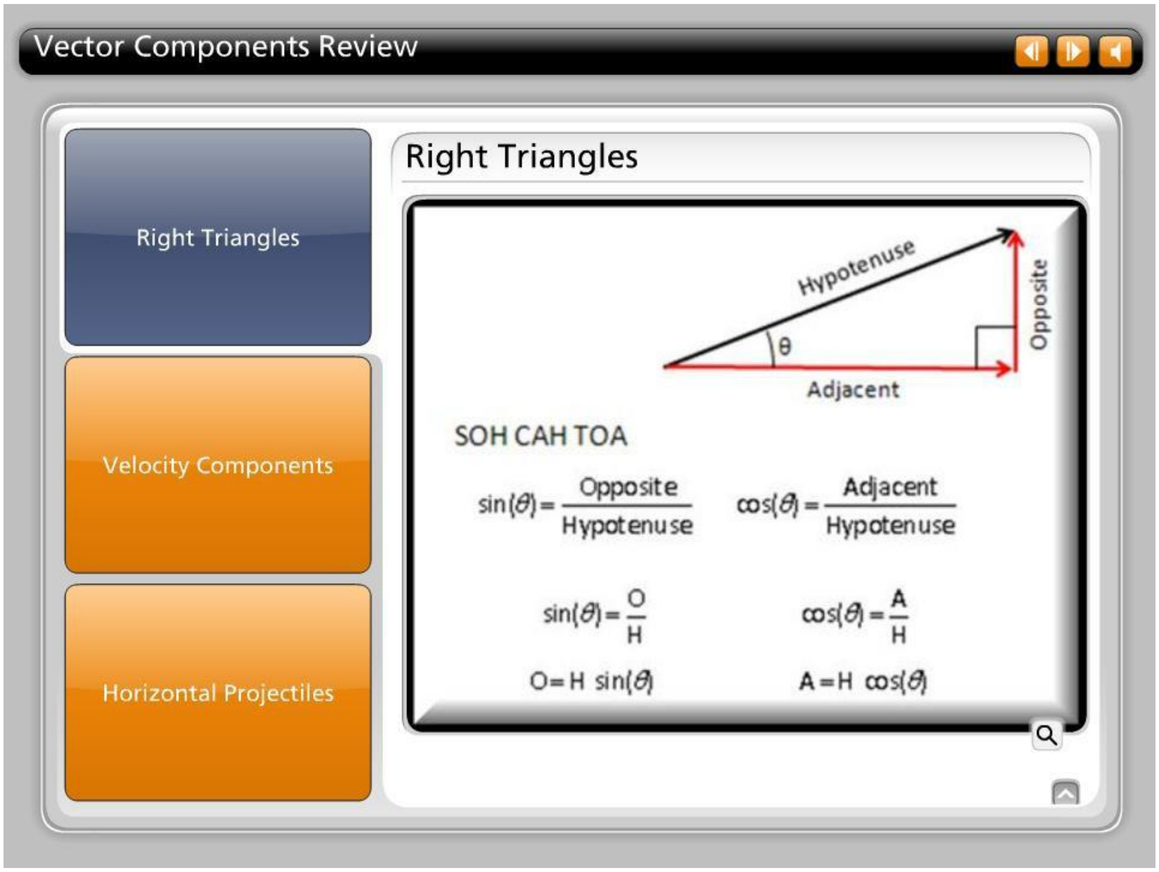
Module 3: Motion in Two Dimensions

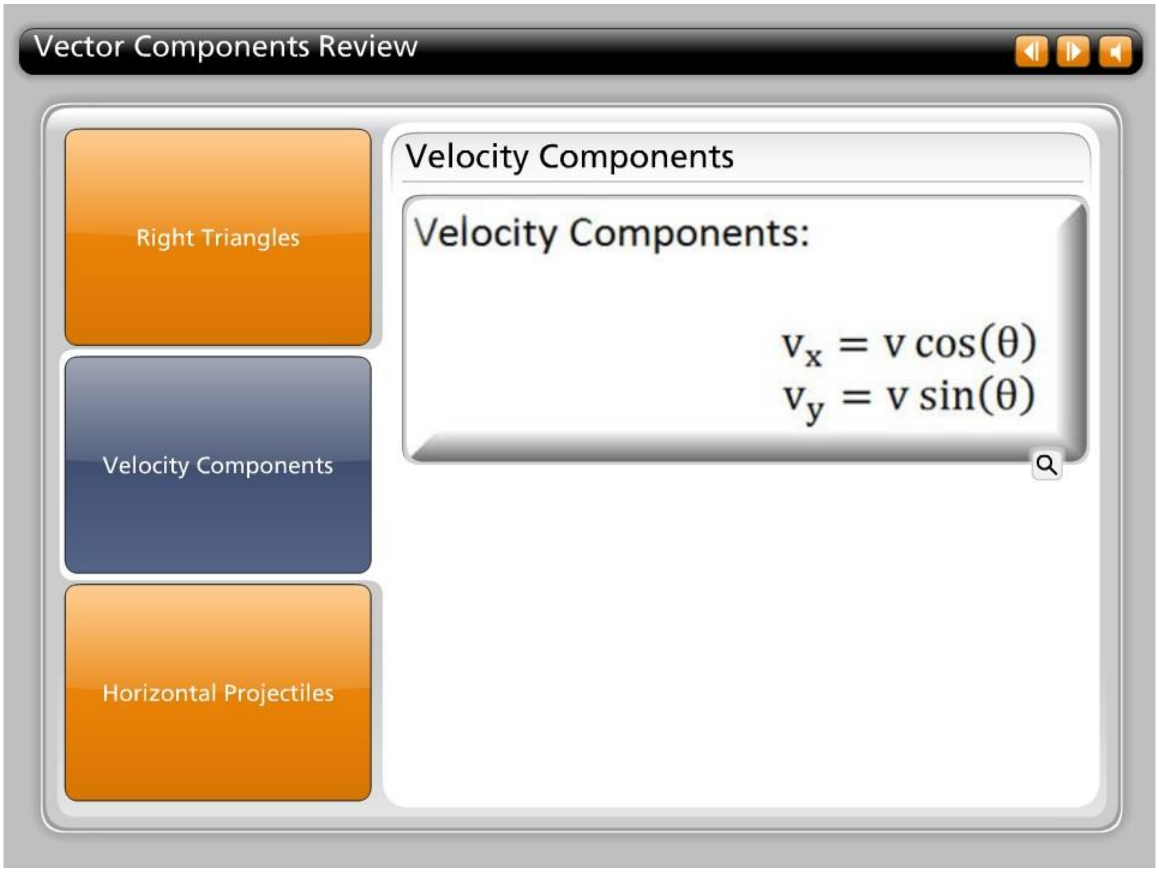
Topic 3 Content: Projectile Angles Presentation Notes

In order to deal with projectiles that are launched at an angle, we will have to separate the velocity into its vertical and horizontal components so that we can analyze the vertical and horizontal motions independently.

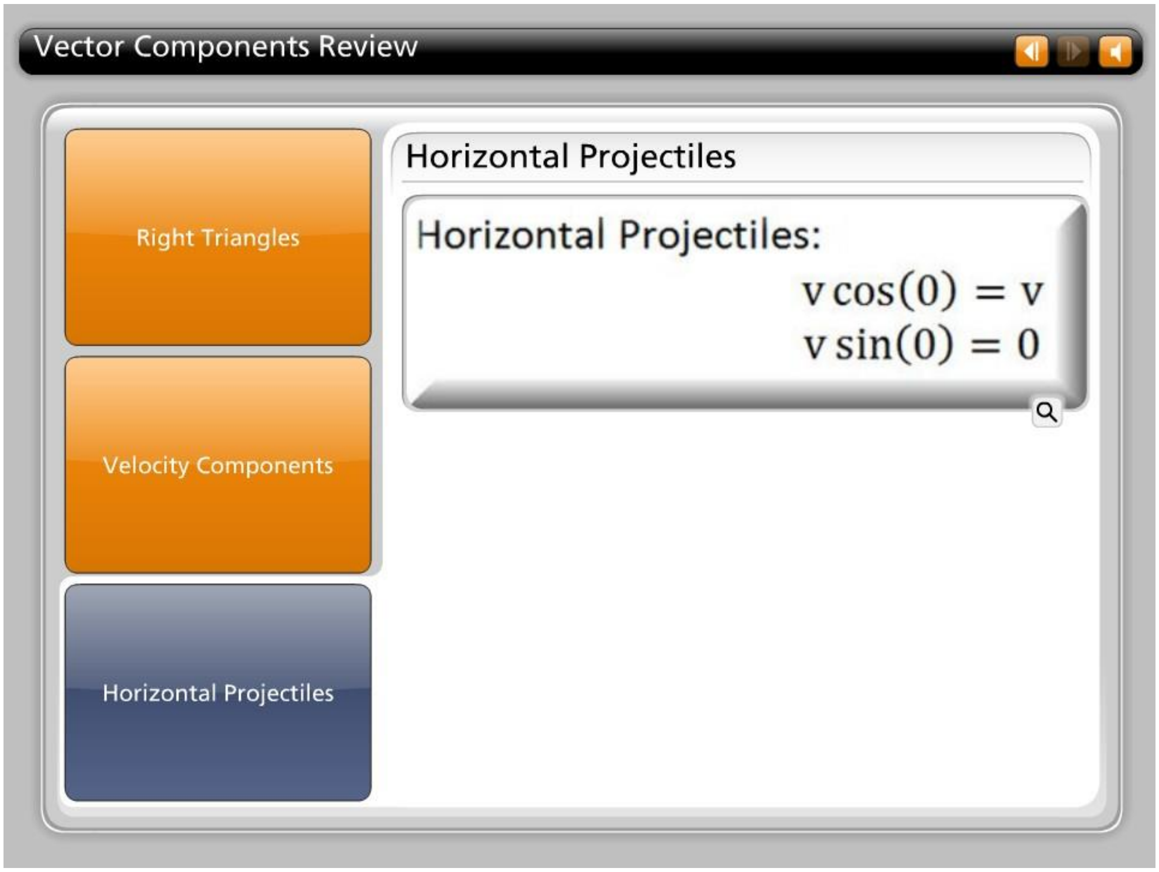
Therefore, you will review vector components prior to continuing on in this lesson.



