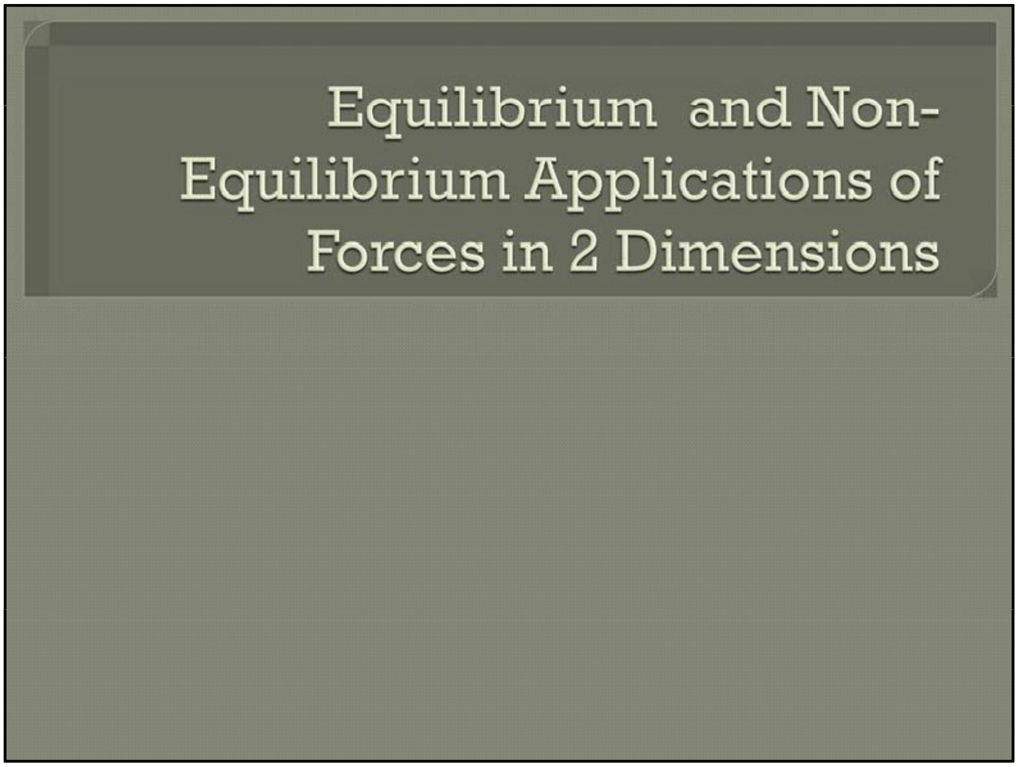
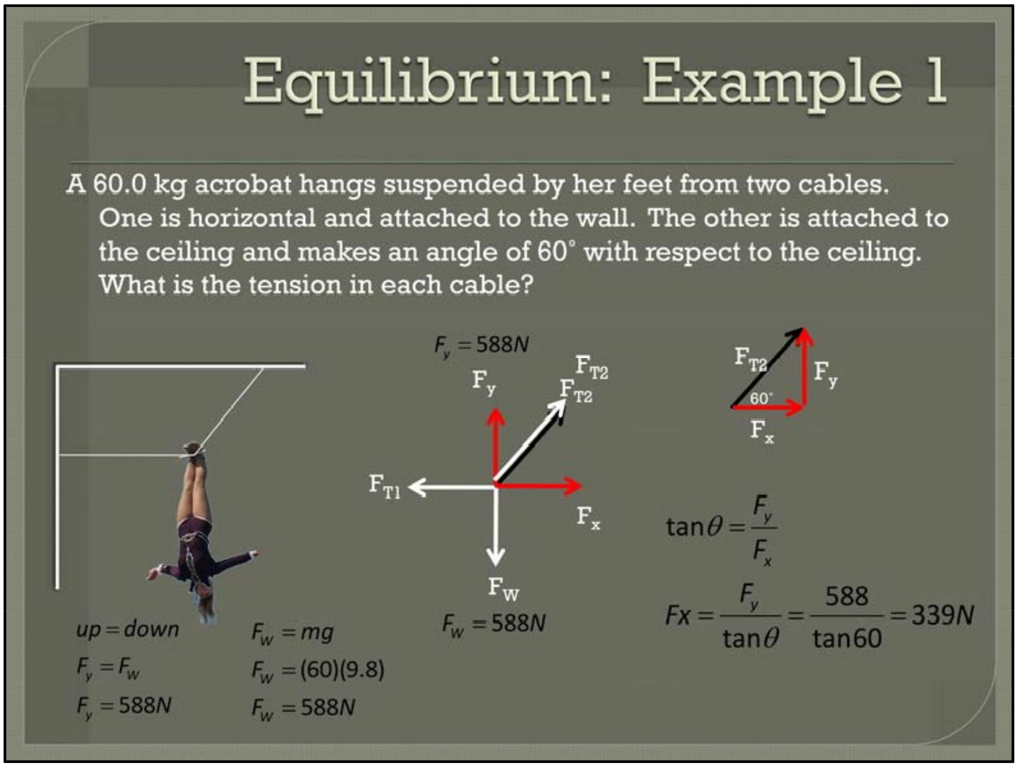


