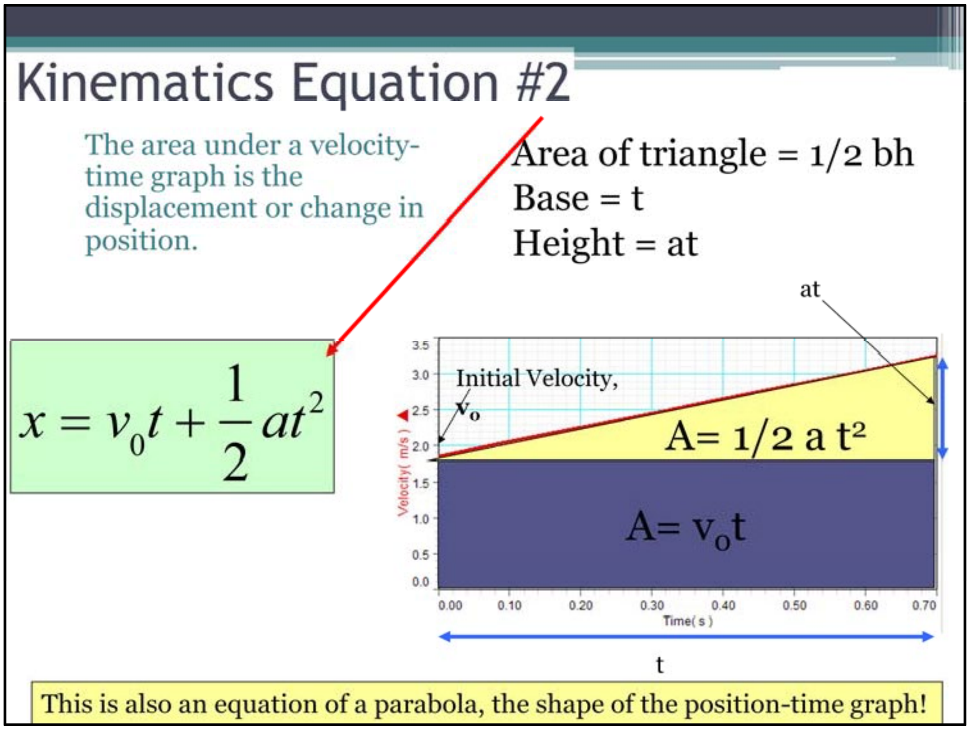
To understand this concept, lets consider a simple velocity-time graph. If velocity is constant, the velocity-time graph is a horizontal line.

We know that velocity is displacement divided by time.

Rearranging this equation, displacement is velocity multiplied by time. What quality of the graph does velocity multiplied by time represent?

If we look at the area under the line, it can be calculated by velocity multiplied by time.

So, displacement is the same as the area under the velocity-time graph. This concept applies even if the graph is more complicated.



How can we find the area if the velocity isn't constant?

We can find the displacement from the area under the velocity-time graph.

To find the area under the red line, we divide it into a rectangle and a triangle. The area of the rectangle is length times width, or v zero times t.

The area of the triangle is one-half base times height, or one-half t times a times t.

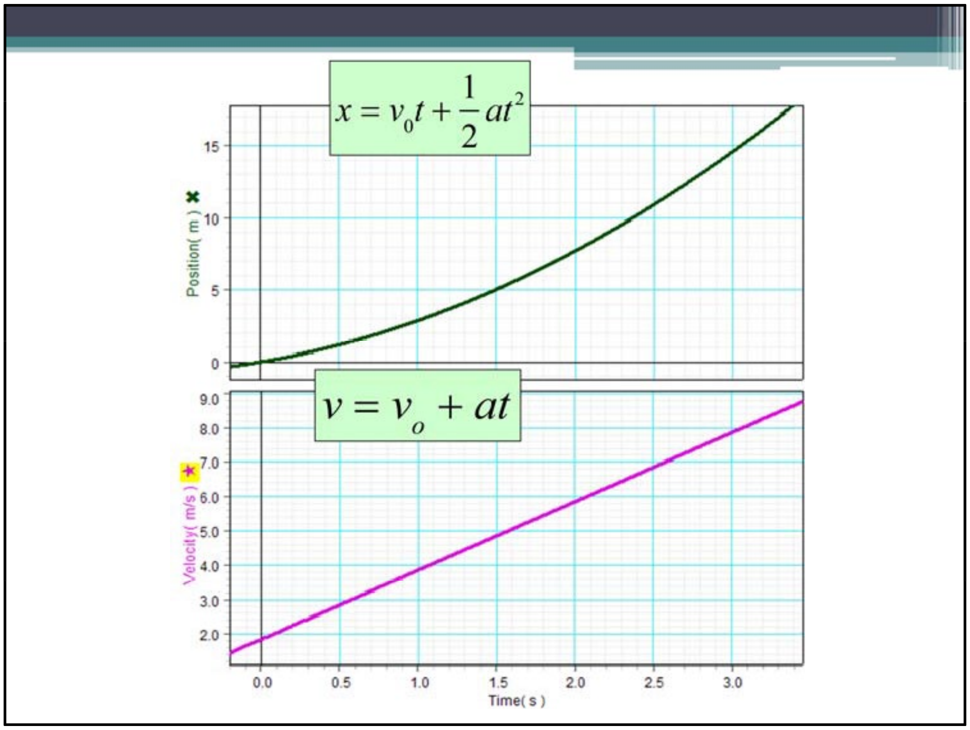
The height of the triangle is the difference between v and v zero, which is a,t from our previous equation.

Multiplying, we get one-half acceleration times time squared.

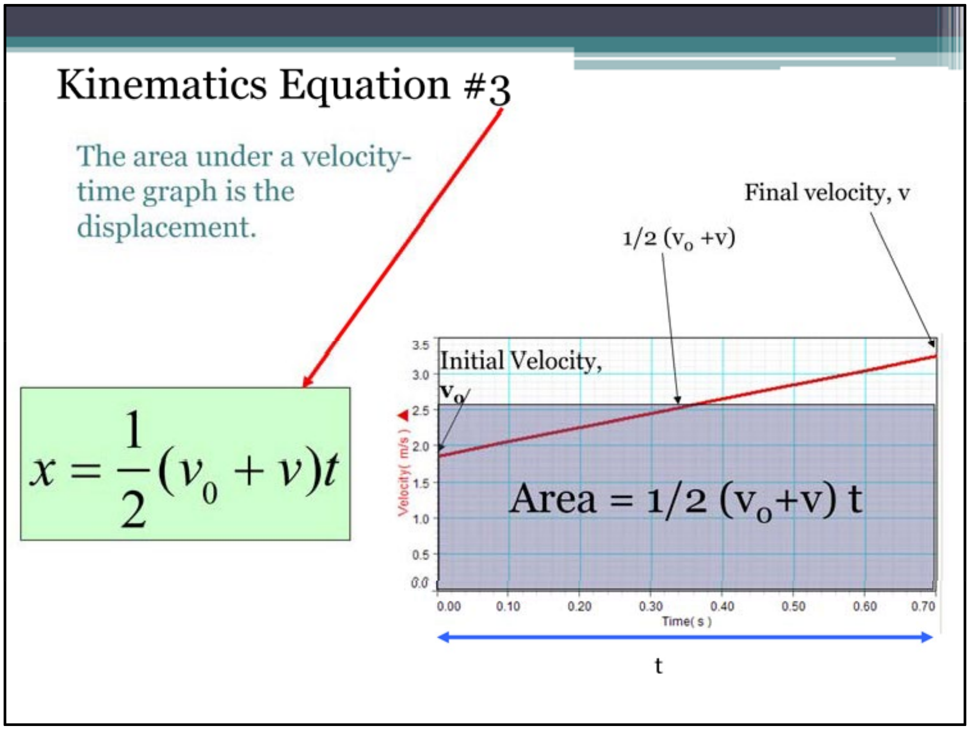
This gives the second kinematics equation, displacement equals initial velocity times t plus one-half a, t squared.

