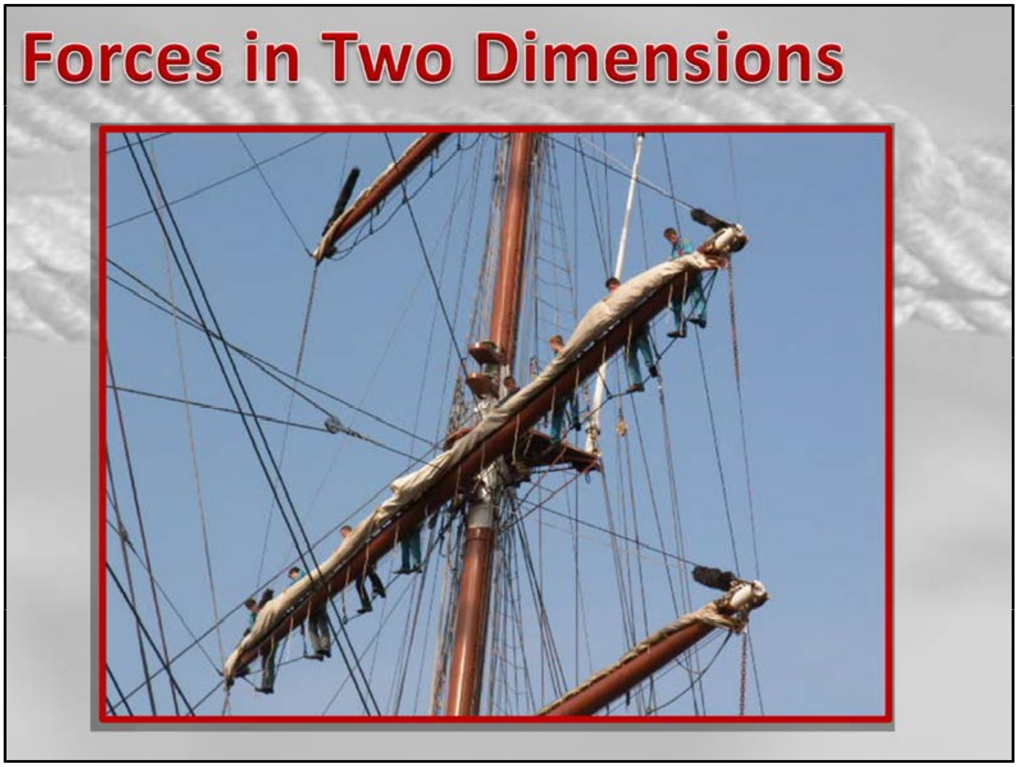
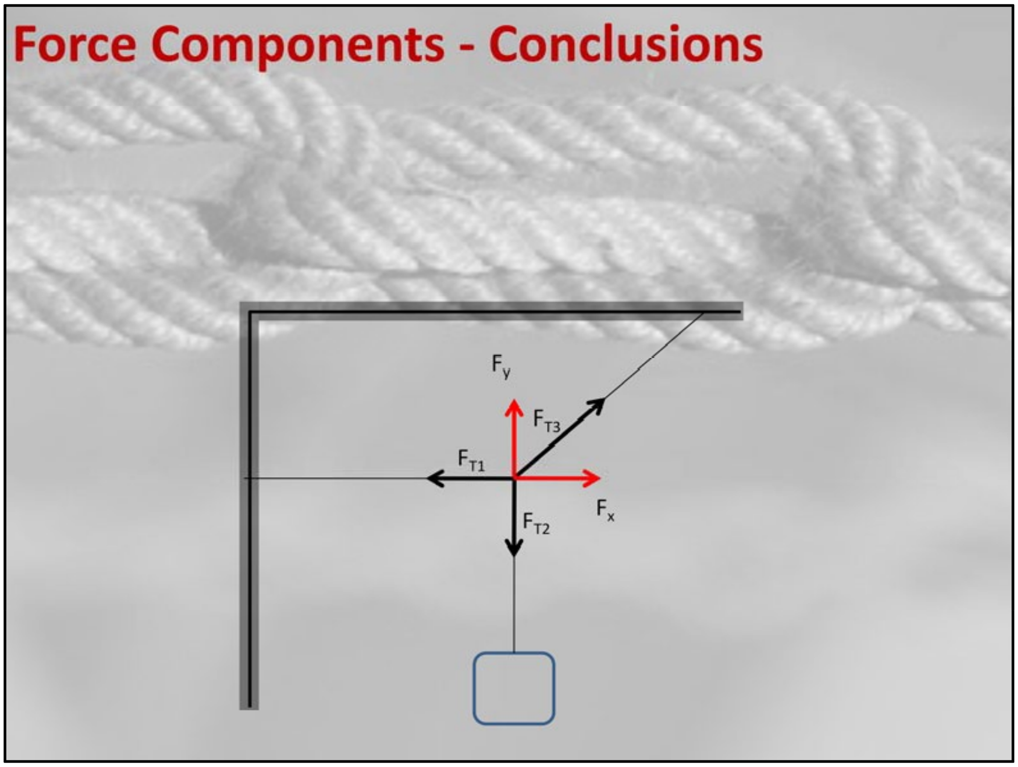
Module 2: Forces and Newton's Laws

