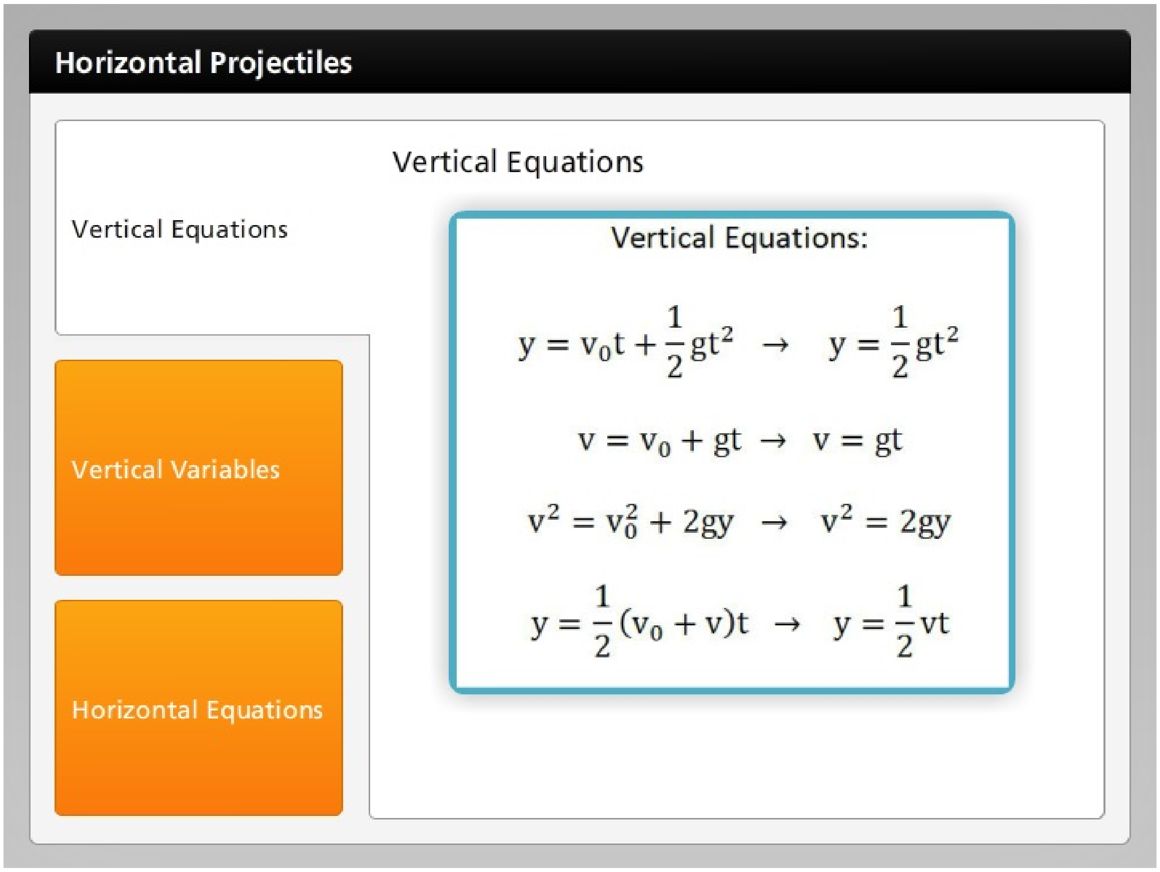
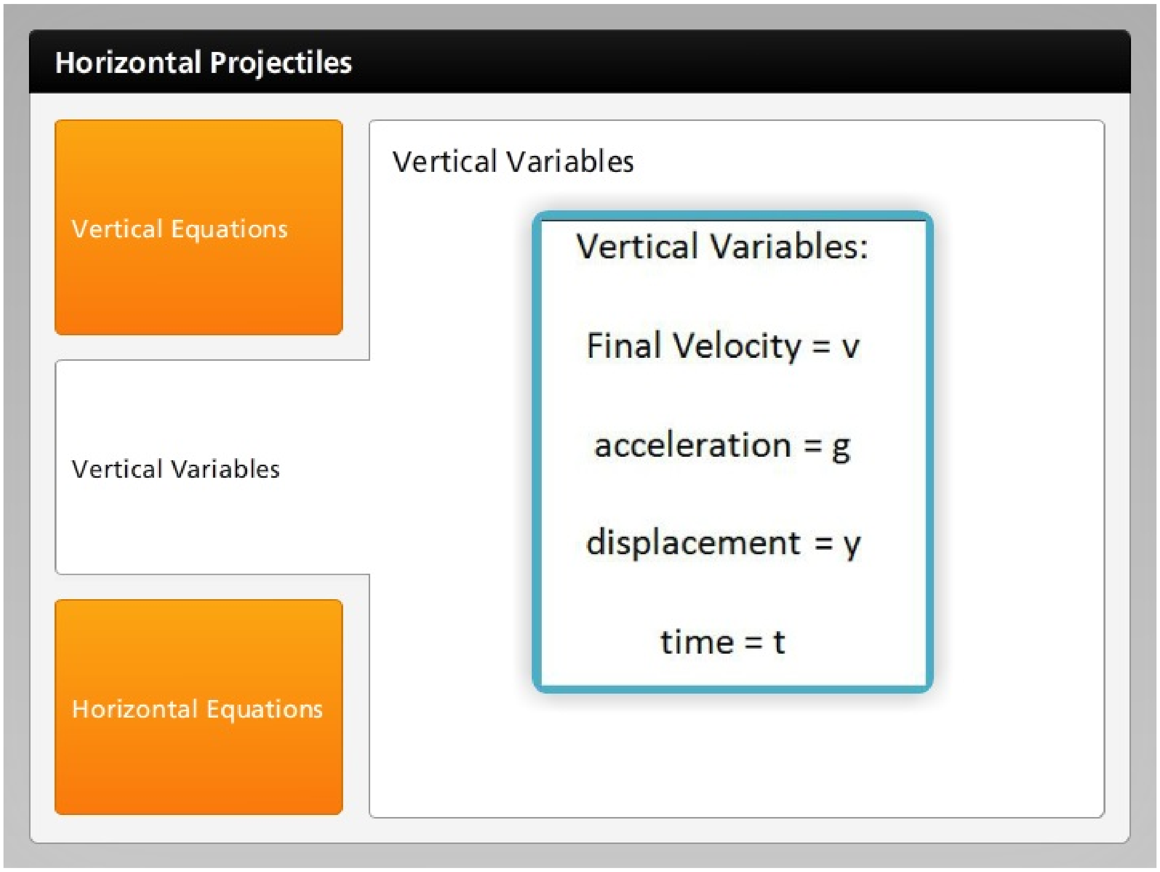
Module 3: Motion in Two Dimensions

Topic 2 Content: Horizontal Projectiles Presentation Notes



In the vertical direction, we have the case of freefall. But since a horizontal projectile will always start with an initial vertical velocity of zero, we can revisit our freefall equations to see what they become in this special case.

With no initial velocity, our first equation simplifies to y equals one half g t squared. The second equation becomes v equals g t. The third can be written as v squared equals two g y, and the fourth becomes y equals one half v t.



You will notice that our normal list of five different kinematics variables for motion has been reduced by one since the initial velocity is zero. So with horizontal projectiles, when dealing with vertical motion, we only need to focus on final velocity, acceleration, displacement and time.