Notice that this is a second-order equation, since the time is squared. What should the shape of a second order polynomial be? If you said parabola, you are right!

You should have noticed in our previous experiments that accelerated motion created parabolic position-time graphs.



Note that parabolic position-time graphs represent the same motion as linear velocity-time graphs. The value of the velocity is the slope of the position-time graph at that point. As time goes on, the parabola gets steeper and the velocity increases.



We can also find the area under the red line by drawing a horizontal line at the midpoint of the red line and finding the area under that line.

The area of the blue rectangle is the same as the area found when we divided the area under the red line into a rectangle and a triangle.

The midpoint of the line is the average of final velocity and initial velocity. This is the height of the rectangle.

The length of the rectangle is the time.

The area of the rectangle is average velocity times time. This is

kinematics equation number three.

Kinematics Equation #4

Solve #1 for t $$t = \frac{v - v_0}{a}$$

Substitute into #3 $$x = \frac{1}{2}(v + v_0)t$$

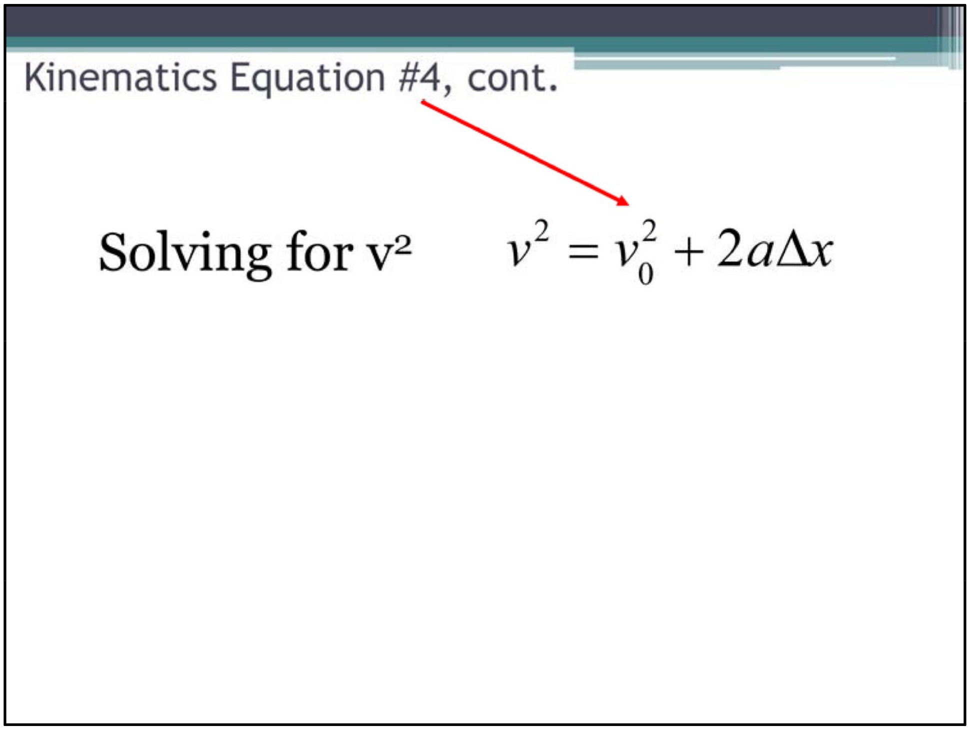
$$x = \frac{1}{2}(v + v_0)\frac{(v - v_0)}{a}$$

$$x = \frac{1}{2}\frac{(v^2 - {v_0}^2)}{a}$$

Sometimes we don't know the time for the motion. We can create an equation to use in this type of situation by combining two other equations.

First, we solve kinematics equation number one for time.

Then we will substitute it into the third kinematics equation. Then we need to do some algebra to put the equation in a nicer form.



The standard form for this equation is when it is solved for velocity squared.

The final velocity squared equals the initial velocity squared plus two times the acceleration times the displacement.

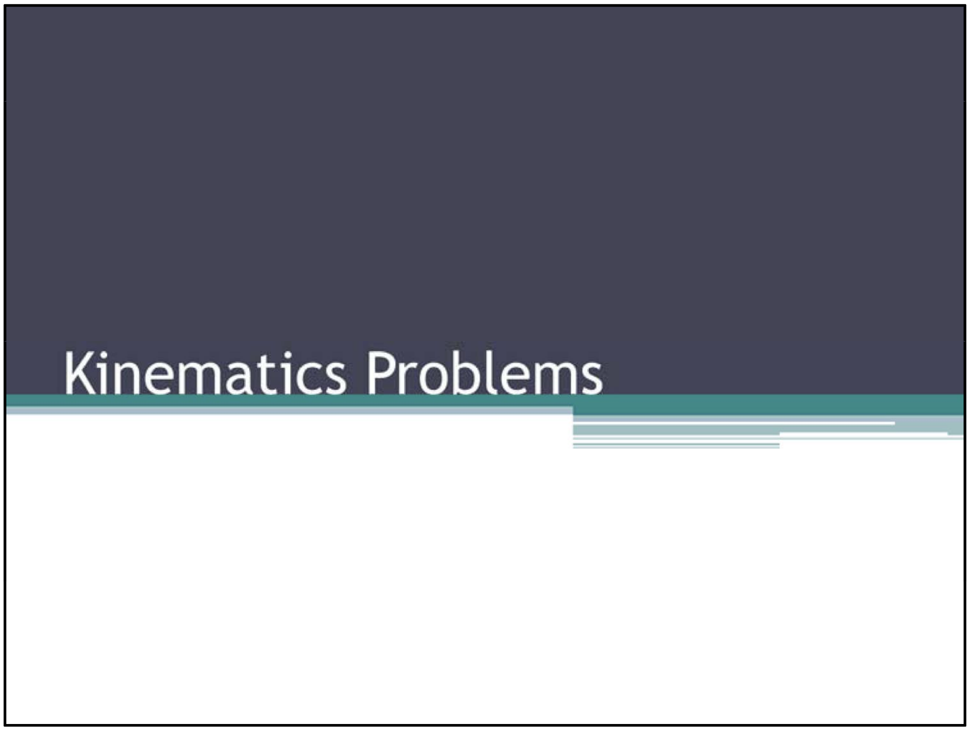
This is the fourth kinematics equation.