Topic 4 Content: Forces in Two Dimensions Presentation Notes



Until now, all the forces, motion and acceleration we've considered have been in one dimension, either horizontal or vertical. When we combined horizontal and vertical forces, one dimension has always canceled out, leaving the net force either horizontal or vertical, so our analysis was reasonably straightforward. But what happens when forces are applied at an angle or neither dimension cancels out?



We will use our understanding of Newton's first and second laws to draw some conclusions about forces.

Let's look at a situation where a mass hangs from a cable that is attached to two others. One connects horizontally to the wall and the other connects at an angle to the ceiling.

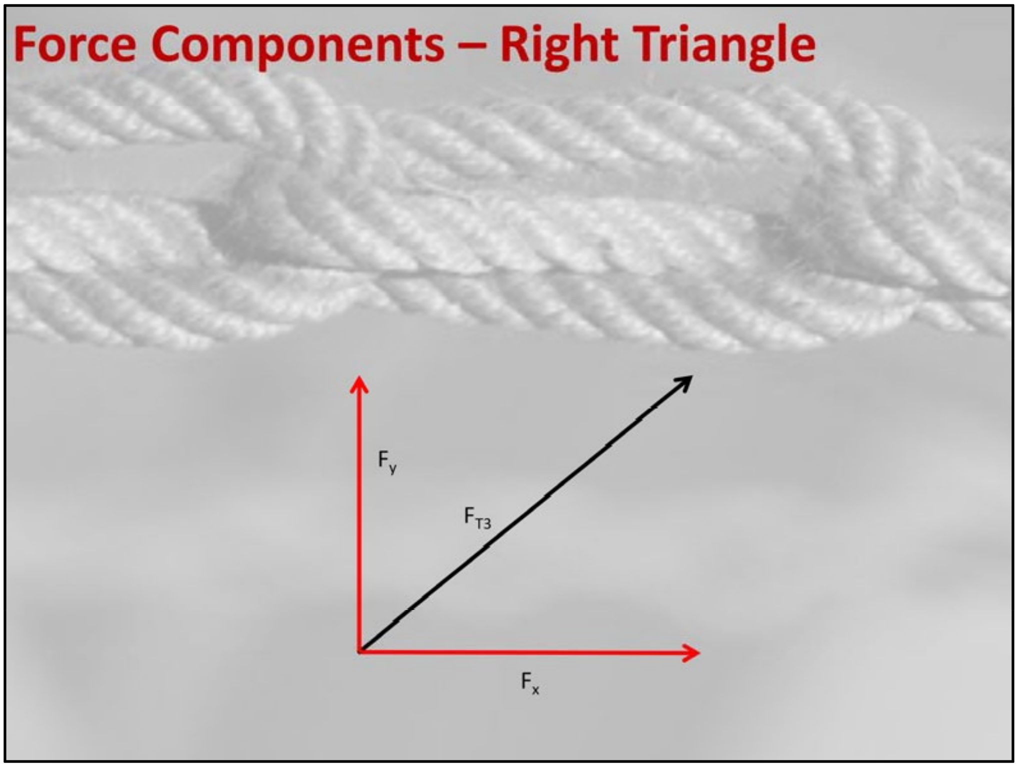
Since the system is at equilibrium, we know that the forces must be balanced. We'll look at the balanced forces on the point where the three cables meet.