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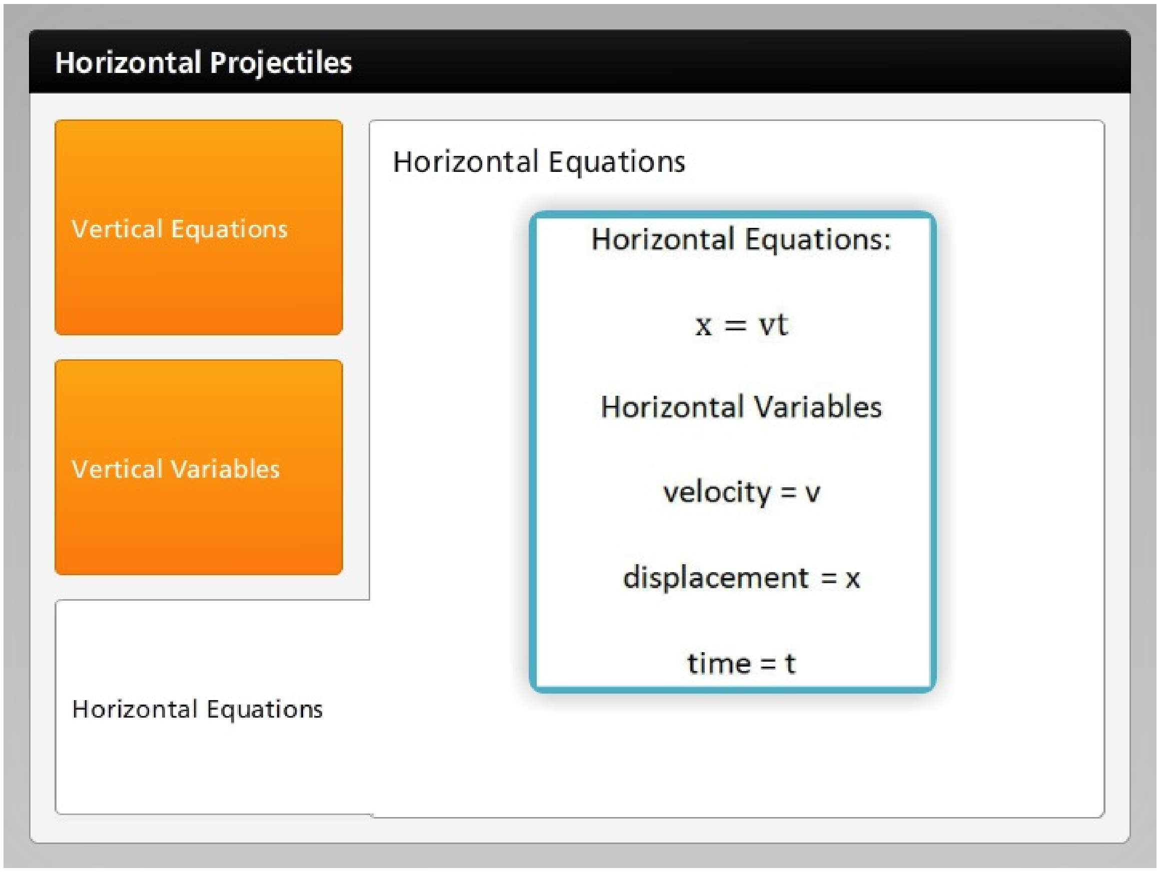
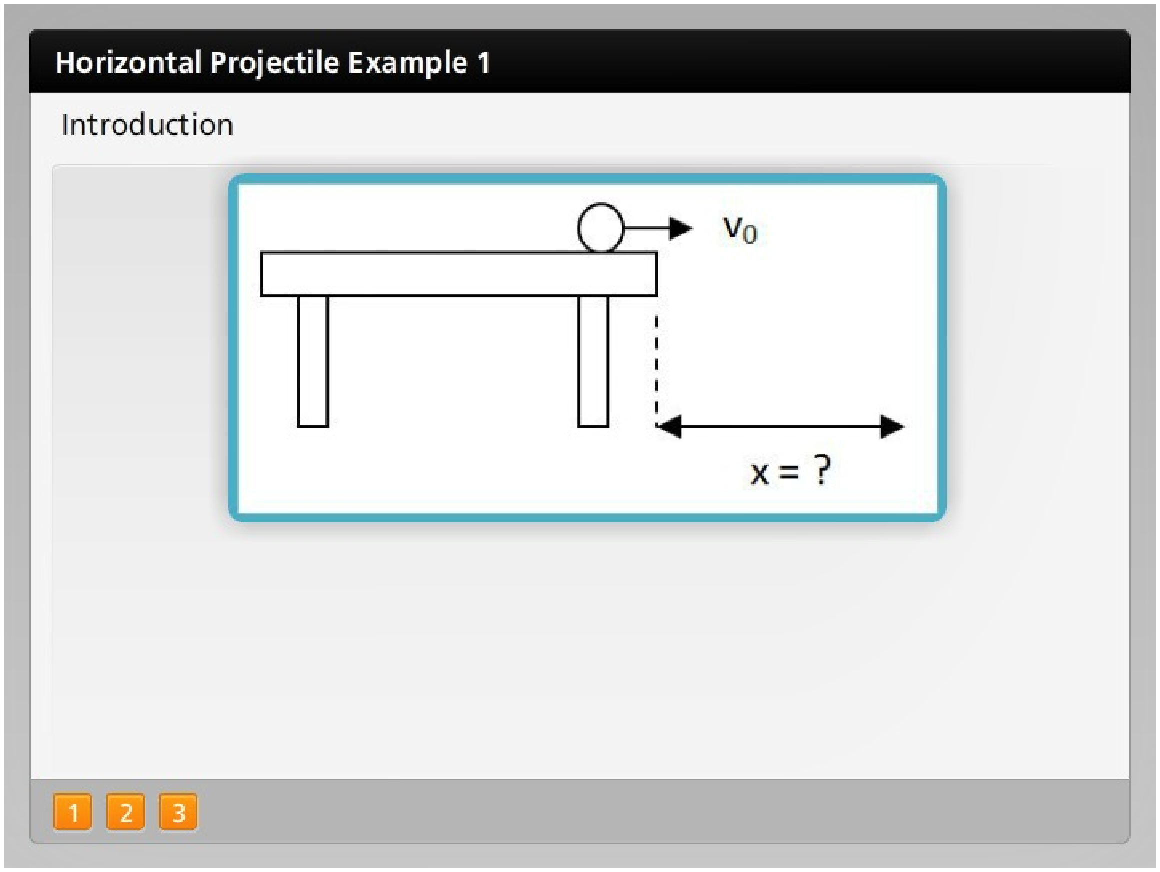


Recall that in the horizontal direction, with no acceleration, the velocity is constant. So we simply have motion at a constant velocity, which is summarized in the equation x equals v to or horizontal displacement equals horizontal velocity times time.