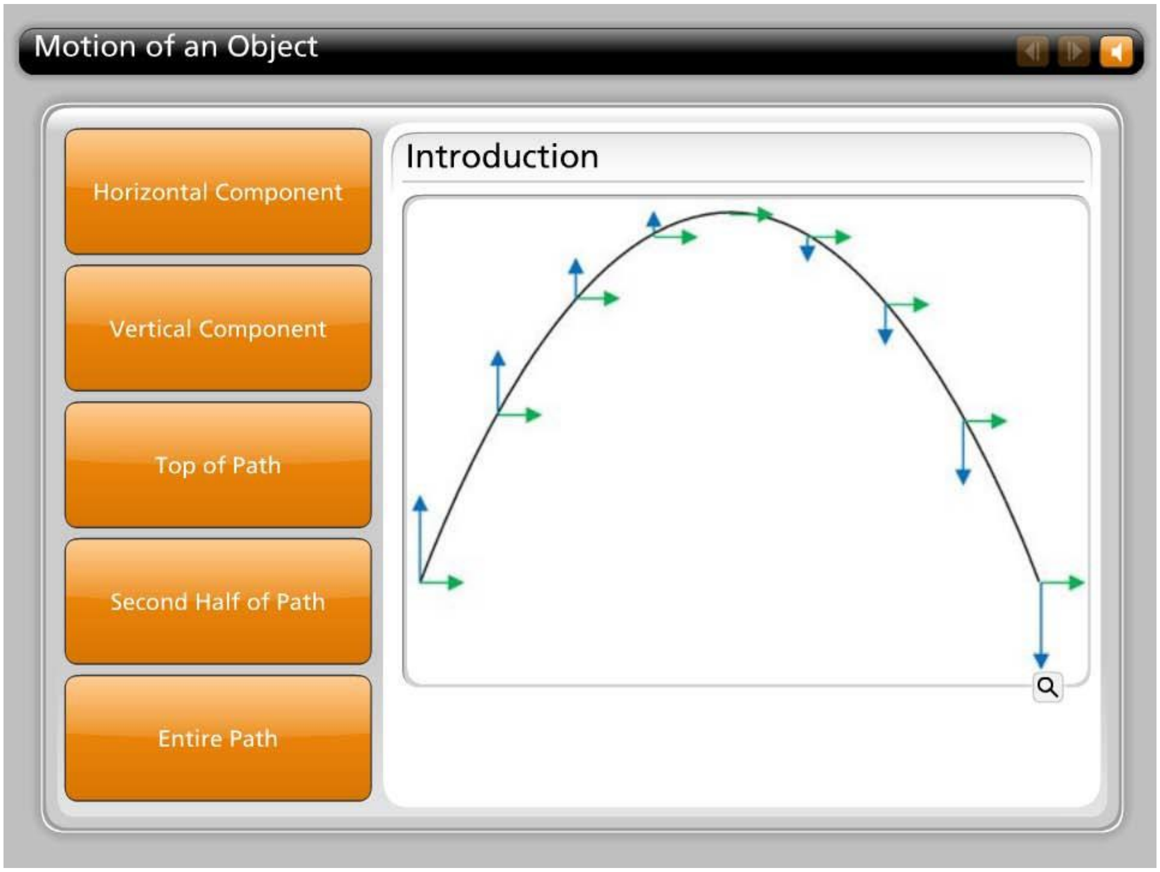
Remember that any vector can be split into its horizontal and vertical, or its x- and y-components using right-angle trigonometry. The horizontal component will be the magnitude of the vector times the cosine of the angle that the vector makes with the horizontal, and the vertical component will be the magnitude of the vector times the sine of the angle the vector makes with the horizontal.



In this case, we're dealing with velocity vectors, so we can write v x equals v cosine theta and v y equals v sine theta.

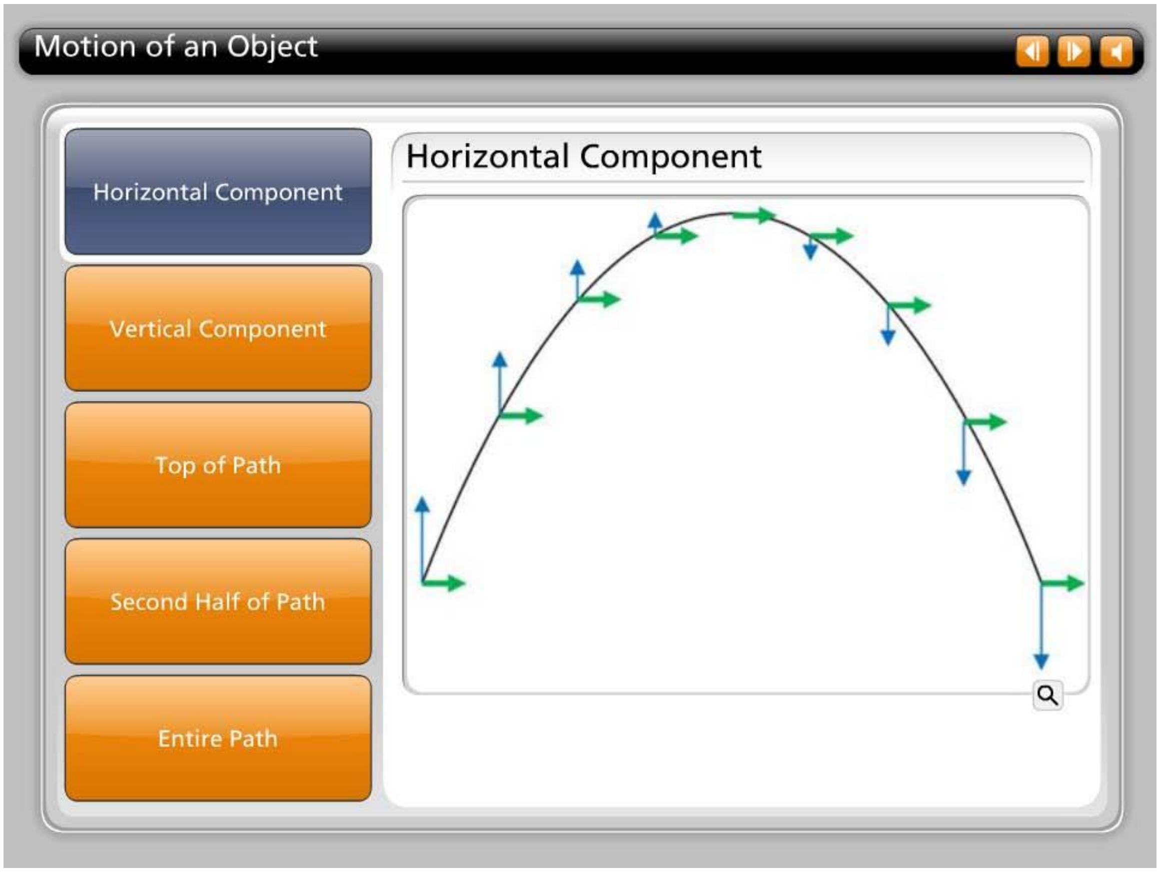


Earlier, we were dealing with projectiles that started out with only horizontal motion, where theta is zero. So the initial horizontal velocity was equal to the initial velocity, since the cosine of zero degrees is equal to one, and the initial vertical velocity was equal to zero since the sine of zero degrees is zero.

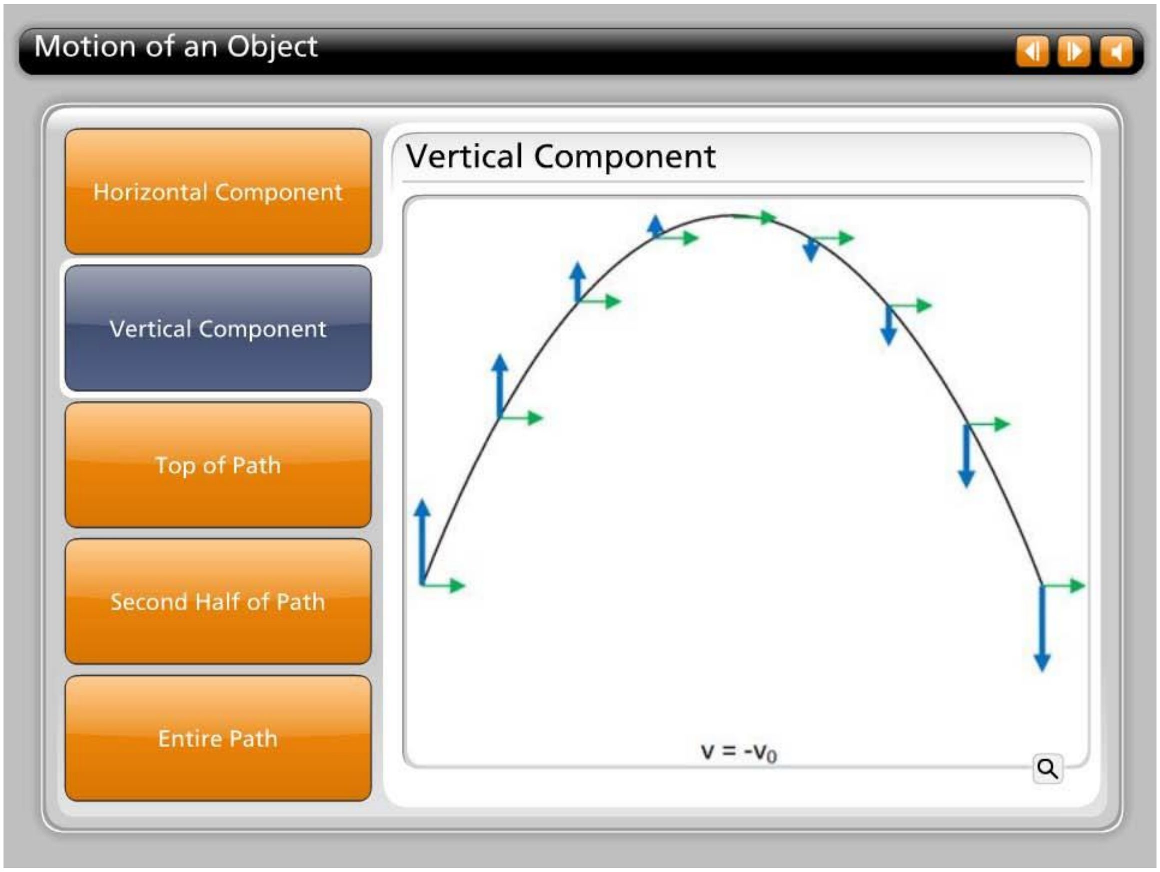


You will find that solving projectile motion problems is quite challenging if you are merely hunting for equations, but becomes much easier as you become familiar with the basic concepts and are able to focus on what is happening at some key points in the motion of the object.

So let's look at the motion of an object that is launched at an angle on level ground. The arrows represent the horizontal and vertical components of the velocity over time.