There is a tension pulling to the left, a tension pulling down and a tension pulling at an angle up and to the right.

Conveniently, the phrase "up and to the right" reflects correctly that this third tension is actually pulling both up and to the right.

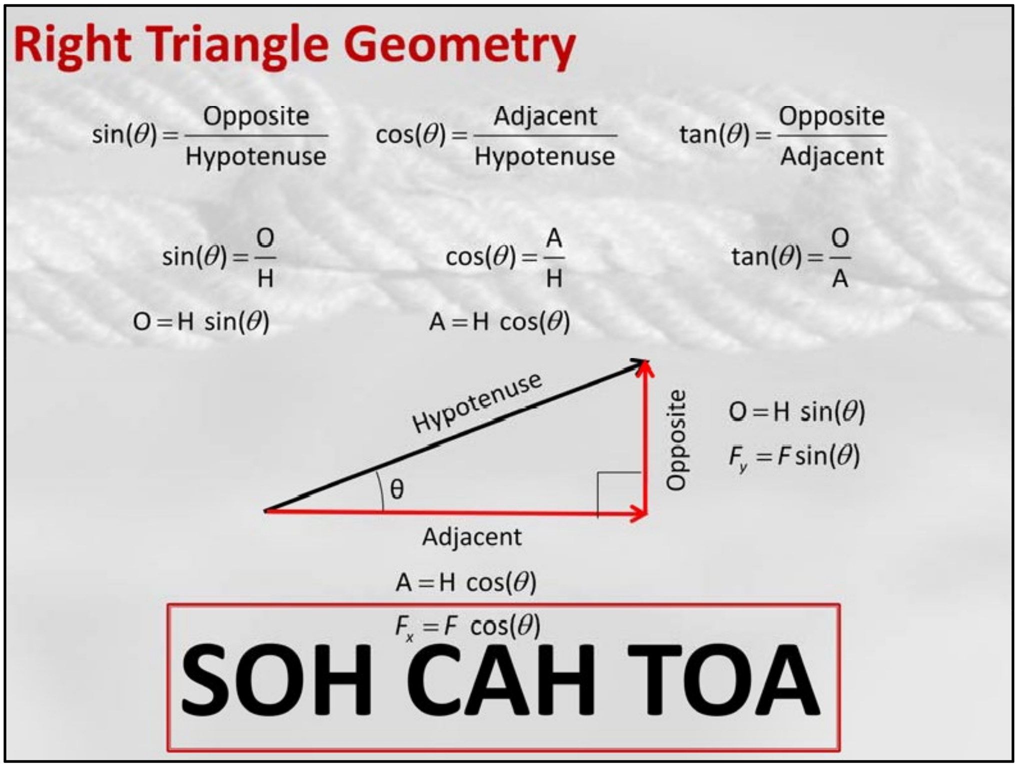
Somehow the third tension force must be pulling to the right with a magnitude equal to F T one so that horizontal forces are balanced and pulling up with a force equal in magnitude to F T two so that vertical forces balance.

We will refer to these forces as F x and F y to represent how much the third tension is pulling in the x and y directions respectively. F y is called the y-component or the vertical component of the force, and F x is called the x-component or horizontal component of the force. We call these x and y referring to our normal placement of the x and y axes.



Let's take a closer look at only this force and its components. These can be rearranged by sliding the y-component to the right to form a right triangle.

Now that we have a right triangle, we can see that right triangle geometry can show the relationships between the force and its x and y components.



When we look at a triangle made up of a force and its horizontal and vertical components, we're looking at a right triangle. It turns out that this analysis will work for any vector quantity, but we're going to begin by analyzing forces.

For any right triangle, you've got a hypotenuse and two legs. If you pick one of the interior angles, other than the right angle, you will see that there is one leg that touches that angle and one leg that is opposite that angle. The symbol we use for an angle in physics is typically the Greek letter theta.