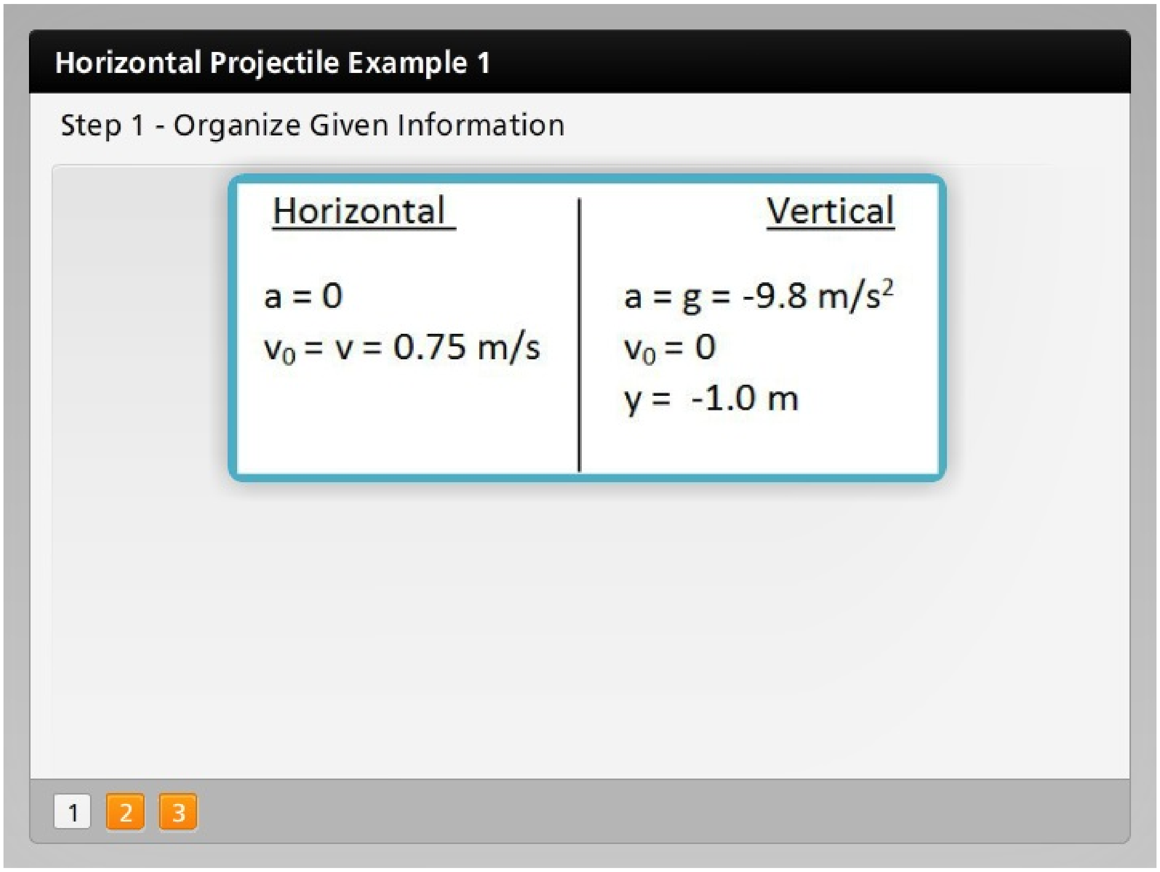
The time is the direct link between vertical and horizontal motion, and is the only value that will always be common to both.



When solving problems of horizontal projectiles, or any projectile motion problem, it will be very important to separate horizontal from vertical motion, and deal with them independently.

Let's see how this looks by exploring a horizontal projectile problem.

If a baseball is rolling on a flat table at a height of one meter, with a constant velocity of zero point seven five meters per second, how far horizontally from the base of the table will the ball hit the ground after it rolls off?



The first step is to organize what you know into things that are happening horizontally and things that are happening vertically. It is sometimes useful to simply draw a line down the center of the page and put the headers "horizontal" and "vertical" over each side.

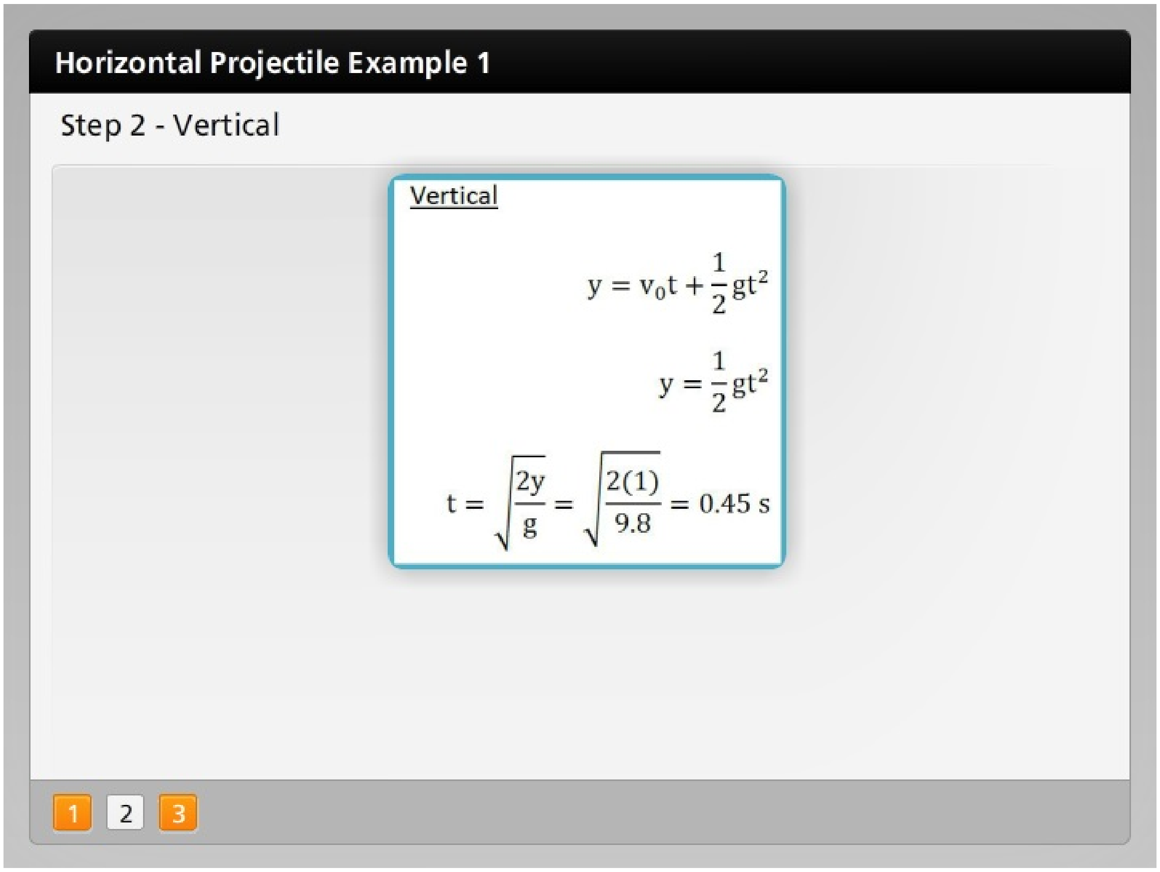
Since this is a projectile motion problem, the only acceleration will be in the vertical direction, so we can write a equals zero in the horizontal column and a equals g equals nine point eight meters per second squared in a negative direction in the vertical column.

Since the motion of the ball is initially horizontal as it leaves the table, the initial velocity can be put in the horizontal column, and we can write v zero equals v equals zero point seven five meters per second. Vertically, the initial velocity is zero.

The other thing we know is that the ball will fall down one meter by the time it hits the ground, so we can write y equals negative one meter.

We are looking for the horizontal displacement, which we could find with the equation x equals v times t, but we don't know the time that the ball will be in the air. How can we solve for time?