Equation Summary

Variables

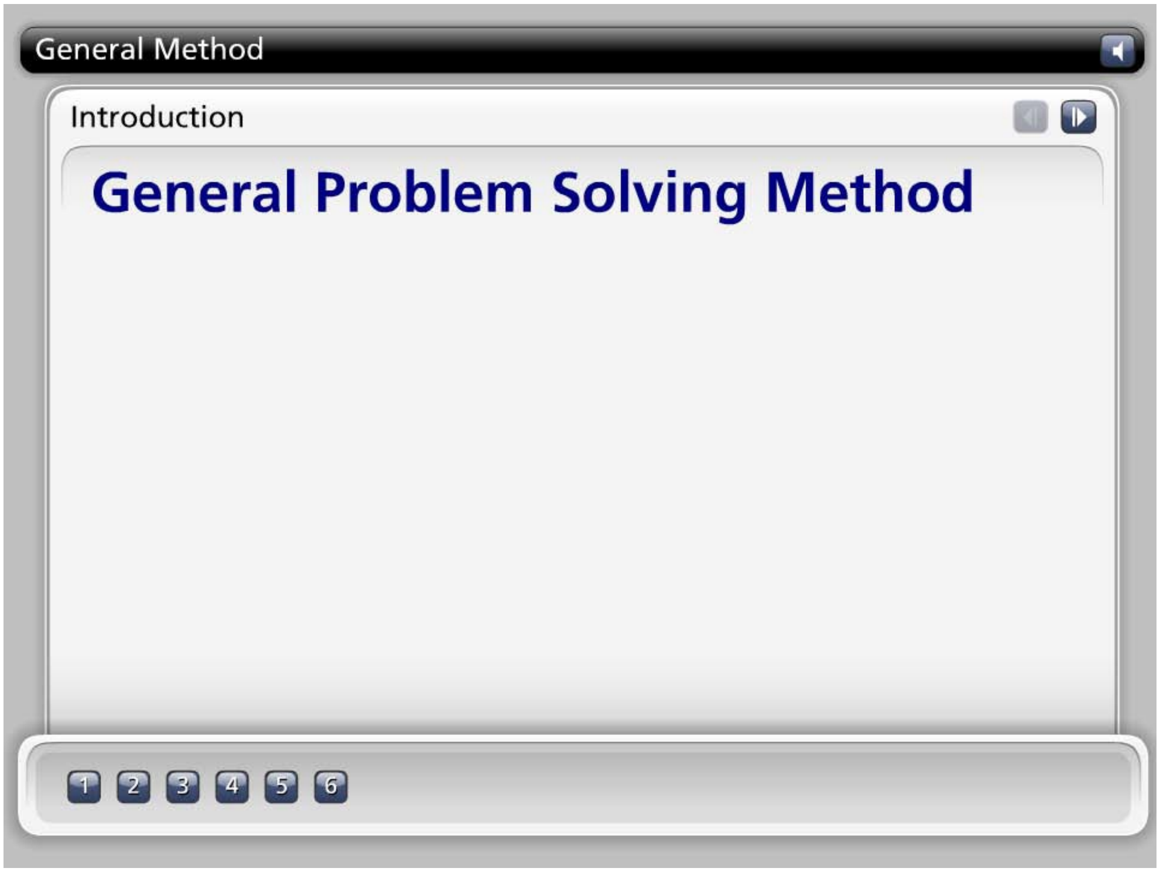
	Equation	x	a	v_0	v	t
1.	$v = v_0 + at$		✓	✓	✓	✓
2.	$x = v_0 t + \frac{1}{2}at^2$	✓	✓	✓		✓
3.	$x = \frac{1}{2}(v_0 + v)t$	✓		✓	✓	✓
4.	$v^2 = v_0^2 + 2ax$	✓	✓	✓	✓	

In summary, we notice that there are five kinematics variables: displacement, acceleration, initial velocity or velocity zero, final velocity and time.
If we look at each equation, we notice that each equation has four of the five kinematics variables.
The four equations each have a unique subset of the five variables.
When we solve problems, we select the equation by identifying the variables that are given in the problem and what is being asked for.
Then, we select the equation that has those four variables in it. In the next lesson we will learn problem solving skills.