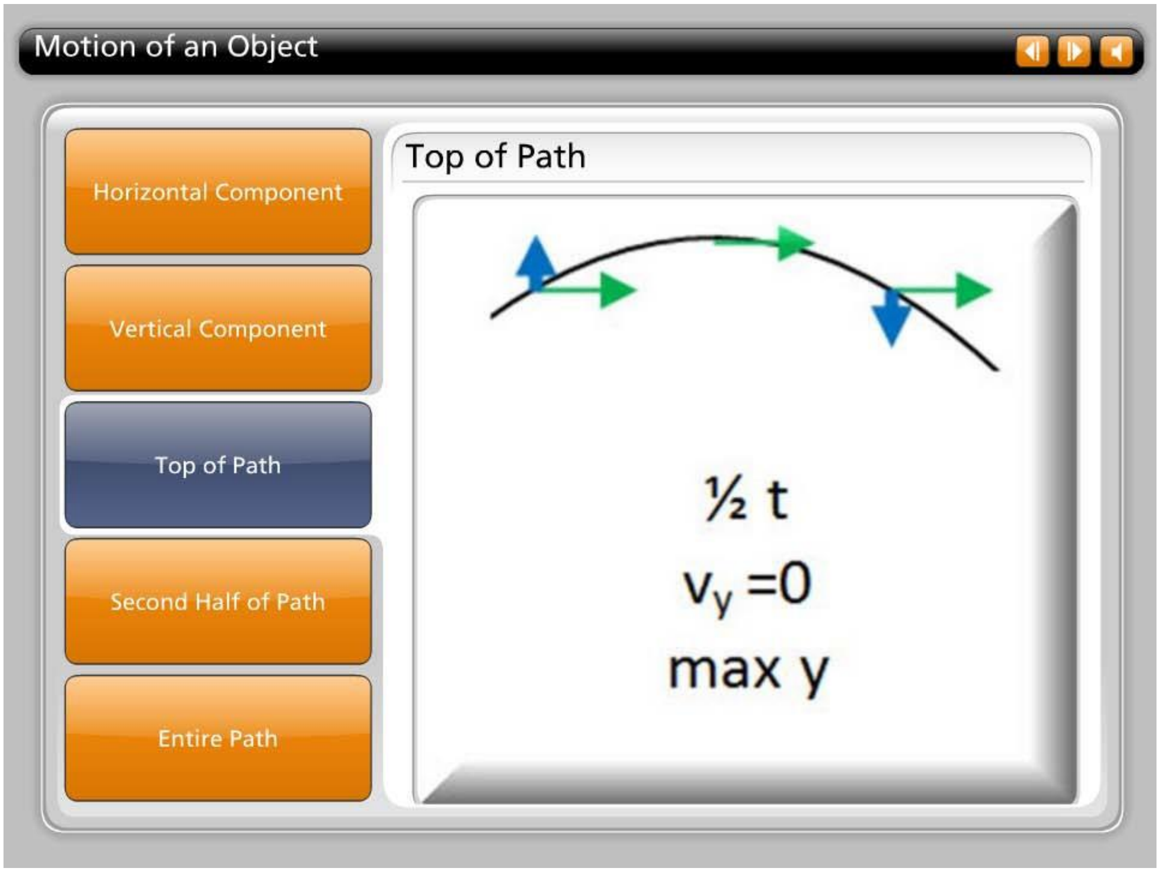
What do you notice about the green arrows? Do you see how they are all the same length? This indicates that the horizontal component of the velocity is constant. Since there is no acceleration in the horizontal direction, the horizontal component of the velocity does not change.



Look at what happens to the blue arrows over the entire flight. The vertical component of the velocity begins positive, but you can see that the length of the arrows initially decreases due to the downward acceleration of gravity. Later, the arrows get longer, but they are pointing downwards, indicating that the vertical component of the velocity is now in a negative direction as the object continues to accelerate downwards due to gravity.

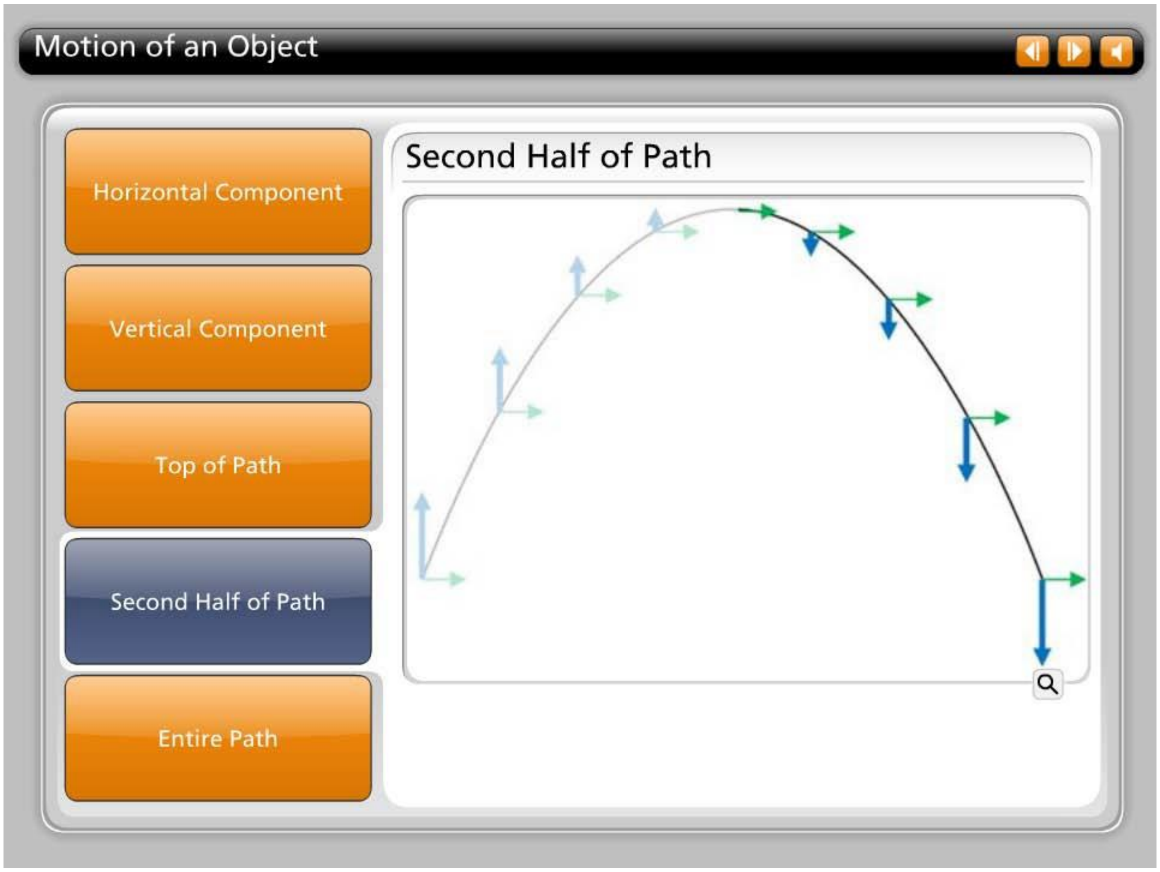
You should recall from your freefall lessons that for an object on level ground, the initial vertical velocity will be equal in magnitude, but in opposite direction to the final vertical velocity with which the projectile returns to the ground. In equation terms, we'd write v equals negative v zero.

This is the case due to the symmetry of the problem. As the projectile rises and falls, the acceleration is constant. Since the distance traveled is the same, it takes the same amount of time for each half of the motion and the changes in velocity are the same.



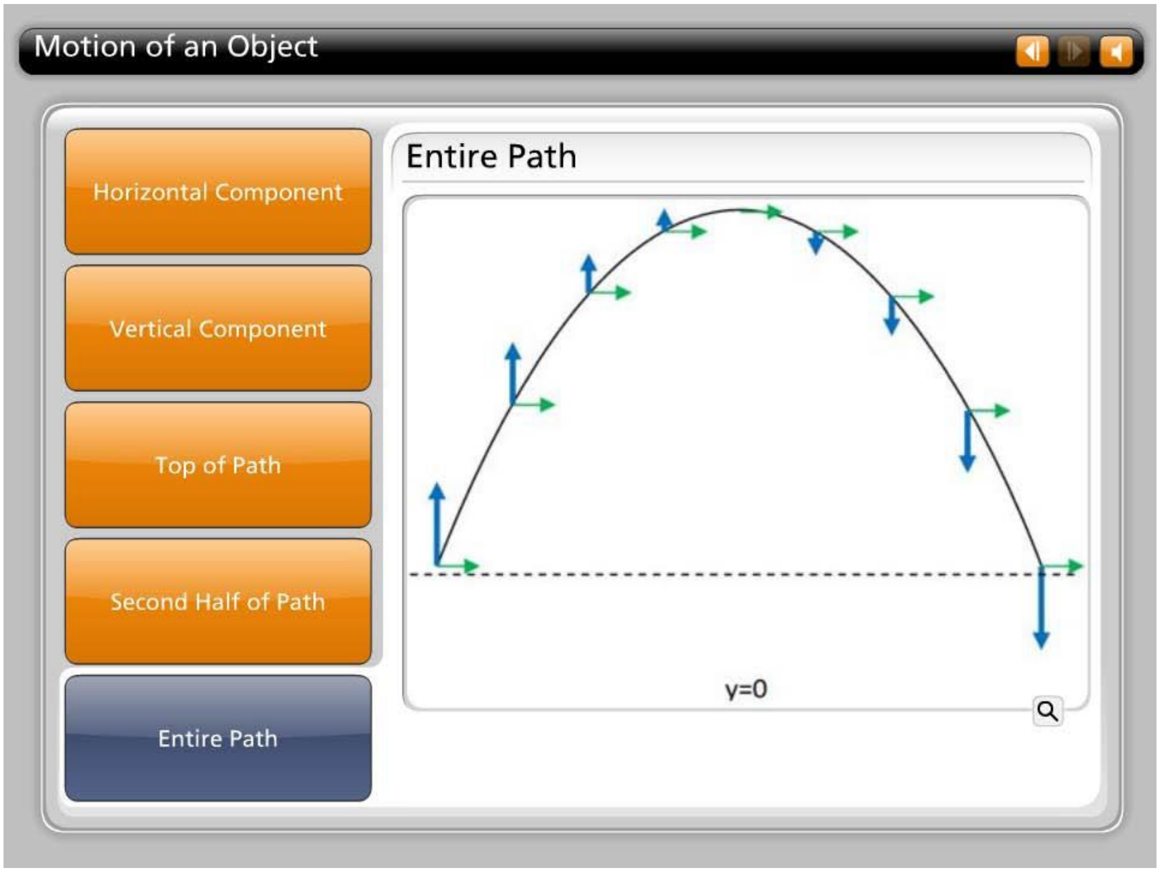
What happens to the blue arrows at the top of the path? You should notice that the vertical component of velocity reaches zero at the highest point of motion. You should recall this from the lesson on freefall.

For projectiles that return to their original height, this happens halfway through the total time of flight. In fact, the maximum height, and therefore the time of flight, depends entirely on the initial vertical velocity.