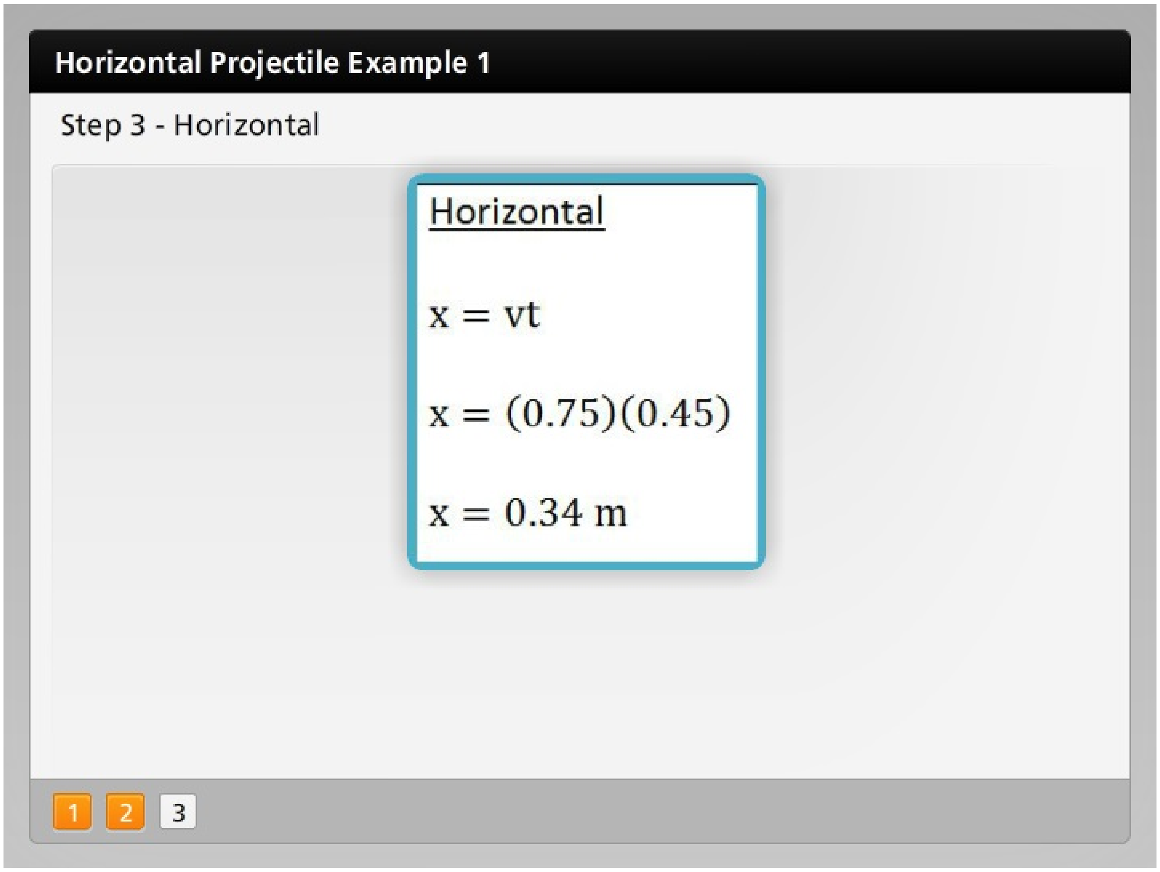
Remember that when you did freefall problems, the hang time was determined by the initial height and the initial vertical velocity. In other words, the time in the air is a vertical motion problem.



So we'll see if we can solve for time in the vertical direction, then bring that time back to the horizontal analysis.

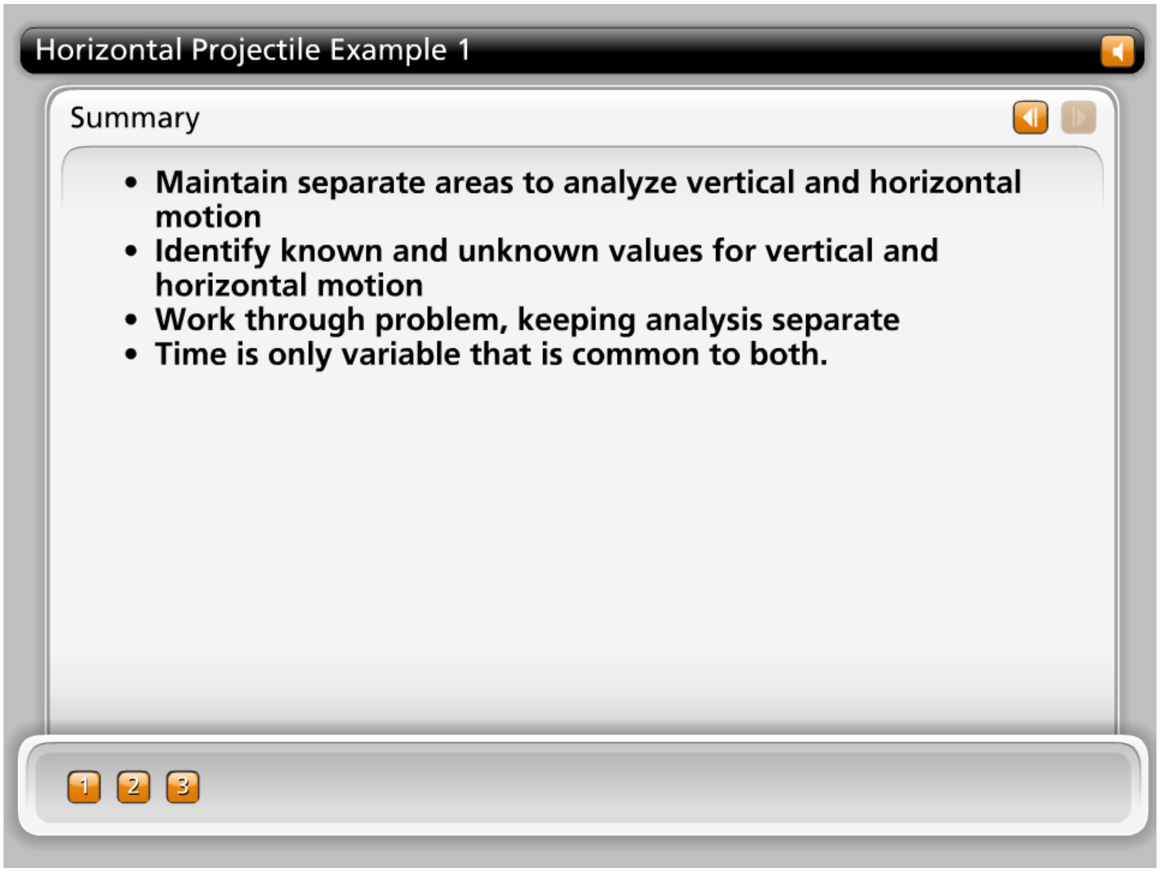
We know the initial velocity, the acceleration and the vertical displacement. We can use the equation y equals v zero t plus one half g t squared, which we've simplified into y equals one half g t squared.

Rearranging, and solving for t, we see that the time is equal to the square root of two y over g. Substituting and solving, we see that the time it takes to fall to the ground is equal to zero point four five seconds.

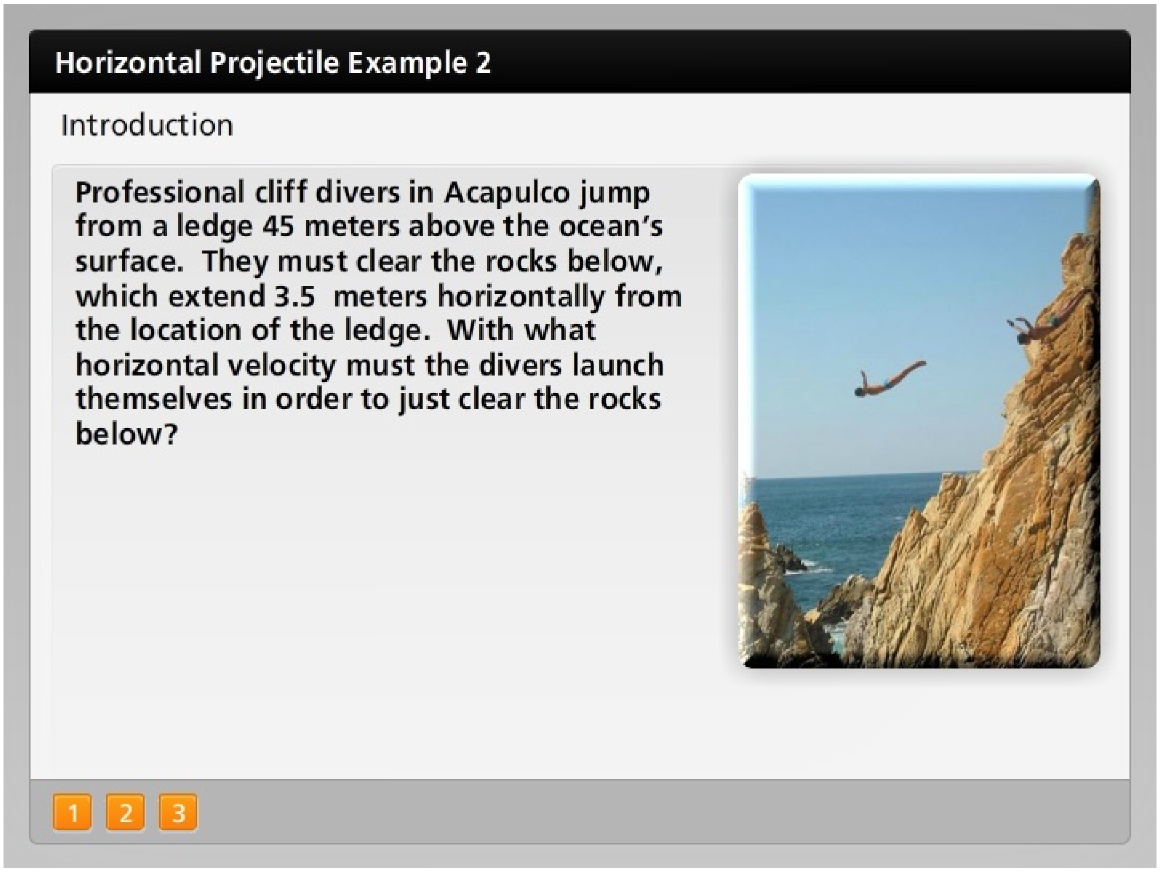


Now, we can return to the our horizontal analysis.