Now that you've had a chance to practice with splitting forces into components and resolving components into resultant net forces, it is time to apply those skills to some more complicated problems involving objects in equilibrium with angled forces acting upon them.



Let's revisit this situation, similar to one from an earlier lesson. A sixty kilogram acrobat hangs suspended by her feet from two cables. One is horizontal and attached to the wall, while the other is attached to the ceiling and makes an angle of sixty degrees with respect to the ceiling. What is the tension in each cable?

First, we should draw a free body diagram of the point where the acrobat's feet touch the cable. We can see that there are three forces acting here. There is the tension in the left cable, which we will call F T one, the tension in the right cable, which we'll call F T two, and the force of gravity on the acrobat, which we will write as F W.

Now we should recognize that the right tension should be represented as two components, which we'll call F y and F x.

Since the acrobat is in equilibrium, we can see that the horizontal component of F T two, F x, must be equal in magnitude to F T one. Also, the vertical component of F T two, F y, must be equal in magnitude to the weight of the acrobat.
Remember that for equilibrium situations, we like to say "up equals down" and "left equals right". First, the weight of the acrobat will equal her mass times the acceleration of gravity. This works out to five hundred eighty eight Newtons.

Do we know enough to solve for any of these yet? We certainly do. Let's look at the vertical forces.

Next, since up equals down F y equals the weight of the acrobat. Therefore, the y-component of the angled tension is equal to five hundred eighty eight Newtons.