Regardless of the overall size of the triangle, there are certain ratios that will always be the same if the angle is specified.

The sine of the angle is defined as the ratio of the length of the opposite side divided by the length of the hypotenuse. The cosine of the angle is defined as the ratio of the length of the adjacent side divided by the length of the hypotenuse. And the tangent of the angle is defined as the ratio of the opposite side to the adjacent side.

To remember these ratios, we use the phrase sohcahtoa *[pronounced so ka toe ah]*

Now, if we know the hypotenuse and the angle theta, how do we determine the magnitude of the opposite side?

Since the sine of theta equals opposite over hypotenuse, we can multiply both sides by the hypotenuse. The result is the opposite side is equal to the hypotenuse times the sine of theta.

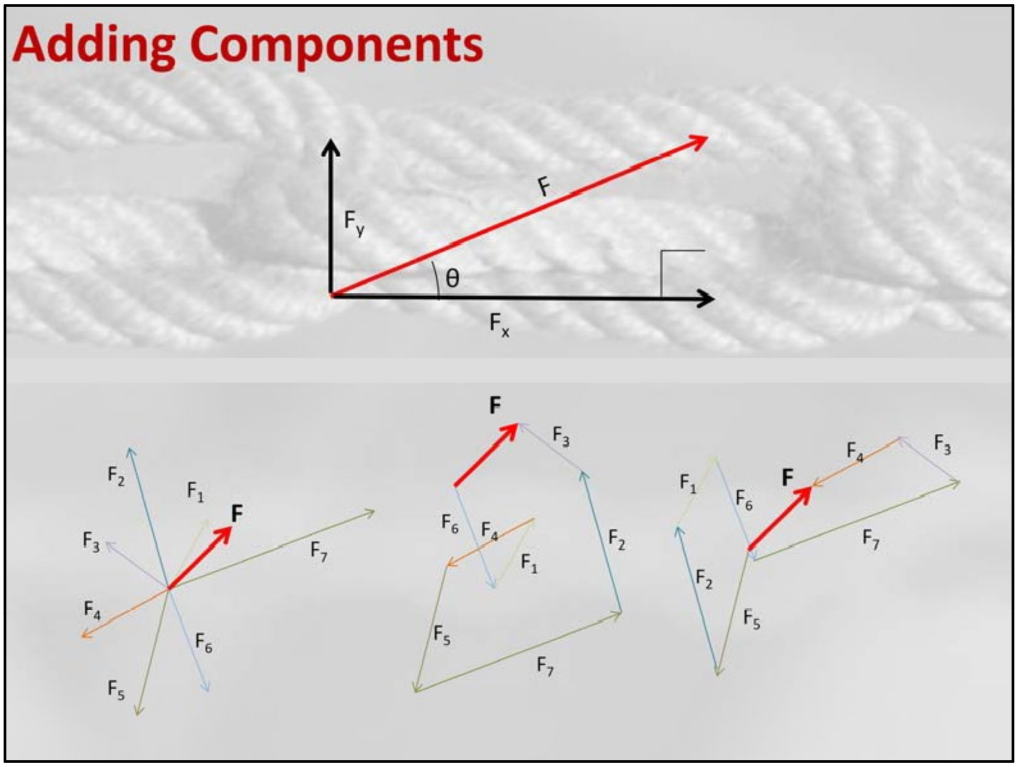
Similarly, to determine the magnitude of the adjacent side, we can rearrange the cosine relationship and see that the adjacent side is equal to the hypotenuse times the cosine of theta.

But we started this discussion with force vectors. If we know the magnitude of any force and the angle that it makes with the horizontal, we can determine the x- and y-components of that force using these relationships. The hypotenuse is the force, the opposite is the y-component and the adjacent is the x-component.

So we get that the y-component of the force is equal to the force times the sine of the angle and the x-component of the force is equal to the force times the cosine of the angle.

This same analysis will work if we are looking at angled velocity vectors, displacement vectors, or any vector quantity.

Just be careful that the angle theta is always measured from the horizontal axis. If you choose the angle from the vertical axis, the sine and cosine functions switch, so you'll be solving for the other component. Also, you must have your calculator in degrees, not radians, as the angles will be measured in degrees.