The horizontal displacement is equal to the horizontal velocity times the time it is in motion. Substituting our values , we see that the ball moves zero point three four meters.

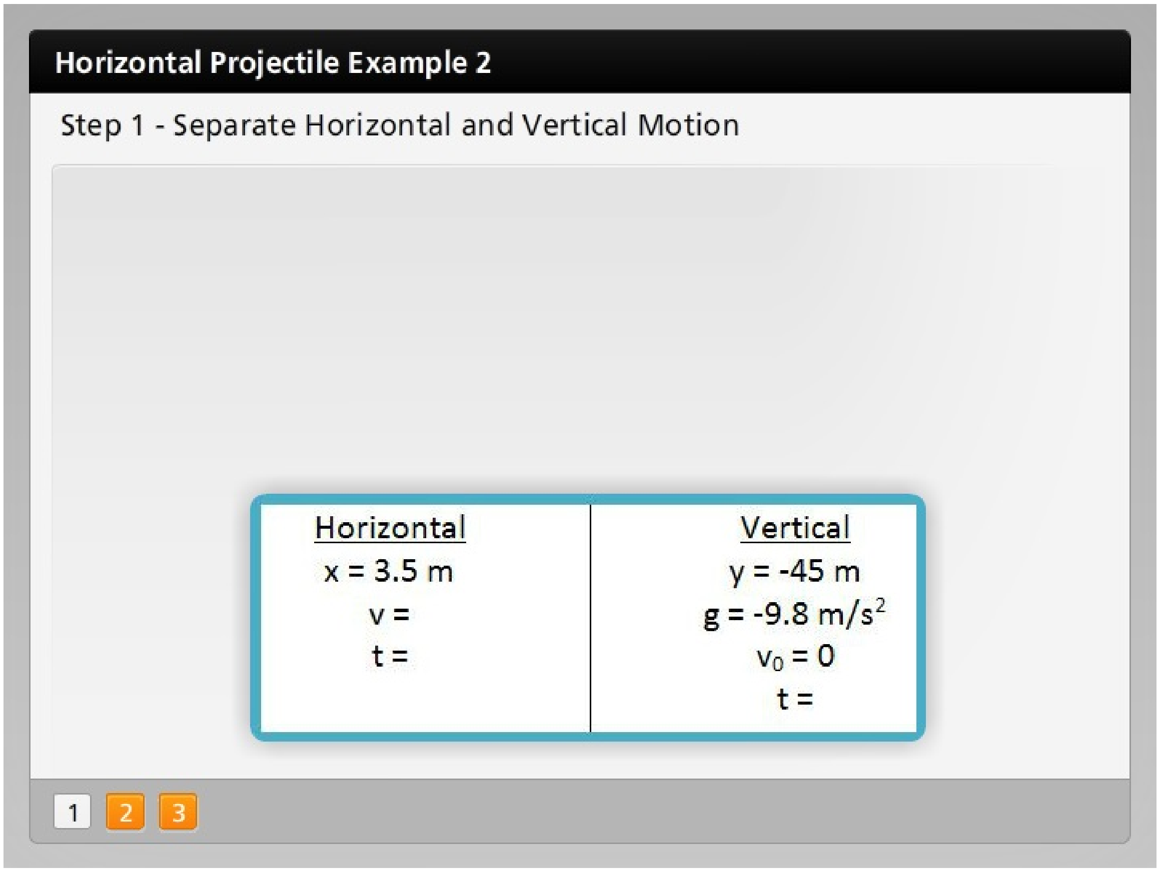


The method we used to solve this problem is similar to the method we will use to solve any projectile motion problem. We first set up areas to record data and analyze vertical and horizontal motion separately. Then we identified both known and unknown values related to vertical and horizontal motion. We then worked through the steps of the problem, being very careful to keep our analysis of vertical and horizontal motion separate, with the time as the only variable that could be used in common for both analyses.



Let's look at another example.

Professional cliff divers in Acapulco jump from a ledge forty five meters above the ocean's surface. They must clear the rocks below, which extend three point five meters horizontally from the location of the ledge. With what horizontal velocity must the divers launch themselves in order to just clear the rocks below?

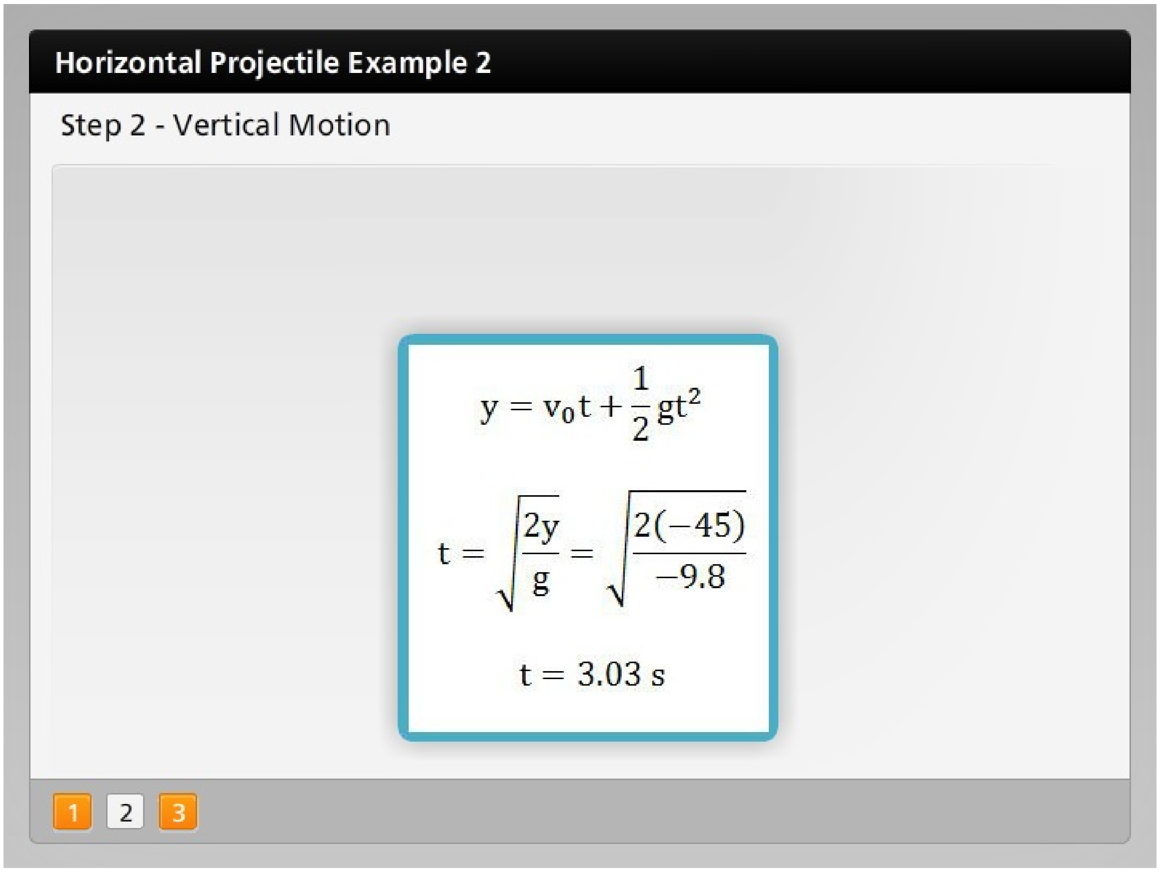


Our first step in solving this problem is to separate our analysis of horizontal and vertical motion.