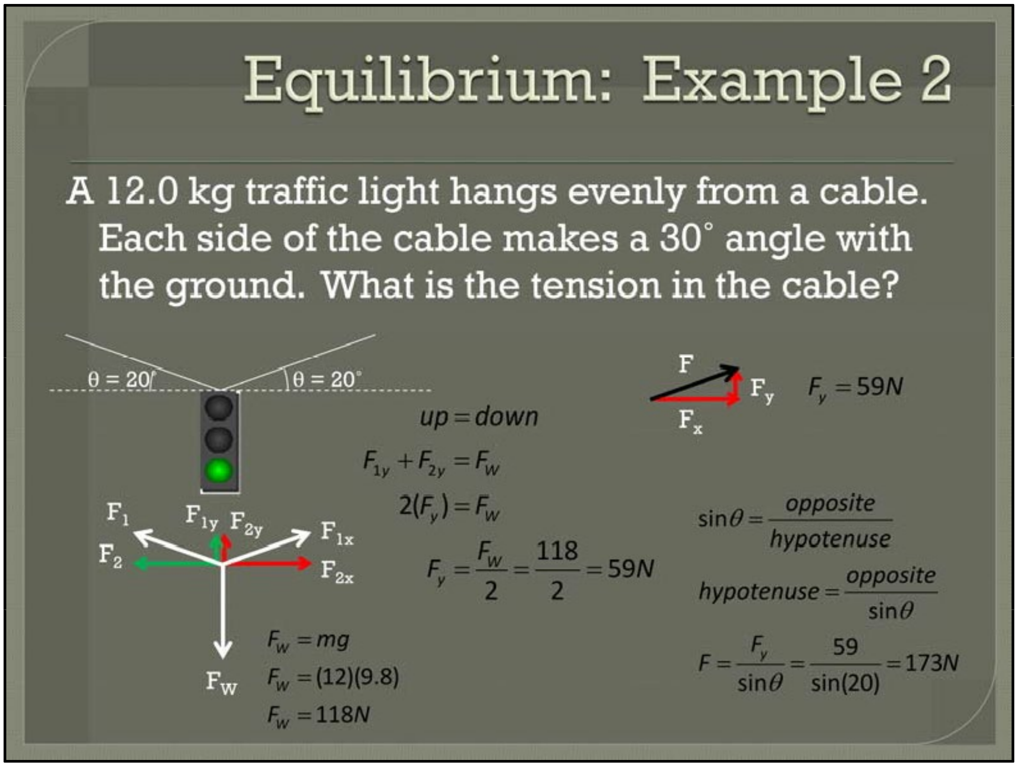
Remember that we can slide the y-component of the angled force over to make a right triangle. Since the angle with the ceiling is sixty degrees, the opposite interior angle, or the angle between F x and F T two must also be sixty degrees.

Now that we have a right triangle, the angle theta and one side, we can solve for the hypotenuse, F T two.

Since the vertical component, F y, equals F sine theta, F must equal F y over sine theta. Substituting, we find that the tension in the right cable equals six hundred seventy nine Newtons.

To solve for the left cable, we can use the tangent function.

The tangent of theta equals F y over F x, so F x equals F y over the tangent of theta.



Now let's consider a twelve kilogram traffic signal that is hanging evenly from a cable. Each side of the cable makes a twenty degree angle with the ground. What is the tension in the cable?

We first should draw a free body diagram of the traffic light. There are three forces acting on the light. There is the weight of the light and the tension from each side of the cable.

Since we have forces at angles, we will have to split up each into an x- and y-component so that we can properly analyze the horizontal and vertical forces.

We now should look at vertical and horizontal directions separately.

In the vertical direction, since we are at equilibrium, we can say "up equals down". We have two forces directed up, the vertical components from each section of the cable, or F one y and F two y. Together, these must equal the weight of the light.

The weight of the light is its mass times the acceleration of gravity, or twelve times nine point eight. This is about one hundred eighteen Newtons.

Now, setting up equals down, we see that F one y plus F two y must equal F W. But since the light is hanging evenly, the vertical components of tension will be equal, so we can say that two times the y-component of tension equals the weight of the light.

Substituting and solving, we find that each vertical component of the tension equals fifty nine Newtons.