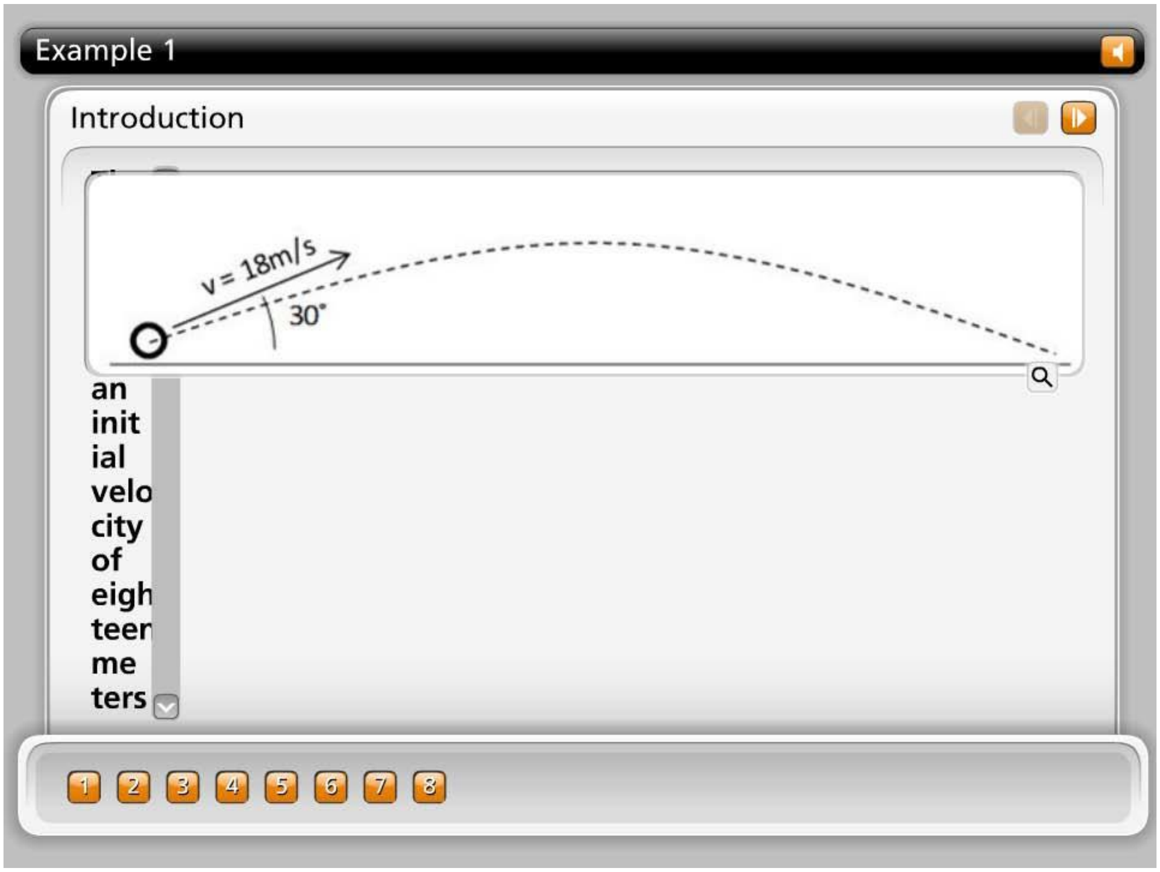


You may also notice that the second half of a level-ground projectile problem looks precisely the same as a horizontal projectile problem, as both begin with zero vertical velocity.

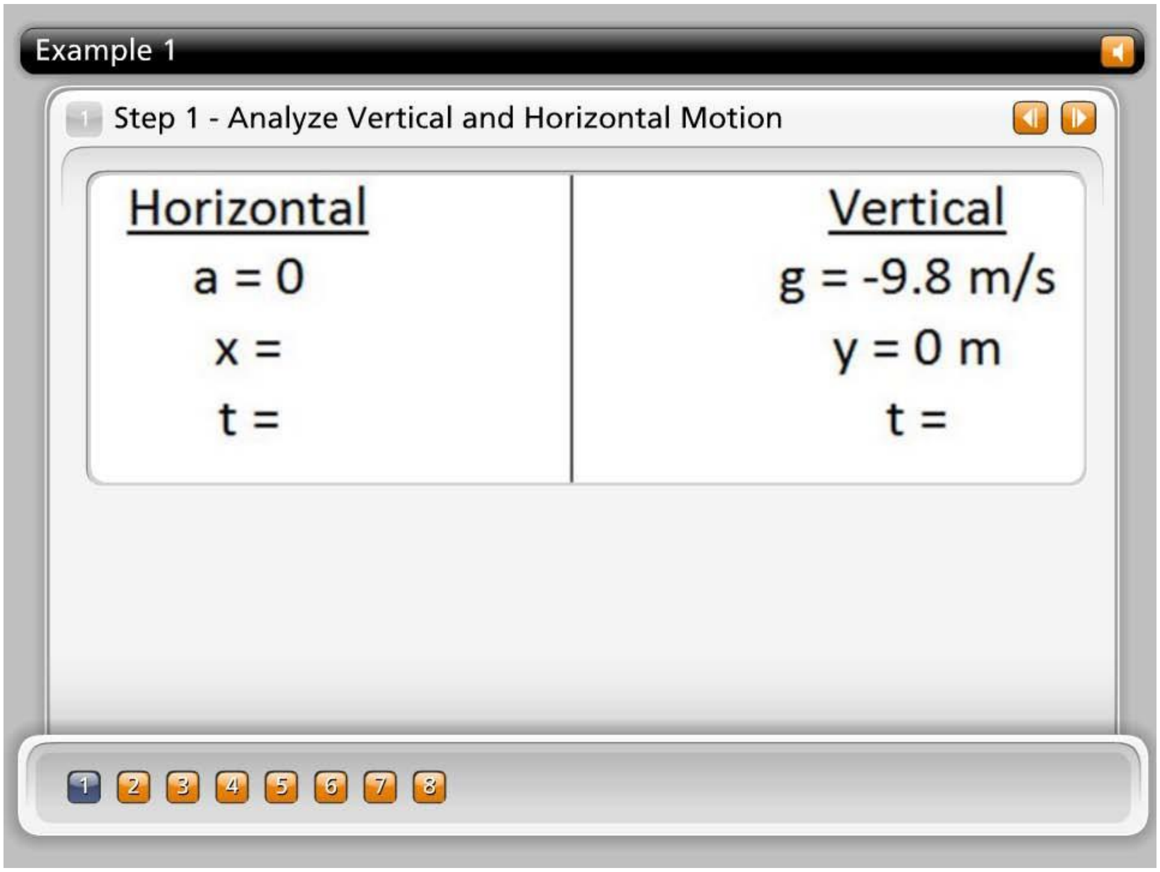
These are key factors that will help you to answer projectile motion problems.



Now look at the entire path of the object. How does the initial height compare to the final height? Since the object returns to the same height from which it began, the total vertical displacement will be zero from the beginning to the end of the flight,. We'd write that in an equation as y equals zero. We'll apply each of these phenomena as we look at several examples of projectile motion.



Let's take a case of a baseball thrown at an angle on level ground. The ball is thrown with an initial velocity of eighteen meters per second at an angle of thirty degrees above the horizontal. How far, horizontally, from the thrower does the baseball hit the ground?

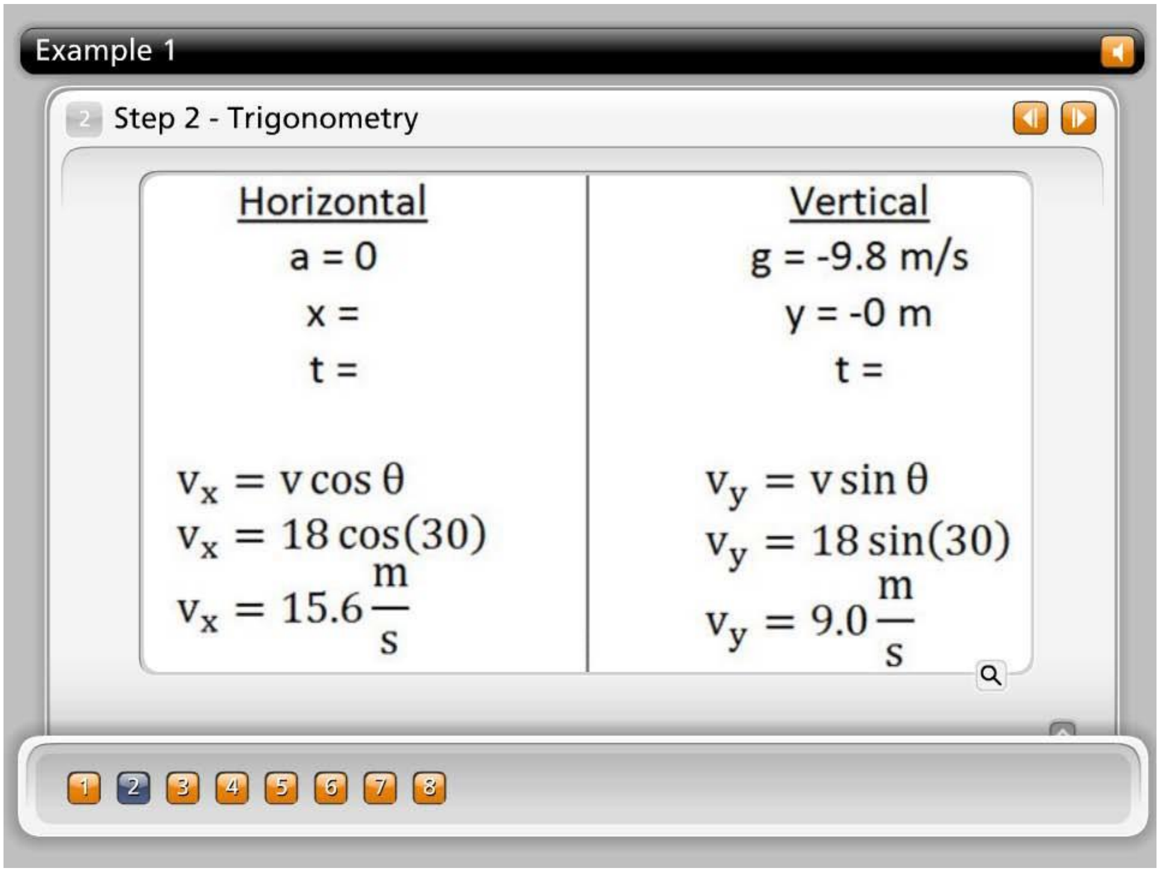


Our first step is to set up our paper to analyze horizontal and vertical motion separately, then write down what we know in each case.

Before writing any other variables down, we know that the horizontal acceleration is zero and the vertical acceleration is the acceleration of gravity, nine point eight meters per second squared.

We also know that once the baseball again reaches the ground, its vertical displacement will be zero. It will have gone up and back down again, returning to its original height.