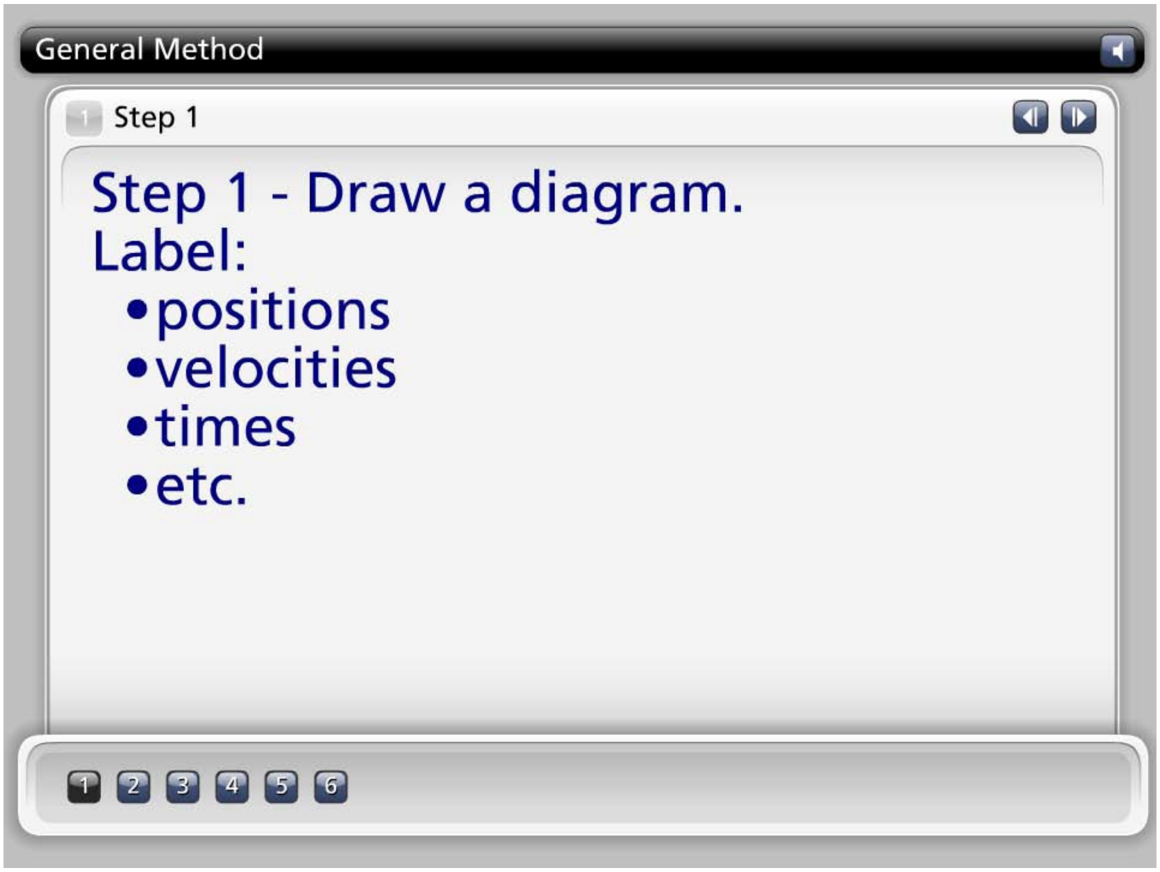


Kinematics Problems

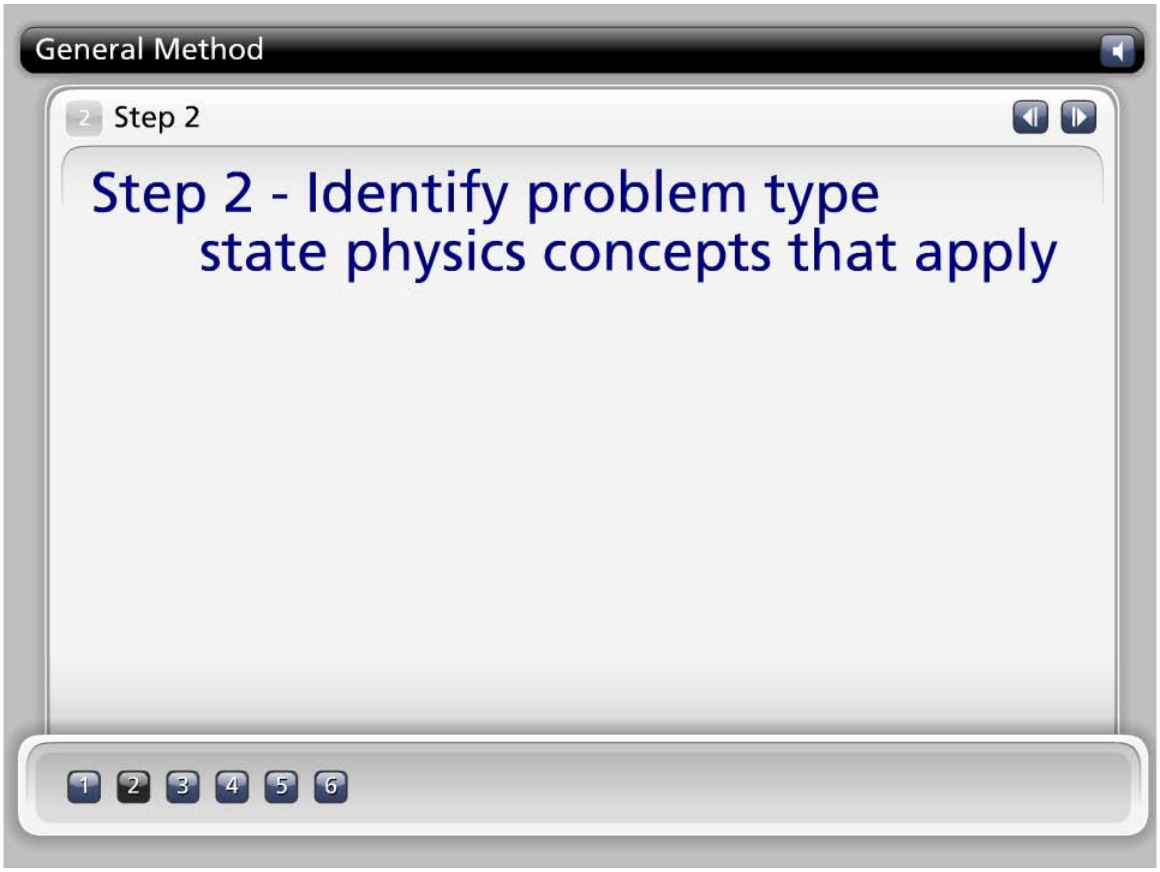
Module 1: Describing Motion
Topic 4 Content: Modeling Motion with Equations



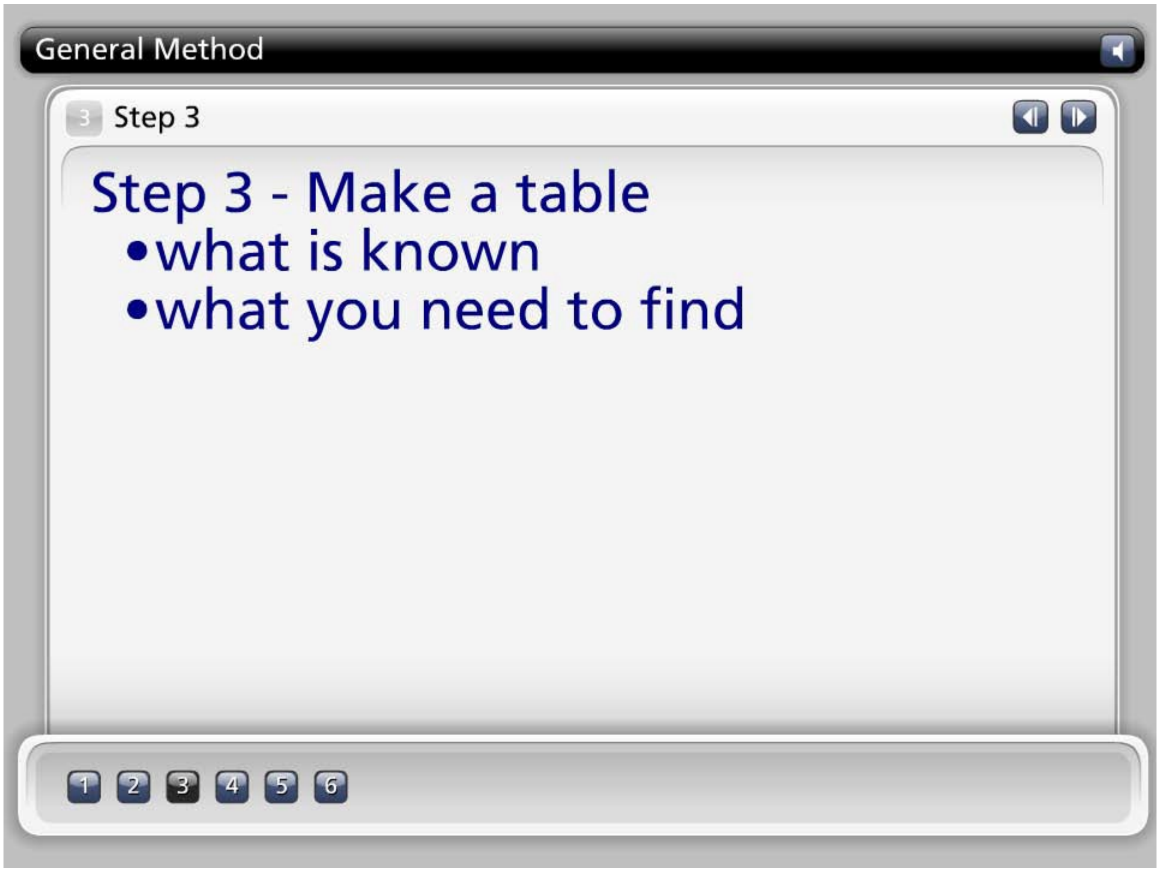
We will be solving problems throughout this course. In the beginning, the problems will be easier. As we progress through our studies of mechanics problems will become more complex. Here we will establish some basic problem solving procedures that will be used as a general method throughout the course.