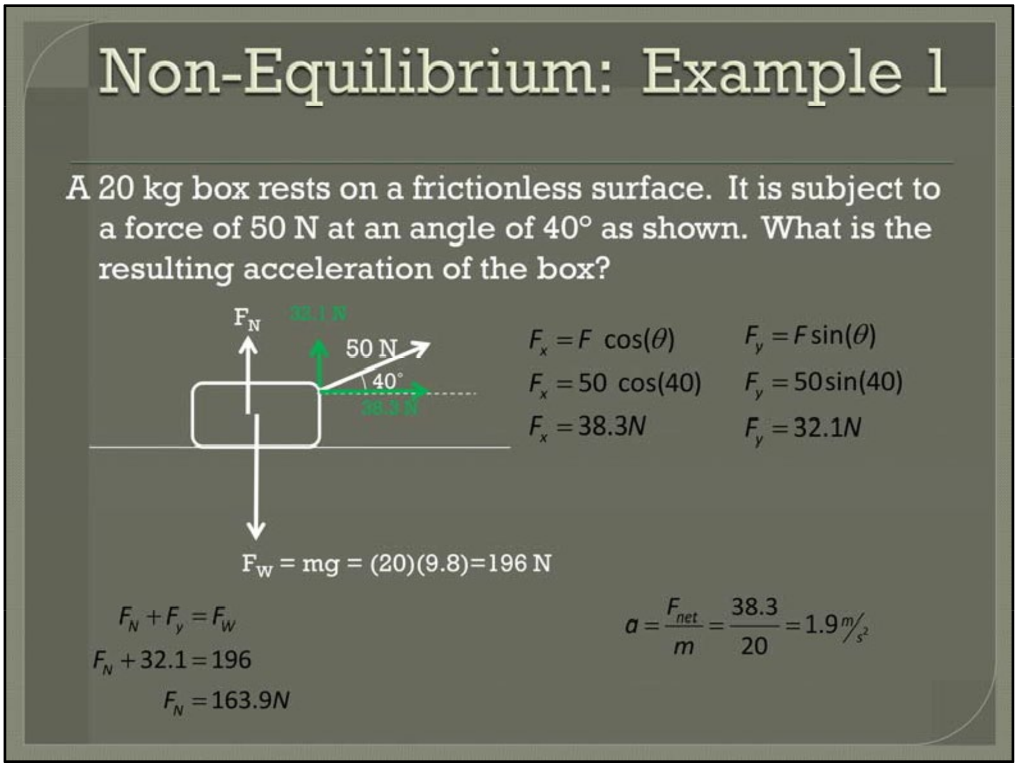
Now, we can use our knowledge of trigonometry to determine the actual tension in the cable.

Sliding the y-component over, we can arrange the horizontal and vertical components head-to-tail to make a right triangle.

We know the angle and one of the sides, so we can use trigonometry to determine the magnitude of the hypotenuse.

The sine of the angle equals the opposite over the hypotenuse, so the hypotenuse equals the opposite divided by the sine of the angle.

Substituting and solving, we see that the tension in the cable is one hundred seventy three Newtons.



Now, you will apply your knowledge of splitting angled force vectors into components and resolving components into resulting net force vectors to solve non-equilibrium problems.

Let's look at an example where a twenty kilogram box on a frictionless surface is subject to a force of fifty Newtons applied at an angle of forty degrees. What is the acceleration of the box?

The angled force needs to be split into its horizontal and vertical components. We see that for the horizontal component, F x equals F cosine theta and F y equals F sine theta.

Calculating these we see that the horizontal component of the force is thirty eight point three Newtons and the vertical component of the force is thirty two point one Newtons.

The other forces acting on the box are gravity and the normal force.