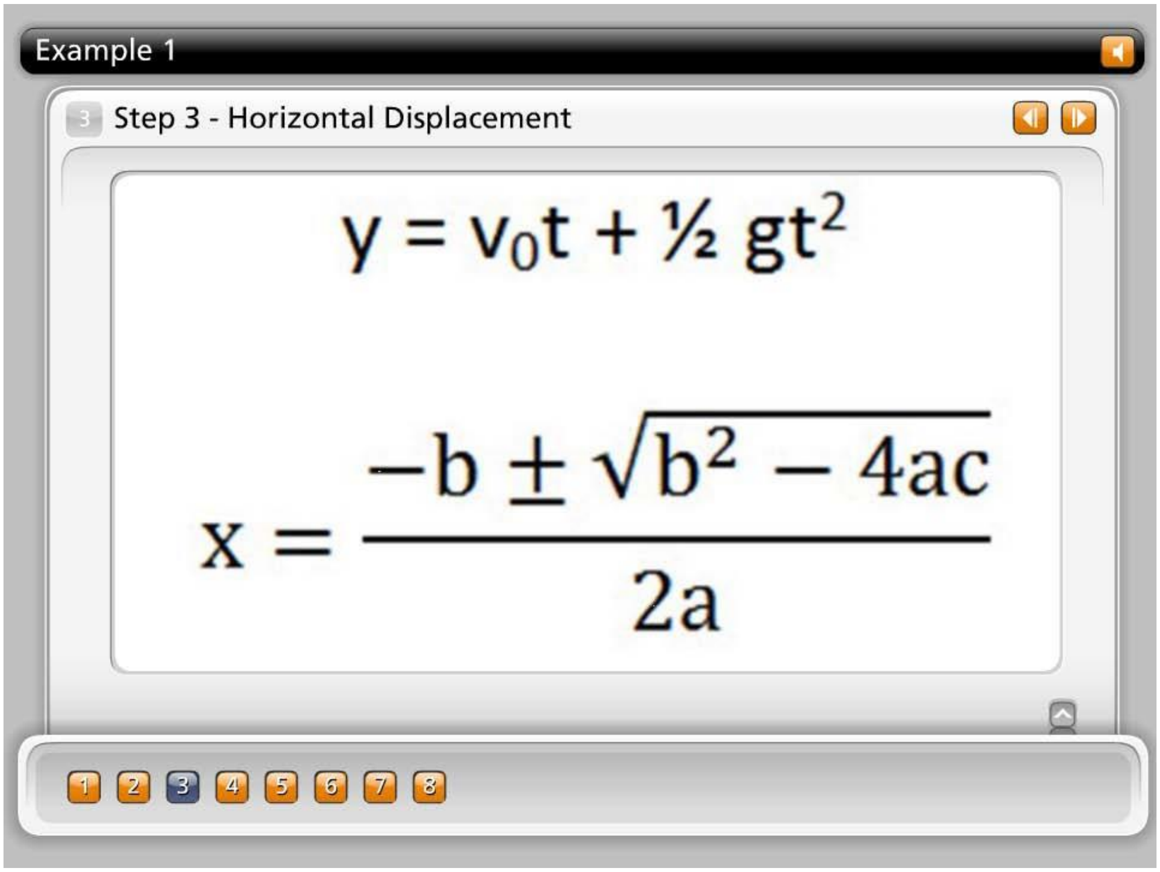
We do not know the horizontal displacement or the time of flight.



However, we do have a velocity. This velocity is not entirely horizontal, nor is it entirely vertical. So we have to use trigonometry to split the velocity into its horizontal and vertical components.

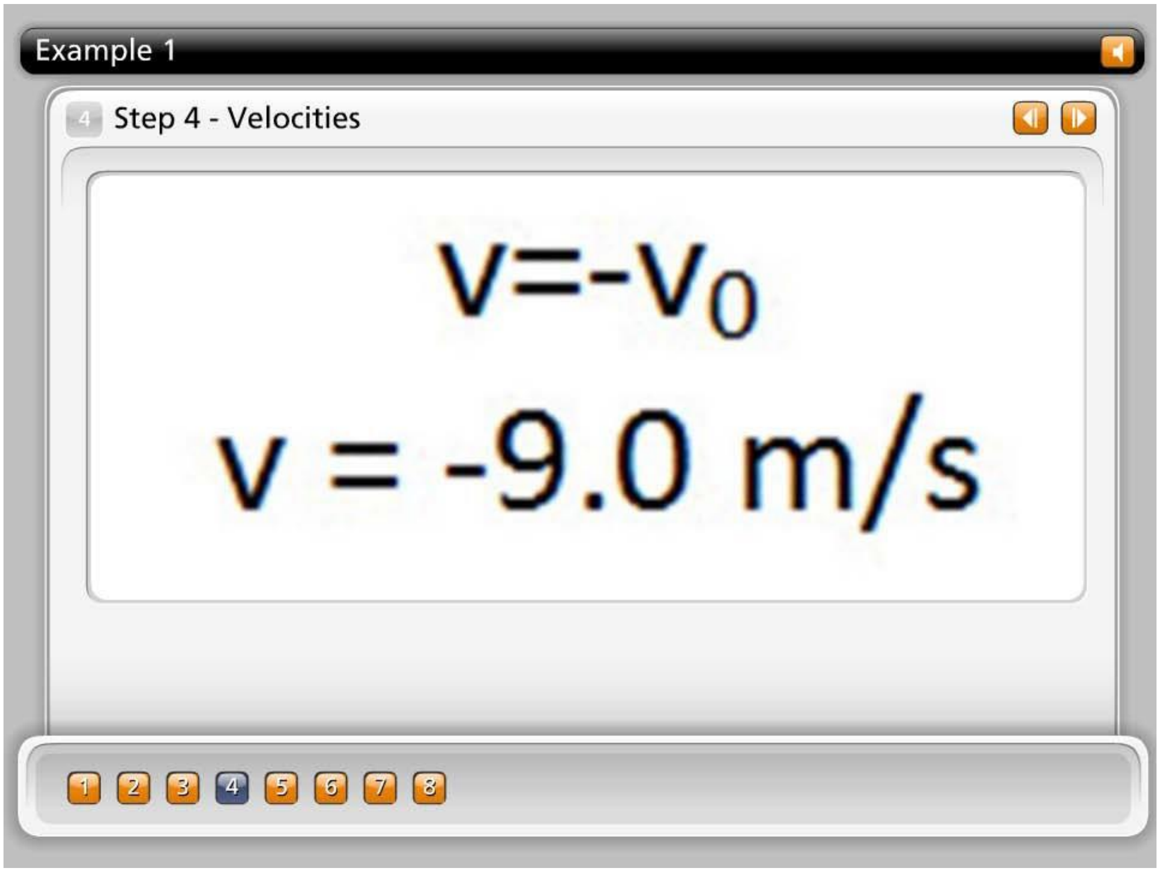
The initial velocity in the horizontal direction equals the true initial velocity times the cosine of the angle and the initial velocity in the vertical direction equals the true initial velocity times the sine of the angle.

Substituting and solving, we see that the initial horizontal velocity equals fifteen point six meters per second, and the initial vertical velocity equals nine meters per second. You should see from the illustration that the vertical velocity should be a positive value.

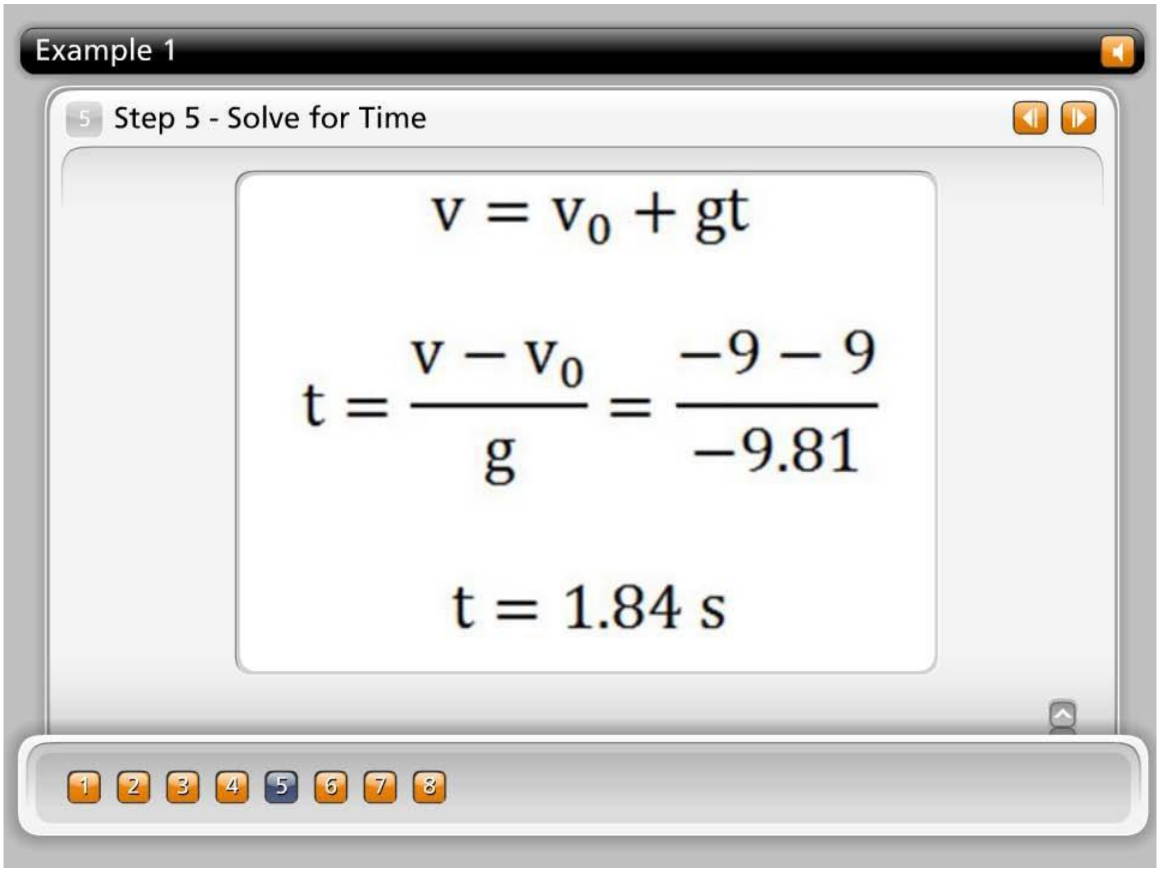


In order to determine the horizontal displacement, we must determine the time of flight, but there is not enough information in the analysis of horizontal motion to determine this, so we have to look at the analysis of vertical motion to determine the time of flight.