We know the horizontal extension of the rocks, so we can write x equals three point five meters. We do not know the velocity or the time.

As is always the case with projectile motion, there is no acceleration in the horizontal direction.

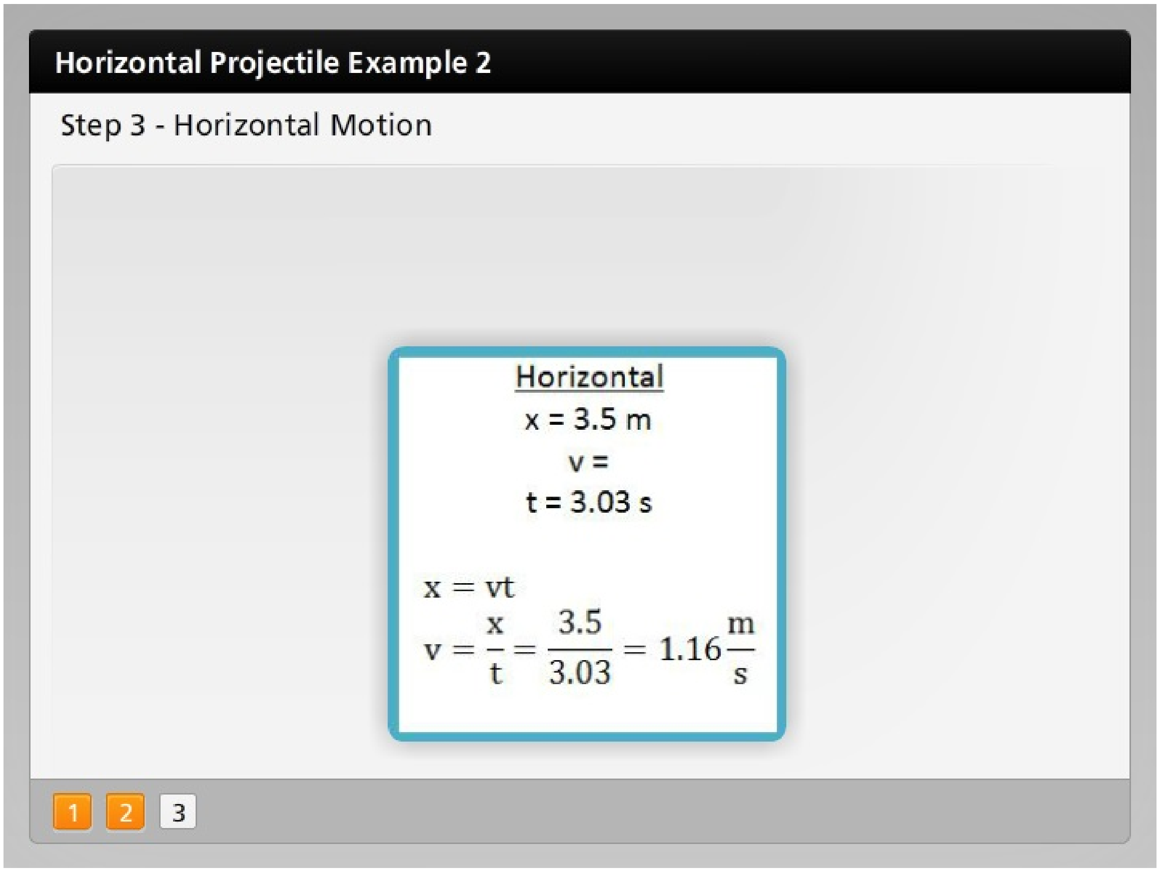
In the vertical direction, we know that the displacement of the diver will be forty five meters in the negative direction when they reach the water, so we write y equals negative forty five meters. The vertical acceleration is the acceleration of gravity, so we can write g equals negative nine point eight meters per second squared. The initial vertical velocity will be zero. We do not know the time.



In order to solve for the initial horizontal velocity, we need to know the time, but there is no way to solve for this using only our information from the horizontal direction.

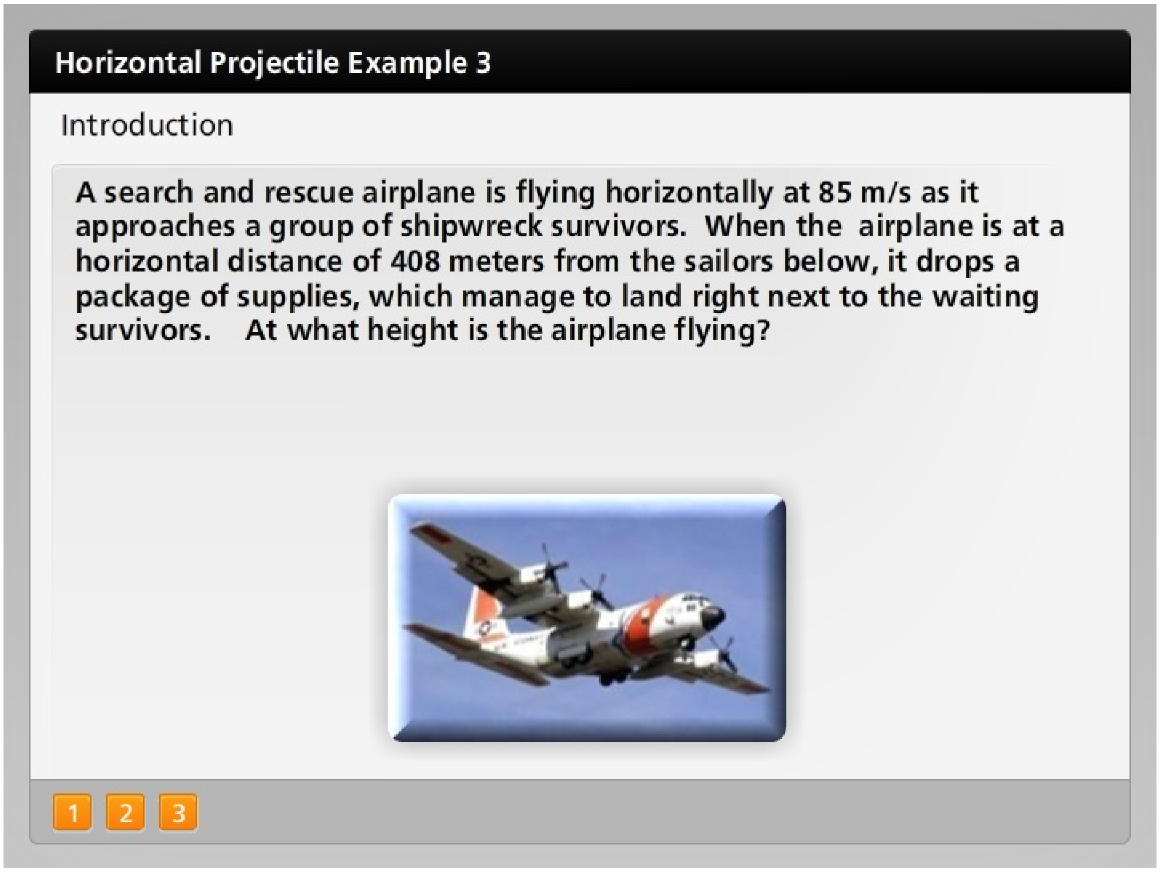
However, if we look at the information we have on vertical motion, we have enough information to use our freefall equations to solve for the time. The equation to use is y equals v zero t plus one half g t squared, which rearranges to t equals the square root of two y over g.