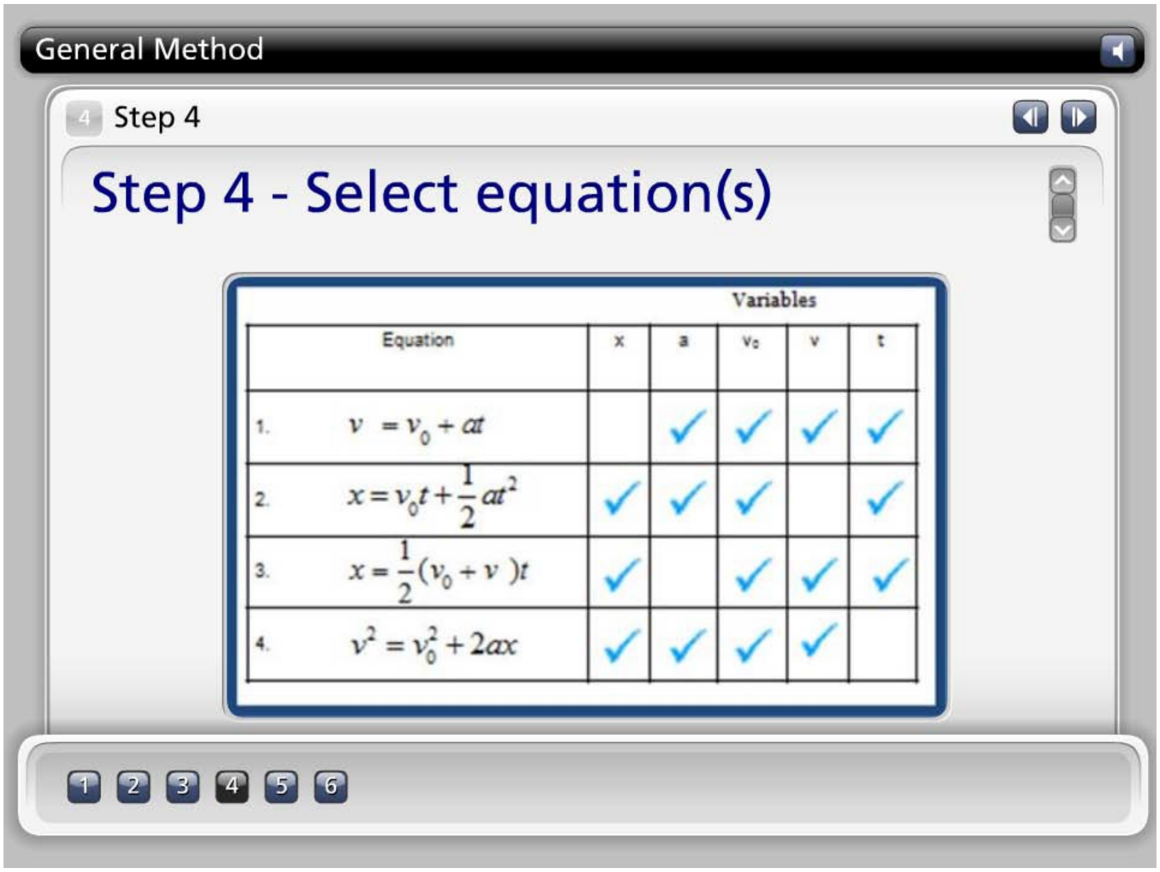
Step one is to draw a diagram. We will label positions, velocities, times and other important information.



Step two will be to identify the problem type. We will do this by stating what physics concepts apply to this situation.

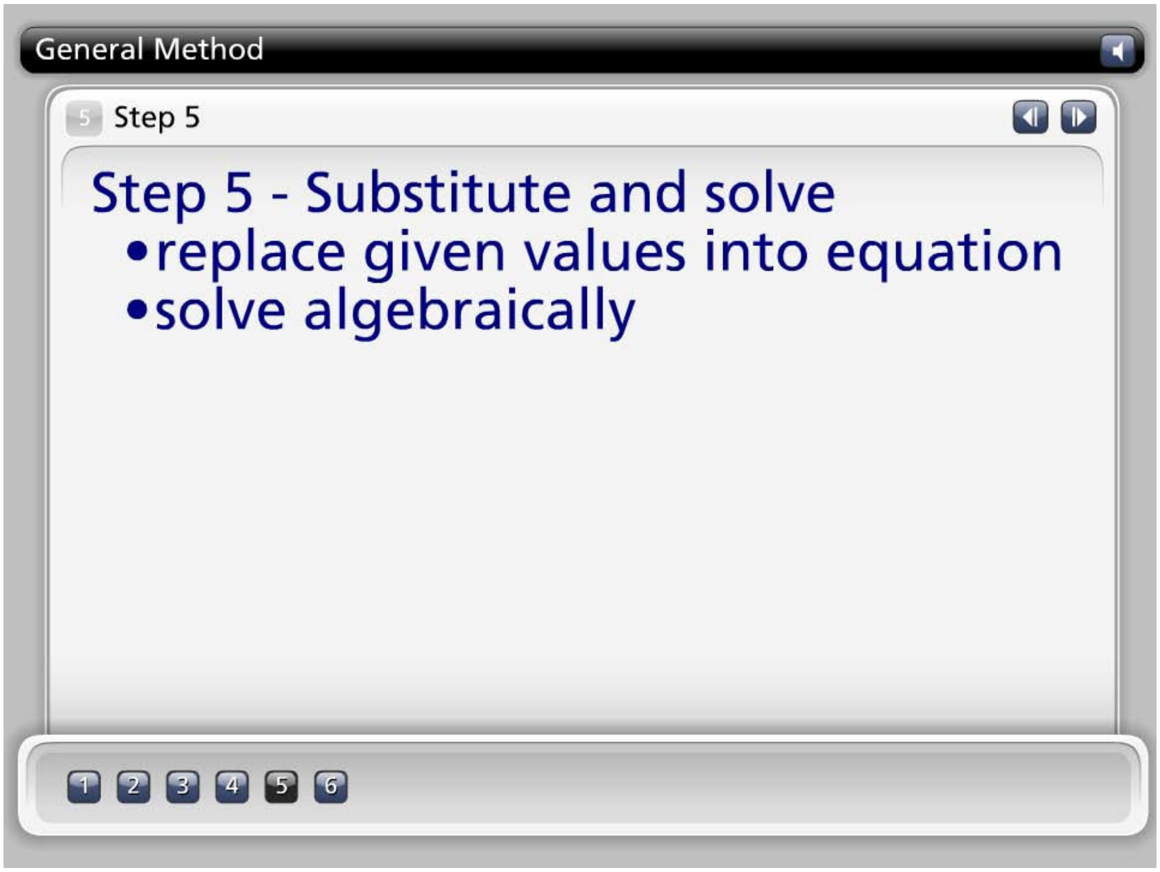


Step three is to make a table of information.