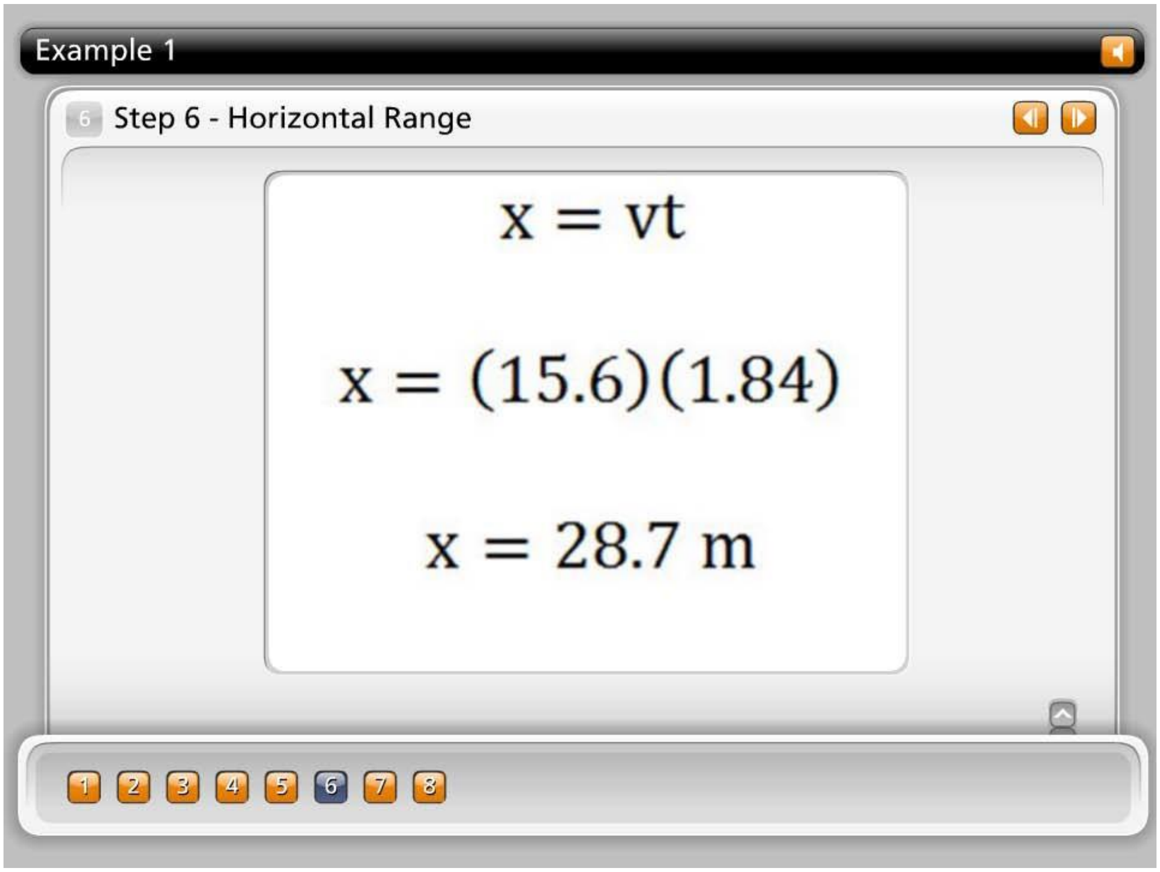
We know the acceleration, the initial velocity and the displacement, so we could use the equation y equals v zero t plus one half g t squared to determine the time of flight. You will quickly discover that solving for t requires using the quadratic formula.



Some students prefer not to use the quadratic formula unless absolutely necessary, and in this case, the symmetry of the situation allows us an easier solution. We know that the final vertical velocity will be the same magnitude but opposite direction as the initial vertical velocity.

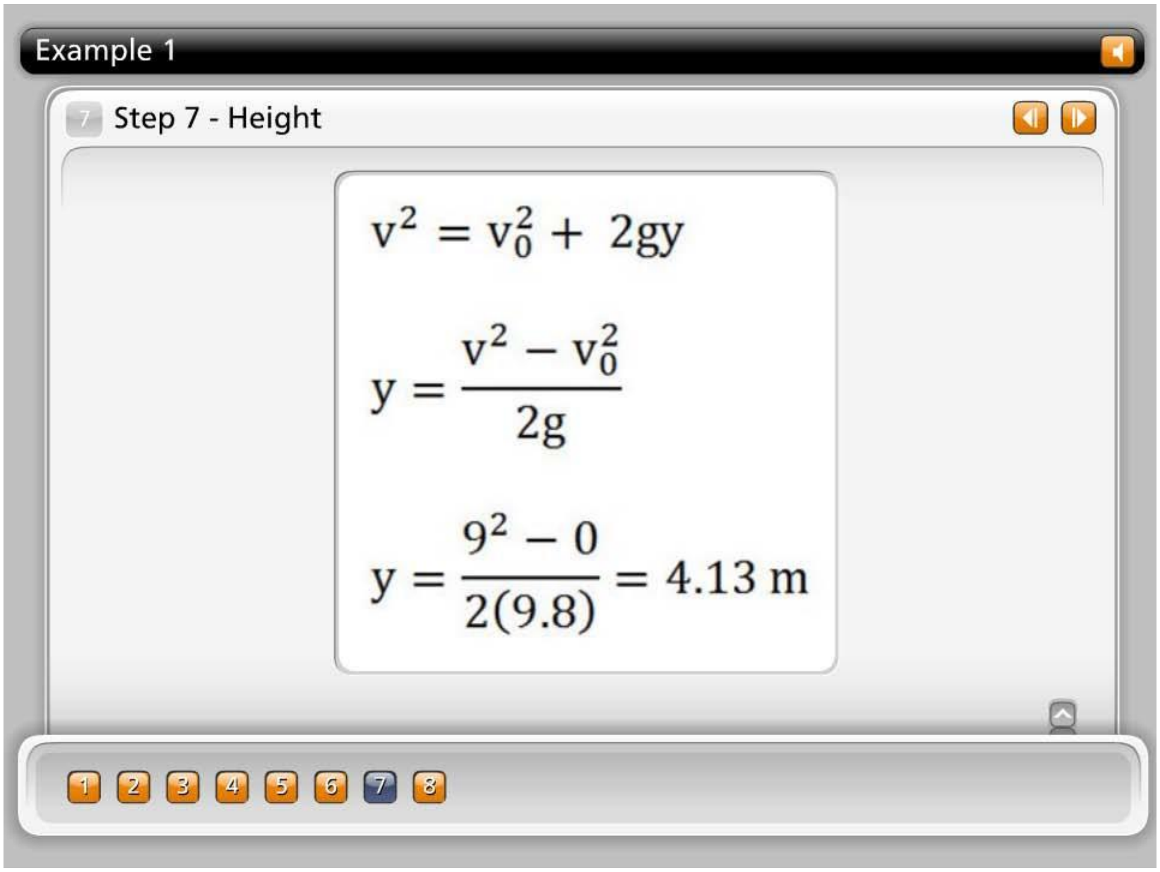


With this additional variable, we can use the formula v equals v zero plus g t to solve for time. Substituting our values, we see that the time of flight is one point eight four seconds.



Now we can return to our horizontal analysis and use the equation x equals v times t to determine the horizontal range of the baseball.

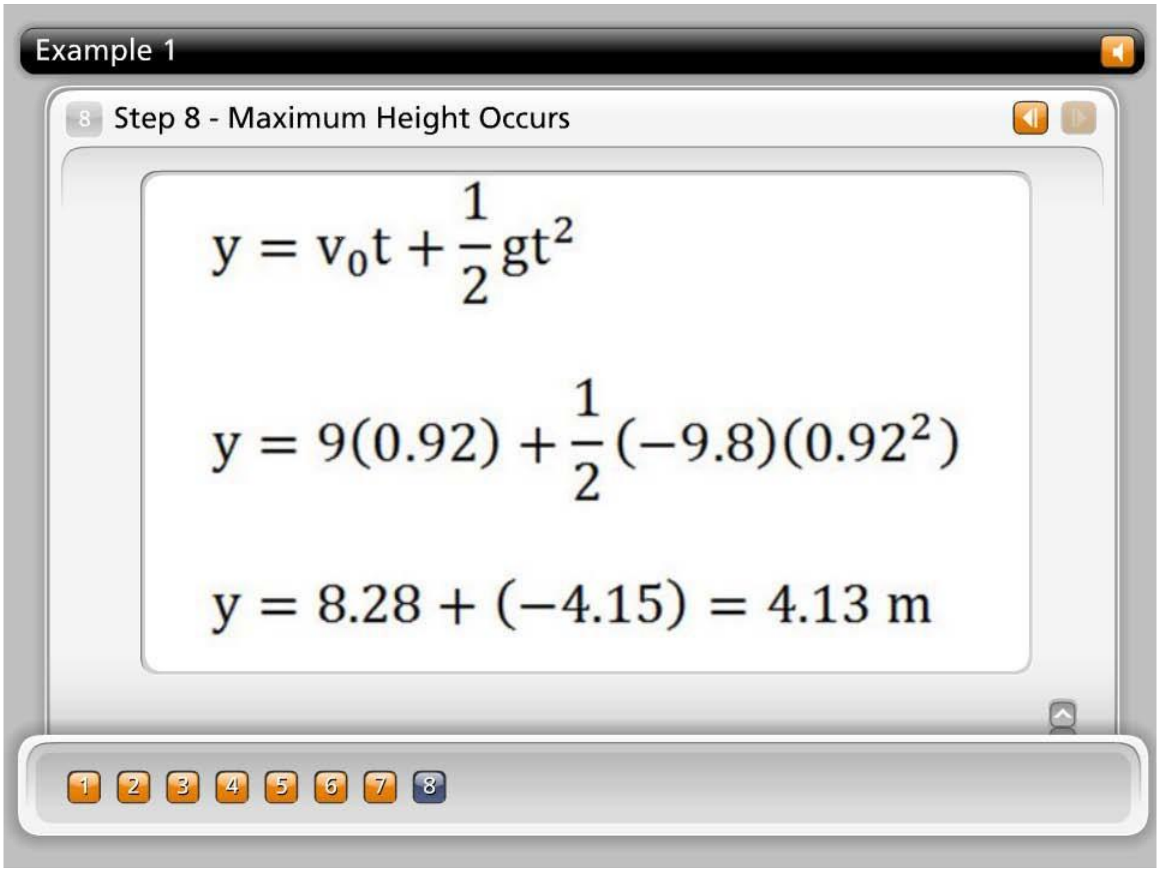
Substituting and solving, we see that the baseball travels twenty eight point seven meters horizontally as it completes its motion.