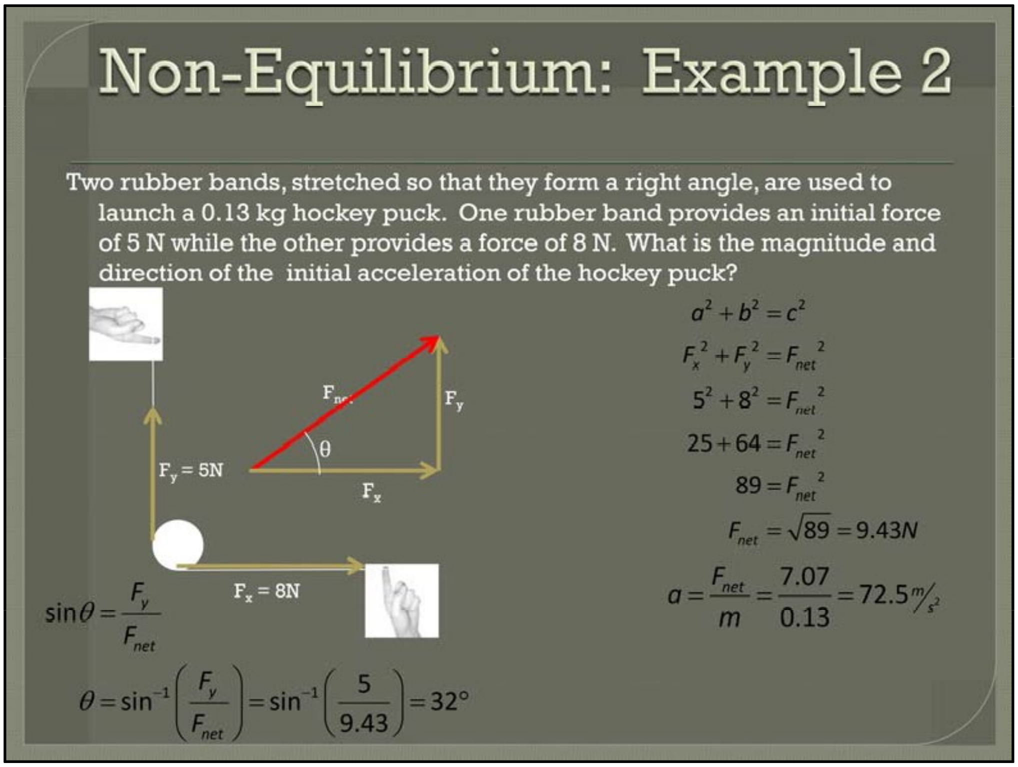
It is interesting to note here that the normal force will not be the same in magnitude as the force of gravity. Since the applied force of fifty Newtons has a vertical component, the surface doesn't have to push back so hard to oppose the force of gravity. In fact, the weight of the object is equal in magnitude to the sum of the normal force and the vertical component of the applied force.

Since the gravitational force is equal to the mass times the acceleration of gravity, it is equal to twenty kilograms times nine point eight meters per second squared, or one hundred ninety six Newtons.

The normal force plus the vertical component must be equal in magnitude to the gravitational force. So F N plus F y equals F W. Substituting and solving, we see that the normal force is equal to one hundred sixty three point nine Newtons. This tells us that vertical forces are balanced. But what about our horizontal forces?

Horizontally, we only have the x-component of the pull force, which is unbalanced and which is the net force acting on the block. Now we can apply Newton's second law which says that acceleration equals net force divided by mass.

Substituting and solving we see that the block will accelerate at a rate of one point nine meters per second squared.



Let’s say that two rubber bands are stretched so that they form a right angle when viewed from above. They are used to launch a zero point one three kilogram hockey puck. One rubber band provides an initial force of five Newtons, while the other provides a force of eight Newtons. What is the initial acceleration of the hockey puck?

We’re only going to consider the forces from each rubber band, as the gravitational force and the normal force are balanced. We can see that there is an x and a y component of force that need to be added to a resultant force. We’ll arrange the x and y components head to tail and draw a hypotenuse to represent the resultant force.

Now we can use the Pythagorean theorem to determine the magnitude of the resulting net force.

a squared plus b squared equals c squared, so F x squared plus F y squared equals F net squared. Substituting and solving, we see that the net force equals nine point four three Newtons.

Now we can calculate the initial acceleration of the puck using Newton’s second law. Acceleration equals net force divided by mass, so when we substitute and solve, we see that the puck will initially accelerate at seventy two point five meters per second squared.

Now that we’ve solved for the net force, let’s determine the angle from the horizontal.

The sine of the angle is F y over F net, so the angle is equal to the inverse sine of F y over F net.

Substituting and solving, we see that theta is thirty two degrees.

Now, you should be ready to practice with some of these problems on your own.