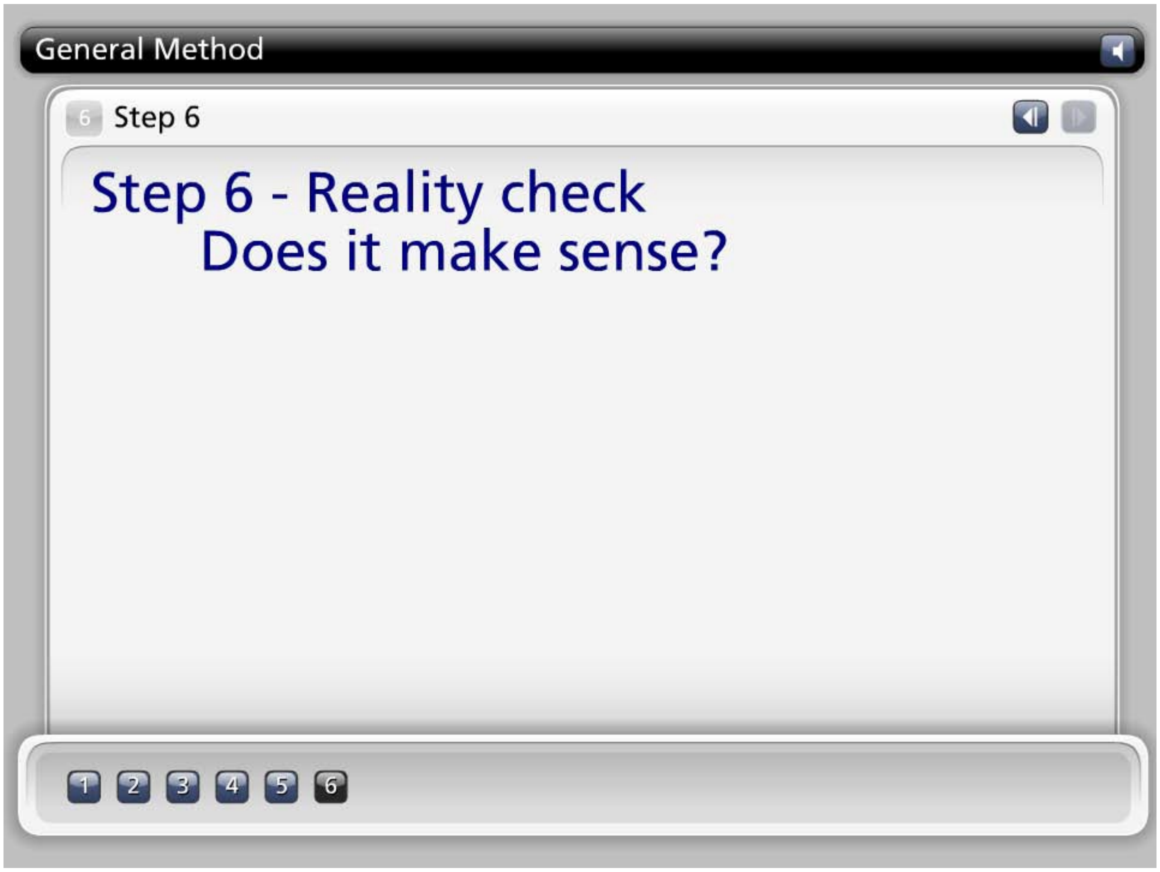


	Equation	x	a	v_0	v	t
1.	$v = v_0 + at$		✓	✓	✓	✓
2.	$x = v_0 t + \frac{1}{2}at^2$	✓	✓	✓		✓
3.	$x = \frac{1}{2}(v_0 + v)t$	✓		✓	✓	✓
4.	$v^2 = v_0^2 + 2ax$	✓	✓	✓	✓	

Step four is to select appropriate equations.



Step five is the substitute values into the equation and solve.



Step six is to check our answer to see if it makes sense.

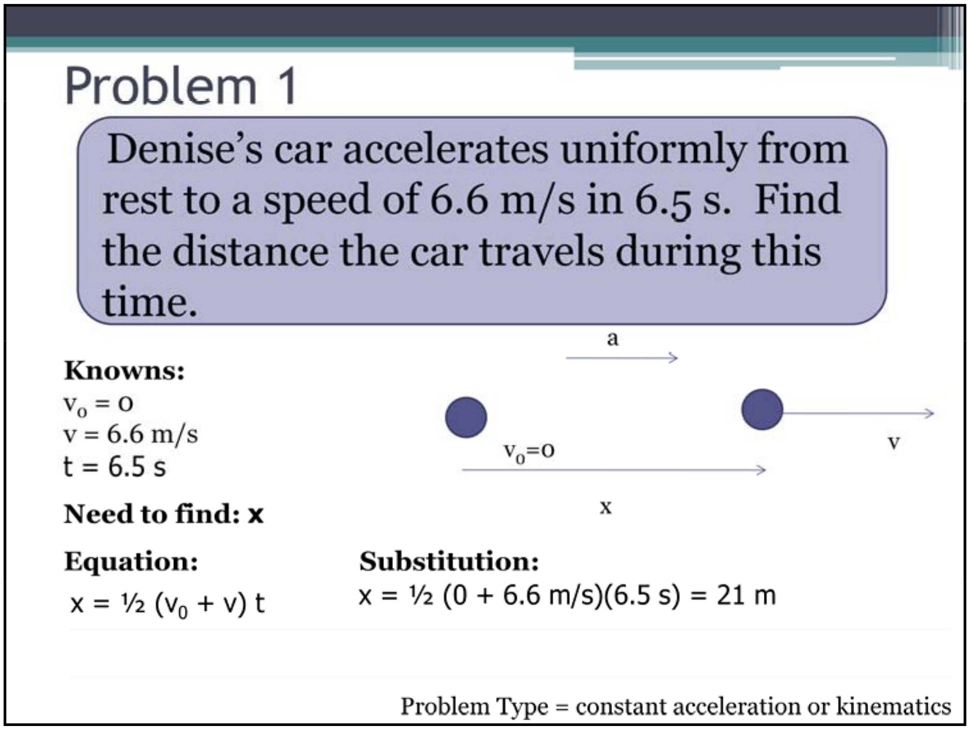
Equation Summary

Variables

Equation	x	a	v_0	v	t
1. $v = v_0 + at$		✓	✓	✓	✓
2. $x = v_0 t + \frac{1}{2}at^2$	✓	✓	✓		✓
3. $x = \frac{1}{2}(v_0 + v)t$	✓		✓	✓	✓
4. $v^2 = v_0^2 + 2ax$	✓	✓	✓	✓	

Here is the list of four equations we developed in the last lesson.

The four equations each have a unique subset of the five variables.
For example, the first equation excludes displacement but includes the other four variables acceleration, initial velocity, final velocity and time.
When we solve problems, we select the equation by identifying the variables that are given in the problem and what is being asked for.
Then, we select the equation that has those four variables in it. In this lesson we will learn problem solving skills.



Denise's car accelerates uniformly from rest to a speed of 6.6 m/s in 6.5 s. Find the distance the car travels during this time.

First, step one, draw a diagram. We will represent the car as a blue dot on our diagram for simplicity. Let's label what we know. The car starts from rest and then accelerates. The initial velocity is zero. The acceleration and the final velocity are in the same direction. The displacement is in the same direction as the velocity and the acceleration.

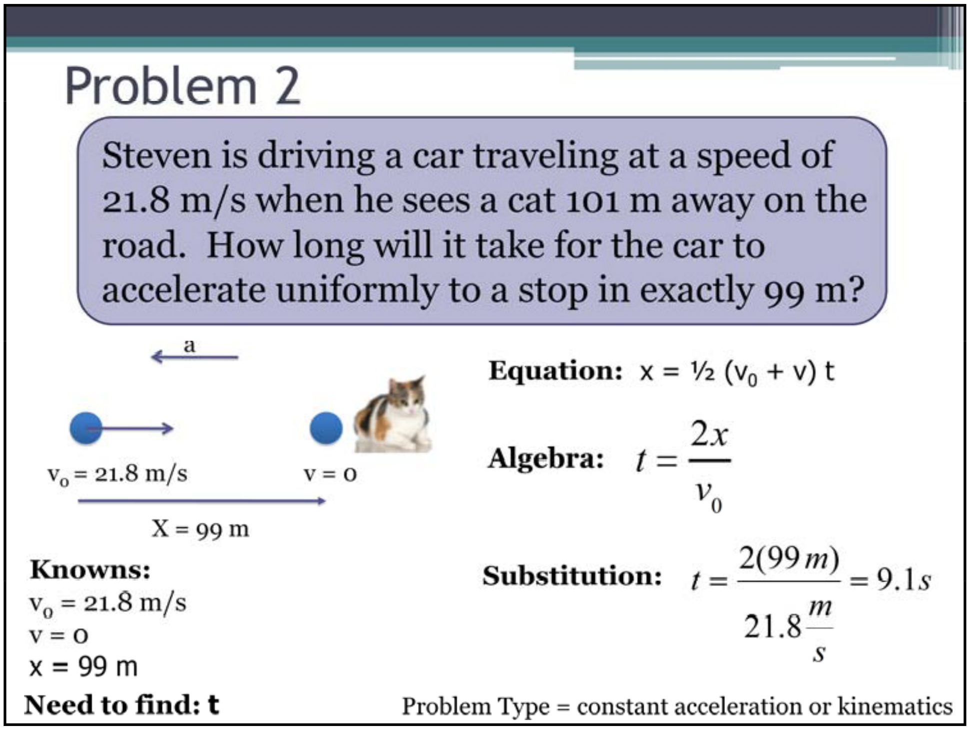
Next, step two, identify problem type. The key phrase in this problem is accelerates uniformly. This tell us the type of problem we are working on. This problem involves uniform acceleration and we can apply the four kinematics equations.