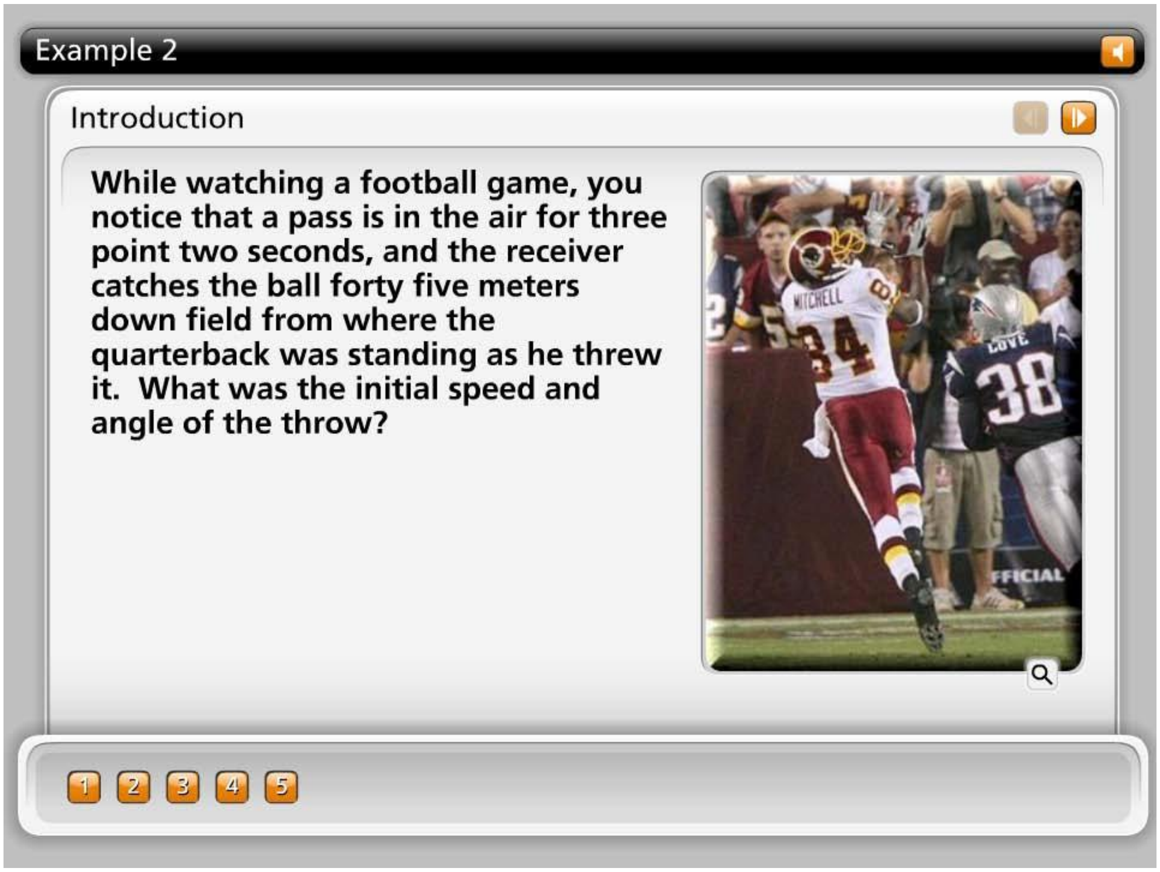
Using the same situation, let's now determine how high in the air the baseball rises at its highest point.

We have a few ways to solve this, and each should give the same answer.

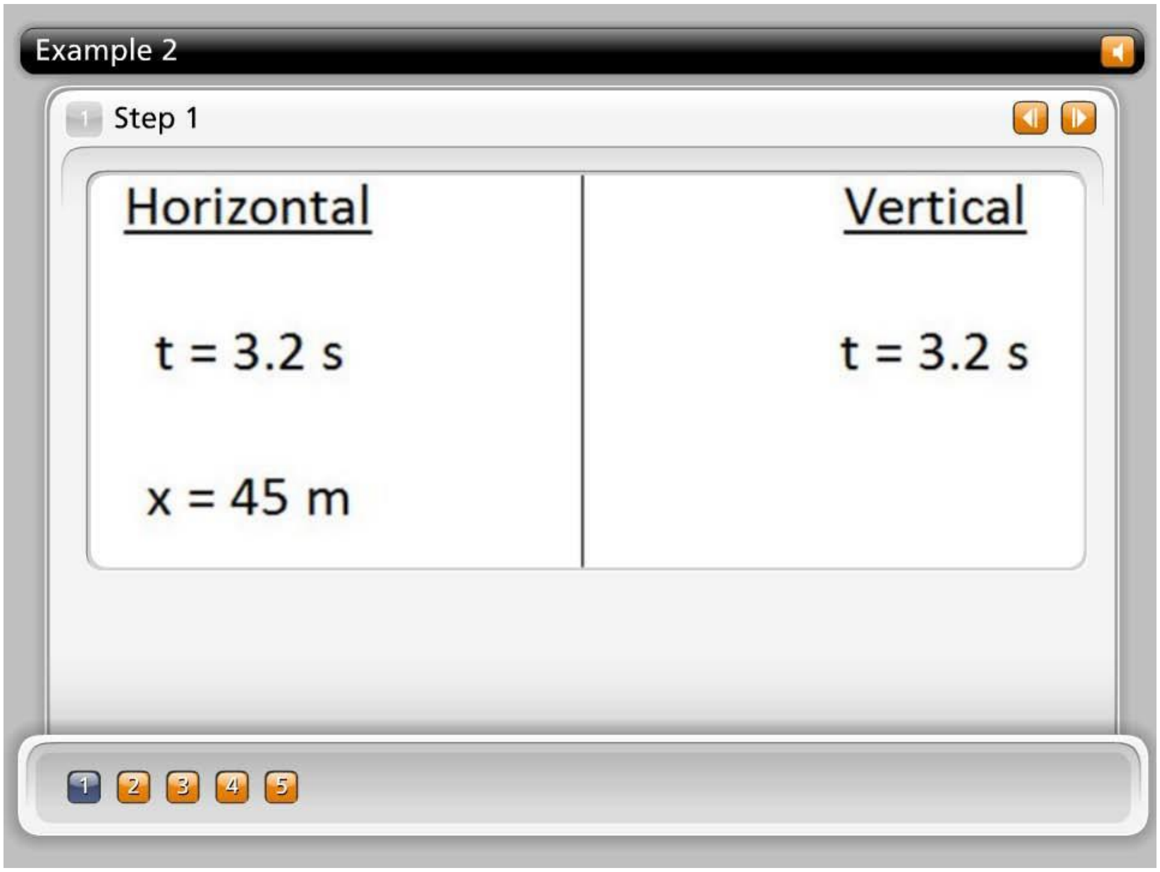
The first method recognizes that when a projectile is at its highest point, the vertical component of the velocity is momentarily zero. Since we know the initial velocity and the acceleration, we can use the equation v squared equals v zero squared plus two g y and solve for y. Substituting our values, we find that the vertical displacement is four point one three meters.



We could also have realized that the maximum height occurs at half the time of flight when the projectile is launched over level ground. Earlier, we determined that the time of flight was one point eight four seconds, so the maximum height would occur at zero point nine two seconds. We can use the equation y equals v zero t plus one half g t squared. Substituting and solving, again we see that the maximum height is four point one three meters.



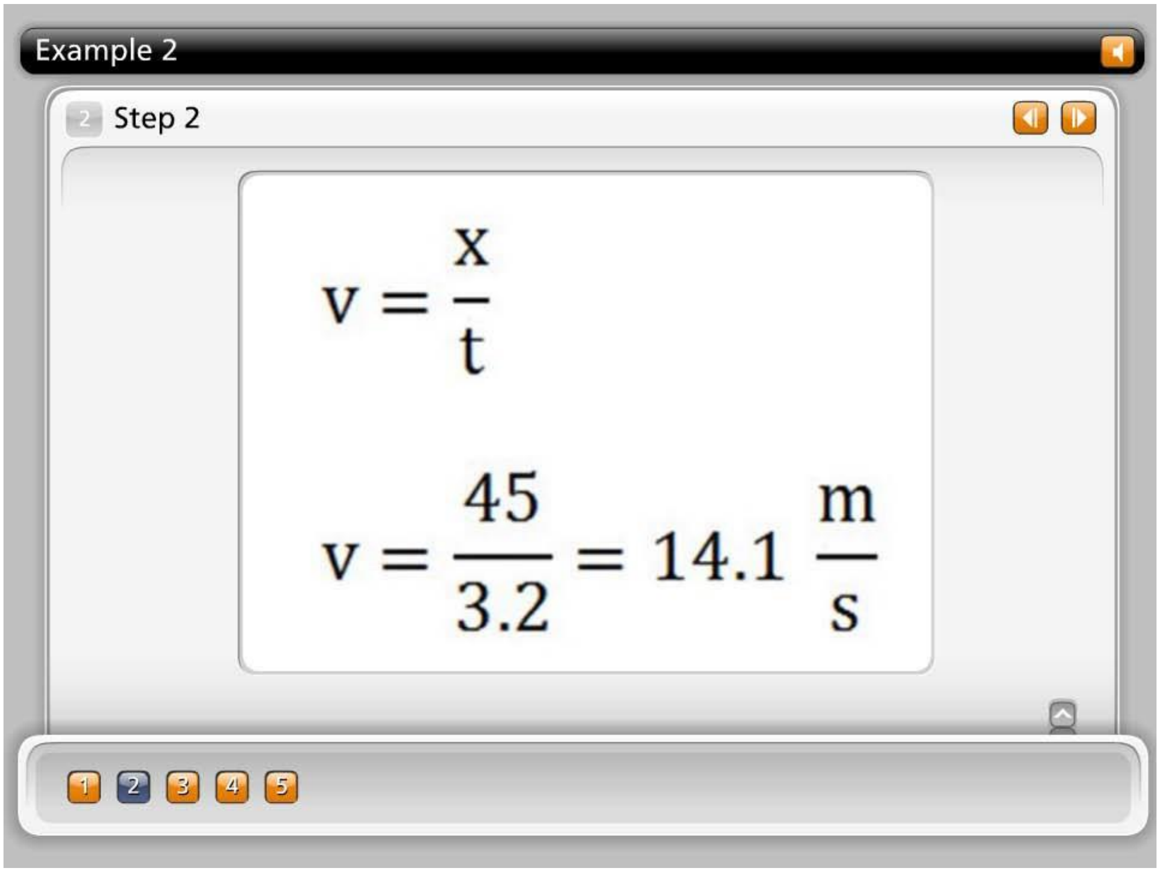
Here's another example. While watching a football game, you notice that a pass is in the air for three point two seconds, and the receiver catches the ball forty five meters down field from where the quarterback was standing as he threw it. What was the initial speed and angle of the throw?