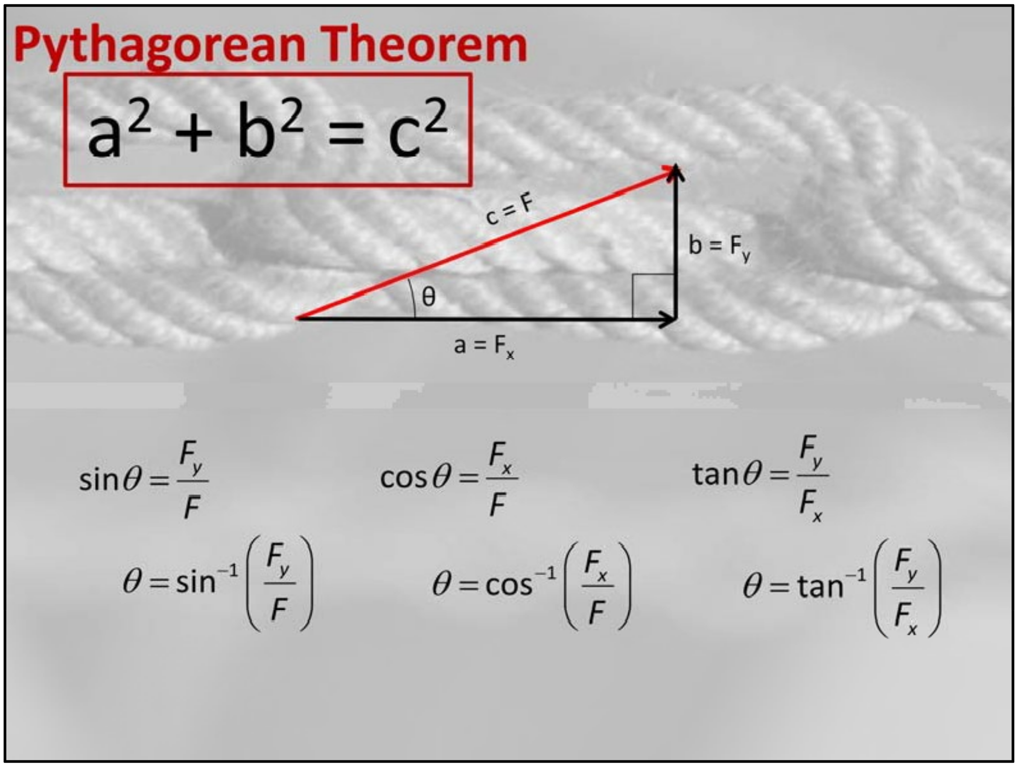
In some situations, instead of splitting a vector into components, you will need to combine horizontal and vertical components, F x and F y into a resulting force.

You should see that if we had the forces F x to the right and F y up as shown, the resulting force, F, would be up and to the right.

To determine the magnitude of this force, we need to again visit our right triangle.

If we slide the vertical component to the right, we can once again make a right triangle. This is referred to as the "head to tail" method of vector addition.
If you line up any two vectors or any series of vectors from head to tail, the resulting path from the beginning of the first vector to the end of the last vector will be the vector that results from the addition of all the components.

The order of the addition does not matter.



Now we can make use of another characteristic of right triangles – the Pythagorean theorem.

The Pythagorean theorem states that the squares of the sides of a right triangle will equal the square of the hypotenuse. The sides are labeled as a and b and the hypotenuse as c so we get a squared plus b squared equals c squared. We can use this to calculate the magnitude of the resulting force given the vertical and horizontal components.

If we want to determine the angle of the resultant net force, we have to revisit our trig functions.

Remember that the sine of the angle is equal to the opposite divided by the hypotenuse. In this case, sine theta equals F y over F.

In order to solve for the angle we need what is called an inverse function. We want the angle whose sine is equal to F y over F. If we calculate the inverse sine of F y over F, we will get theta. You should see the inverse sine or sine to the negative one button just above the sine button on most calculators.

The same applies to the cosine and the tangent functions, in case you need to determine the angle with different unknowns.