Step three is to make a table of values. Here we list all the knowns as well as what we are being asked to determine. In this problem, we know initial velocity, final velocity and time. We need to find displacement.

Step four is to select an equation. We already determined that this is a uniform acceleration problem and that we can use the four kinematics equations. Now we need to decide which one. We need the equation that has initial velocity, final velocity, time and distance in it. We

Step five is to substitute values into the equation and solve. Initial velocity is zero, final velocity is six point six meters per second and time is six point five seconds. We determine that the answer is 21 m.

Step six is to think about if our answer is reasonable. We calculated a positive displacement. Does this make sense? Our original diagram showed the velocity and acceleration both in the positive direction, so the positive displacement is reasonable.



Steven is driving a car traveling at a speed of 21.8 meters per second when he sees a cat 101 meters away on the road. How long will it take for the car to accelerate uniformly to a stop in exactly 99 meters?

First, step one, draw a diagram. We will represent the car as a blue dot on our diagram for simplicity. Let's label what we know. The car starts from an initial velocity of twenty one point eight meters per second and then slows down. The final velocity is zero. The acceleration and the initial velocity are in the opposite direction. The displacement is in the same direction as the velocity but in the opposite direction of the acceleration.

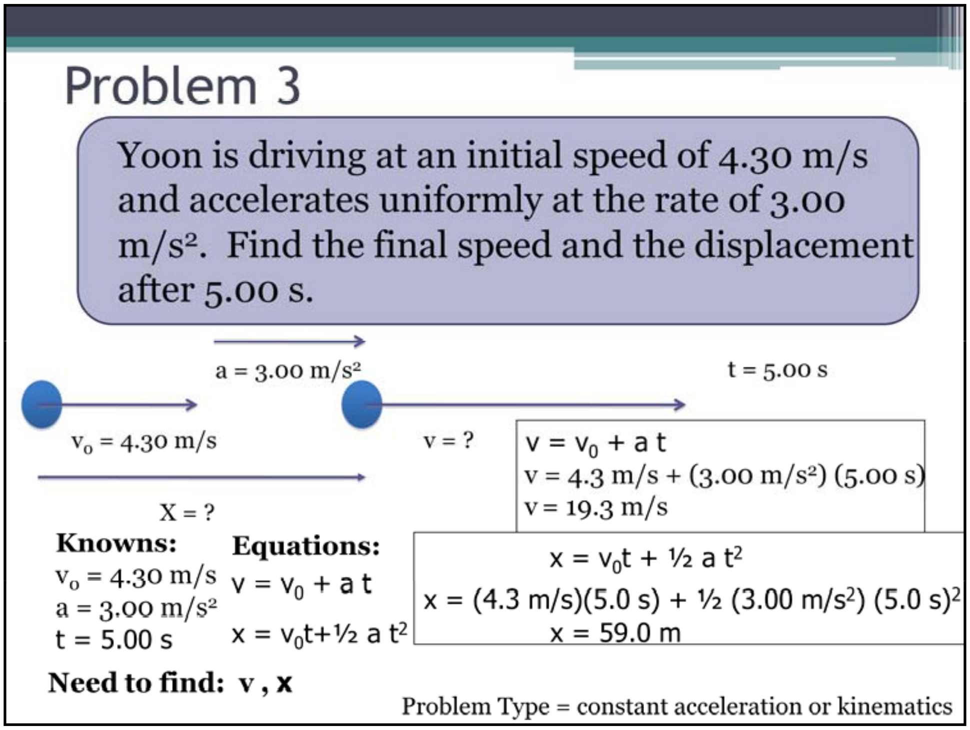
Next, step two, identify problem type. The key phrase in this problem is slows uniformly. This tell us the type of problem we are working on. This problem involves uniform acceleration and we can apply the four kinematics equations.

Step three is to make a table of values. Here we list all the knowns as well as what we are being asked to determine. In this problem, we know initial velocity, final velocity and displacement. We need to find time.

Step four is to select an equation. We already determined that this is a uniform acceleration problem and that we can use the four kinematics equations. Now we need to decide which one. We need the equation that has initial velocity, final velocity, time and displacement in it. We do not want the equation to have acceleration in it. If we look at the list of equations, we can see that only one of them meets these requirements. The equation we will use is displacement equals one half of the quantity initial velocity plus final velocity multiplied by time.

Step five is to substitute values into the equation and solve. Final velocity is zero, initial velocity is twenty one point eight meters per second and displacement is ninety-nine meters. We determine that the answer is nine point one seconds.

Step six is to think about if our answer is reasonable. We calculated a time of about nine seconds for a car to stop. This seems like a reasonable value. If we had gotten an answer that was a very long time then it would not be reasonable.



Yoon is driving at an initial speed of four point three meters per second and accelerates uniformly at the rate of three point zero zero meters per second squared. Find the final speed and the displacement after five point zer zero seconds.

First, step one, draw a diagram. We will represent the car as a blue dot on our diagram for simplicity. Let's label what we know. The car starts from an initial velocity of four point three meters per second and then speeds up. The final velocity is unknown, but since his acceleration is in the same direction as his velocity he should be speeding up. The acceleration and the initial velocity are in the same direction. The displacement is in the same direction as the velocity and the acceleration. The time interval is five seconds.