Substituting and solving, we discover that the time to reach the water is three point zero three seconds.



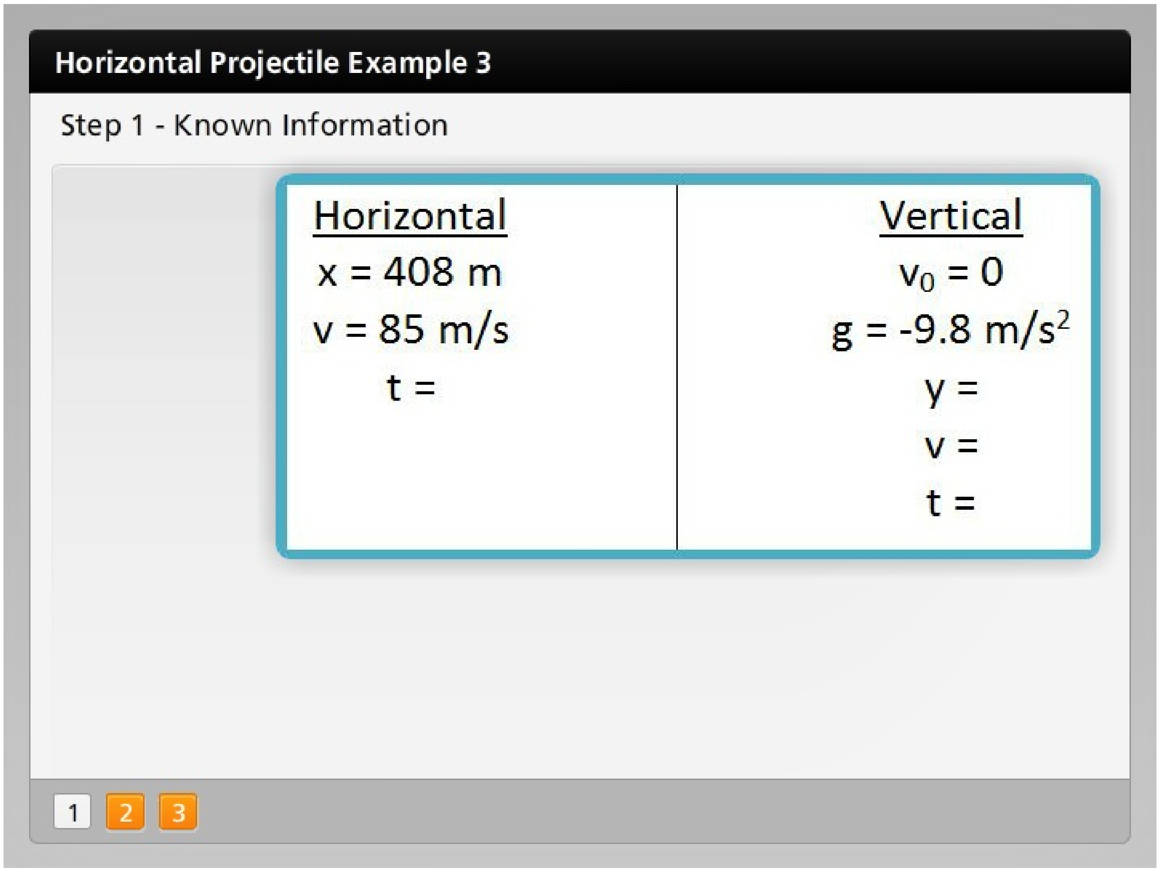
Now, we can look at the horizontal motion.

The time from the vertical analysis is the same as the time in the horizontal analysis, so we can write t equals three point zero three seconds in our horizontal column. Now to solve for v we can rearrange the equation x equals v t to be v equals x over t. Substituting and solving, we see that the initial horizontal velocity must be at least one point one six meters per second in order to clear the rocks below.



Now for a third example.

A search and rescue airplane is flying horizontally at eighty five meters per second as it approaches a group of shipwreck survivors. When the airplane is at a horizontal distance of four hundred eight meters from the sailors below, it drops a package of supplies, which manage to land right next to the waiting survivors. At what height is the airplane flying?

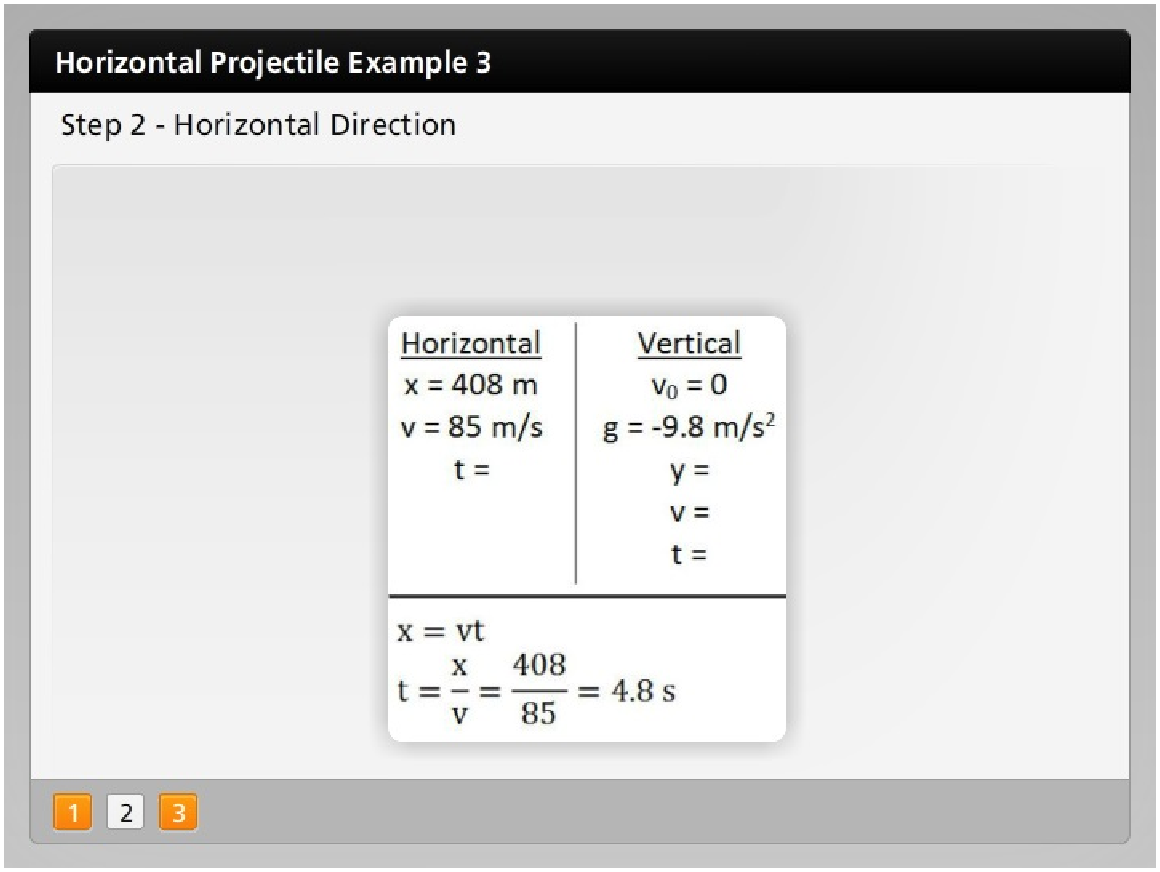


Once again, we will separate what we know about horizontal motion from what we know about vertical motion.

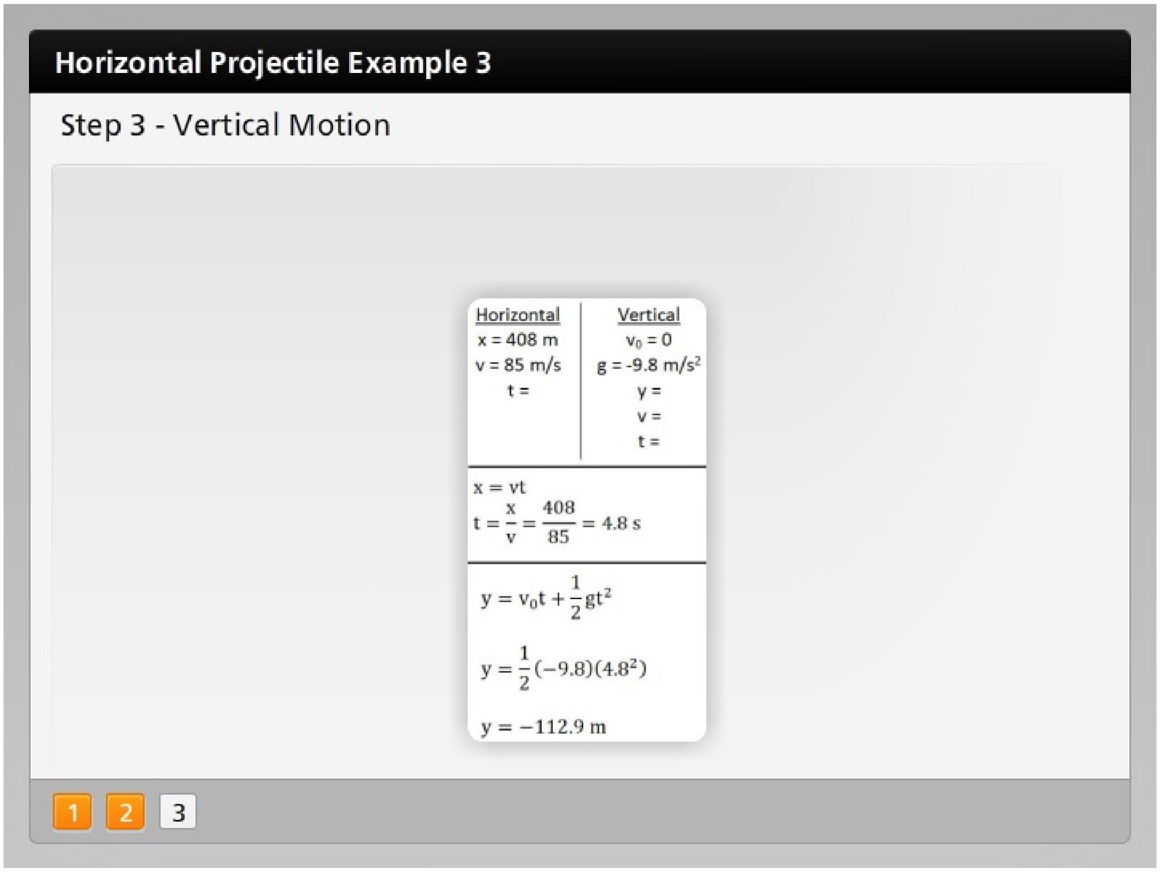
Horizontally, we know the displacement and the velocity, so we can write x equals four hundred eight meters and v equals eighty five meters per second. We do not yet know the time.

Vertically, we have not been given any information, but since this is a horizontal projectile problem, we know the initial vertical velocity is zero and the acceleration is nine point eight meters per second down, so we can write v zero equals zero and g equals negative nine ponit eight meters per second squared. We do not know the displacement, final velocity or time.

The height of the plane will be the same magnitude as the vertical displacement of the package, but to solve for this, we need one more variable.



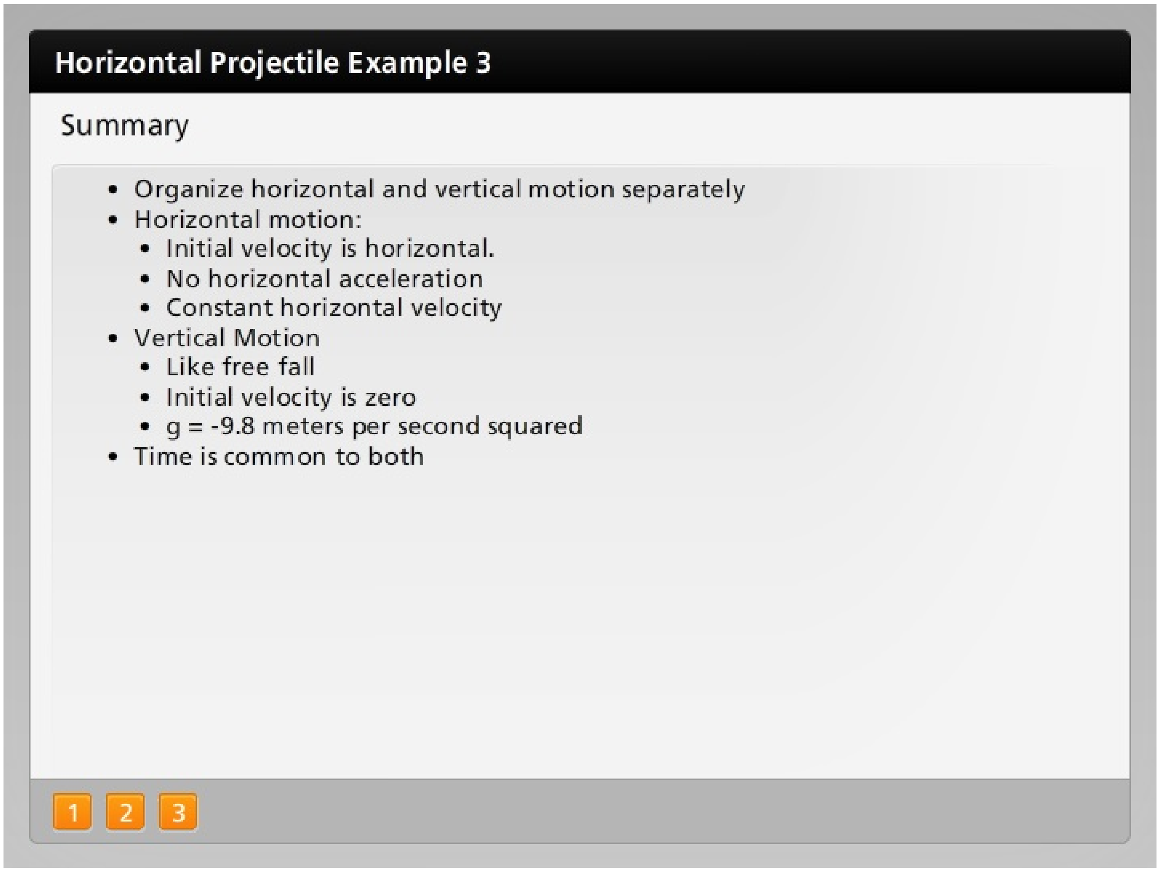
Luckily, we can solve for time in the horizontal direction, knowing the displacement and the velocity. We will rearrange the equation x equals v t and solve for t to get t equals x over v. Substituting and solving, we see that the time of flight is four point eight seconds.



We can now look at the vertical motion and write that time equals four point eight seconds. Knowing initial velocity, acceleration and time, we can solve for the vertical displacement. Again we will use the equation y equals v zero t plus one half g t squared.

Substituting and solving, we see that the displacement is negative one hundred twelve point nine meters.

Why negative? Because that's the direction the package fell, which is what we were basing our calculations on. Clearly this means that the airplane was flying at a height of one hundred twelve point nine meters when it dropped the package.



As you can see, solving projectile motion problems requires some careful organization, but the calculations themselves are no more difficult than you've seen before, and the equations are the same ones you've been using.

Remember, since horizontal and vertical motion are independent of each other, you must organize and analyze motion in the horizontal direction separately from motion in the vertical direction.

For horizontal projectiles, the object starts out with only horizontal velocity. Since acceleration is zero in this direction, the velocity is constant.

In the vertical direction, motion is just like free fall. Initial velocity is zero and acceleration is nine point eight meters per second straight down.

Time is the only variable that is common to both analyses.