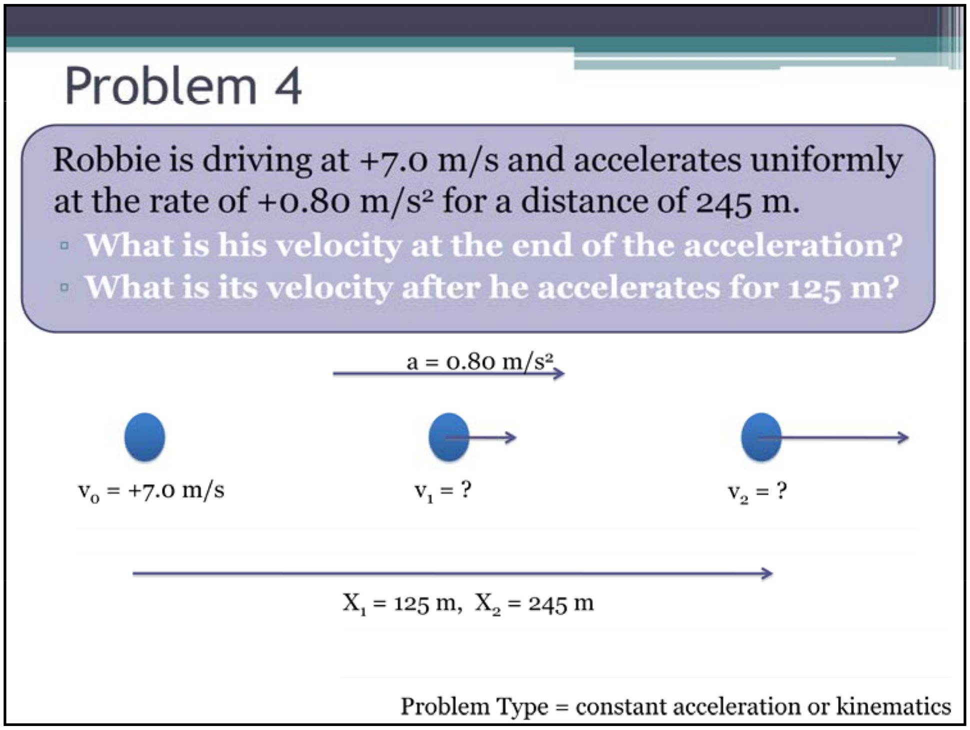
Next, step two, identify problem type. The key phrase in this problem is accelerates uniformly. This tell us the type of problem we are working on. This problem involves uniform acceleration and we can apply the four kinematics equations.

Step three is to make a table of values. Here we list all the knowns as well as what we are being asked to determine. In this problem, we know initial velocity, acceleration and time. We need to find final velocity and displacement.

Step four is to select an equation. We will find final velocity first. We already determined that this is a uniform acceleration problem and that we can use the four kinematics equations. Now we need to decide which one. We need the equation that has initial velocity, acceleration, time and final velocity in it. We do not want the equation to have displacement in it. If we look at the list of equations, we can see that only one of them meets these requirements. The equation we will use is velocity equals velocity zero plus acceleration times time. Then we will want to find displacement. Since we know four of the five kinematics variables now, we could use more than one of the equations. We can use the equation displacement equals velocity zero times time plus one-half acceleration times time squared.

Step five is to substitute values into the equation and solve. To find velocity we substitute: inital velocity is four point three meters per second, acceleration is three point zero zero meters per second squared and time is five point zero zero seconds. To find displacement we substitute the same values into the equation for displacement. We calculate a velocity of nineteen point three meters per second and a displacement of fifty nine point zero meters.

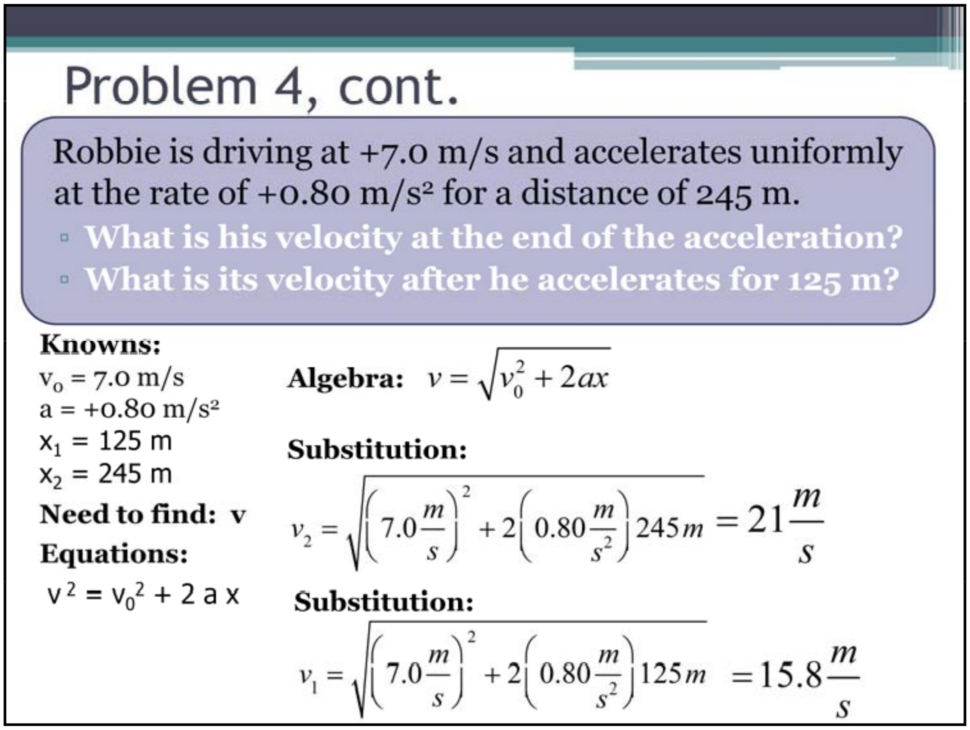
Step six is to think about if our answer is reasonable. Our final velocity is positive nineteen point three, which is bigger than the initial velocity. This makes sense because Yoon is speeding up. The displacement is positive which makes sense because Yoon moves in the positive direction the entire time.



Robbie is driving at positive seven point zero meters per second and accelerates uniformly at the rate of positive zero point eight zero meters per second squared for a distance of two hundred and forty five meters. We want to find his velocity at two different displacements, one hundred twenty five meters and two hundred forty five meters.

First, step one, draw a diagram. Since we have two different displacements we will label the three positions as zero for the initial position, one for the first displacement and two for the second displacement. We will represent the car as a blue dot on our diagram for simplicity. Let's label what we know. The car starts from an initial velocity of seven point zero meters per second and then speeds up. The velocities at position one and two are unknown, but since his acceleration is in the same direction as his velocity he should be speeding up and velocity should be getting bigger. The acceleration and the initial velocity are in the same direction. The displacement is in the same direction as the velocity and the acceleration. The acceleration is given as zero point eight zero meters per second squared. The two displacements of interest are one hundred twenty five and two hundred and forty five meters. The time interval is unknown.

Next, step two, identify problem type. The key phrase in this problem is accelerates uniformly. This tell us the type of problem we are working on. This problem involves uniform acceleration and we can apply the four kinematics equations.



Step three is to make a table of values. Here we list all the knowns as well as what we are being asked to determine. In this problem, we know initial velocity, acceleration and displacement. We need to find final velocity.

Step four is to select an equation. We will find final velocity first. We already determined that this is a uniform acceleration problem and that we can use the four kinematics equations. Now we need to decide which one. We need the equation that has initial velocity, acceleration, displacement and final velocity in it. We do not want the equation to have time in it. If we look at the list of equations, we can see that only one of them meets these requirements. The equation we will use is final velocity squared equals initial velocity squared plus two times acceleration times displacement.

Step five is to substitute values into the equation and solve. To find velocity we substitute: initial velocity is seven point zero meters per second, acceleration is zero point eight zero meters per second squared and displacement is two hundred forty five meters. We calculate a velocity at position two of twenty one meters per second.

Step six is to think about if our answer is reasonable. Our final velocity is positive twenty one, which is bigger than the initial velocity. This makes sense because Robbie is speeding up.

We repeat step five for position one.

Step five is to substitute values into the equation and solve. To find velocity we substitute: inital velocity is seven point zero meters per second, acceleration is zero point eight zero meters per second squared and displacement is one hundred twenty five. We calculate a velocity at position two of twenty one meters per second.

Step six is to think about if our answer is reasonable. Our final velocity fifteen point eight meters per second, which is bigger than the initial velocity but smaller than the velocity at position two. This makes sense because Robbie is speeding up the entire time.