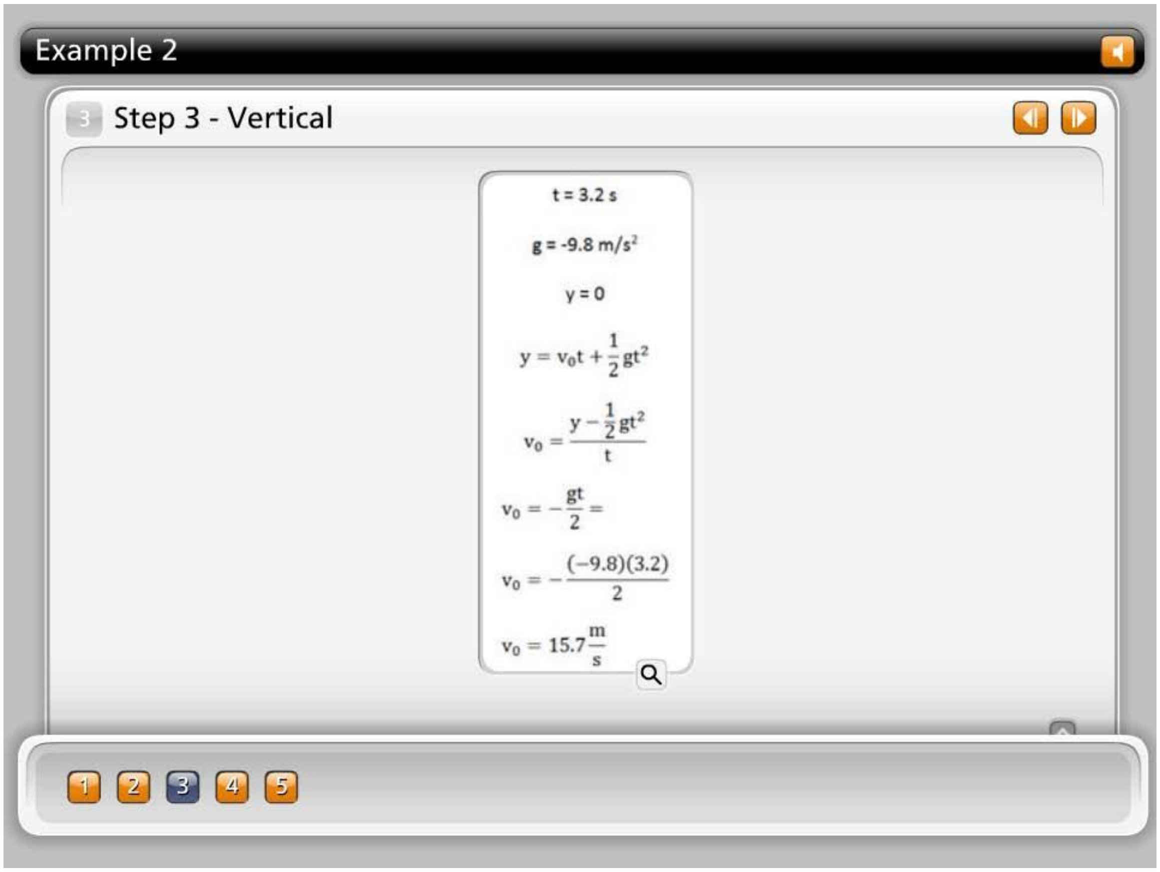
As we always do with projectile motion problems, we're going to separate the horizontal and vertical motion and analyze them separately.

In order to determine the initial speed and angle of the throw, we'll need to know both the initial horizontal and initial vertical velocity. From these, we will be able to re-combine them using trigonometry to determine the original velocity.

We know the ball was in the air for three point two seconds, and we can place this both in the horizontal and vertical analysis. We also know that the ball traveled a horizontal distance of forty five meters down the field. So we can write x equals forty five meters.



Now, we know enough to find the horizontal component of the velocity. With no acceleration, velocity equals distance over time, or v equals x over t. Substituting, we find that the horizontal component of the velocity equals fourteen point one meters per second.



Now, let's see what we can determine in the vertical direction.

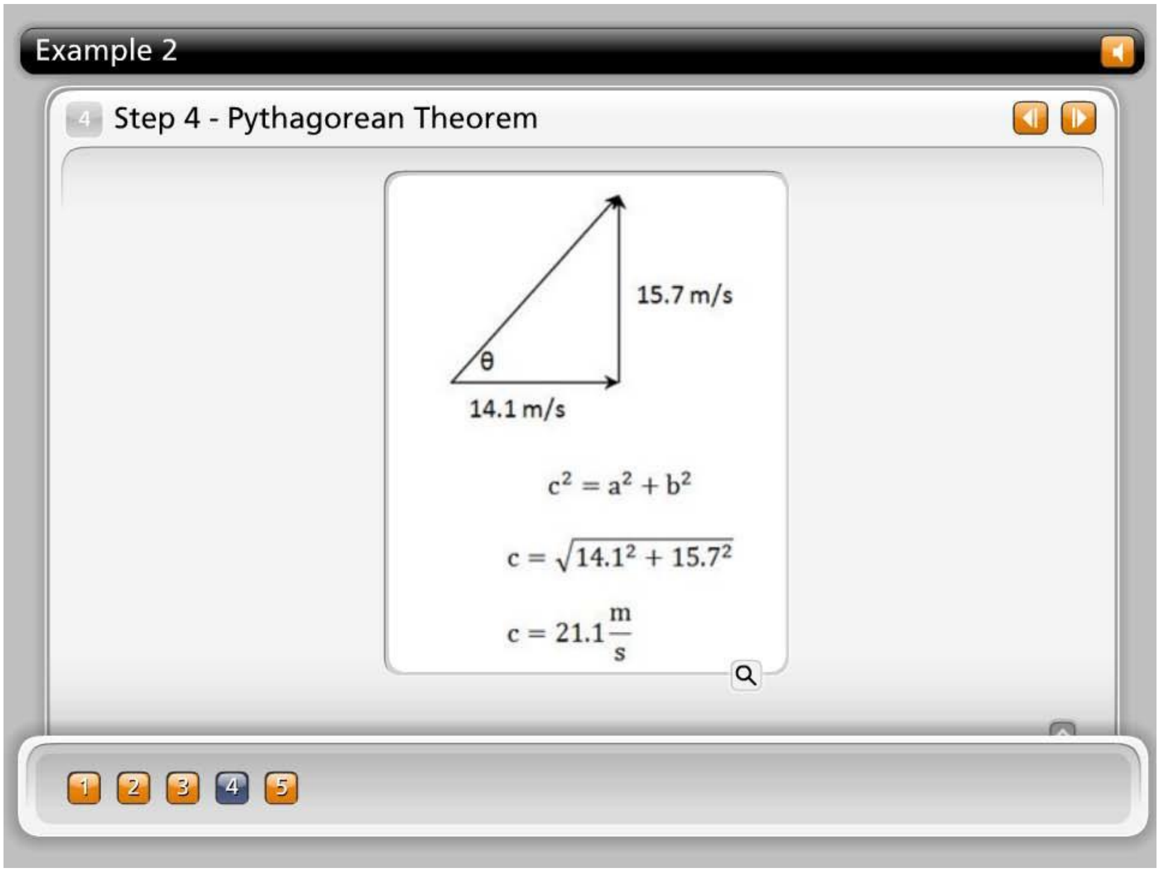
The time is equal to three point two seconds. The acceleration is equal to nine point eight meters per second squared in the negative direction.

This does not seem to be enough information to determine the initial velocity, because all of our kinematics equations require three variables to determine the fourth. But if we recognize that the ball goes up and then comes back down to its original height, we know that the vertical displacement is zero. So we can write y equals zero.

Which equation uses time, acceleration, displacement and initial velocity? It is y equals v zero t plus one half g t squared.

We can now rearrange this equation to solve for v zero.

Recognizing that y equals zero, we can simplify the equation even further. Substituting and solving, we find that the initial velocity equals fifteen point seven meters per second.

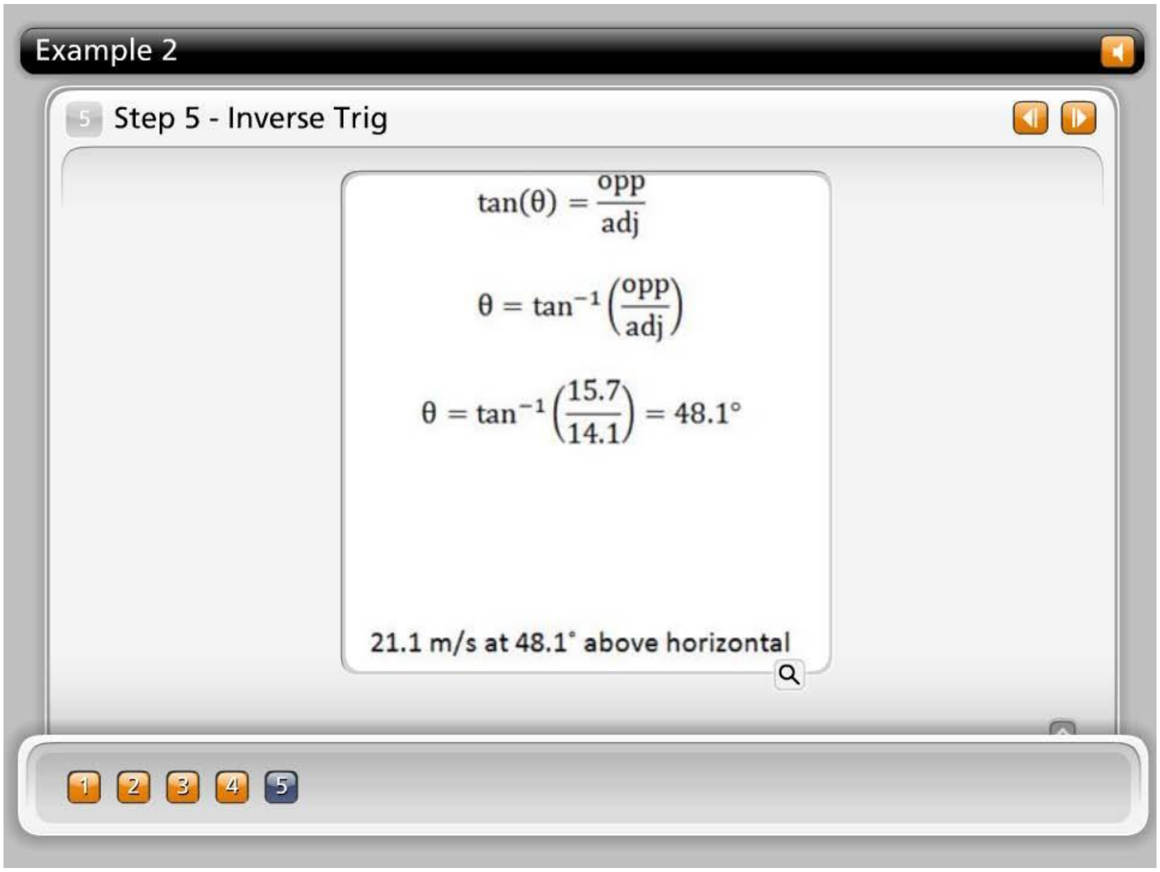


Now that we know the initial horizontal and vertical components of the velocity, how do we combine them to determine the original initial velocity?

Since we used trigonometry earlier to convert a vector into its horizontal and vertical components, we can again use trigonometry to recombine vector components into the original vector.

We can arrange the horizontal and vertical components head to tail to form the two legs of a right triangle with the hypotenuse representing the original vector.

The magnitude of the hypotenuse can be found from the Pythagorean theorem.

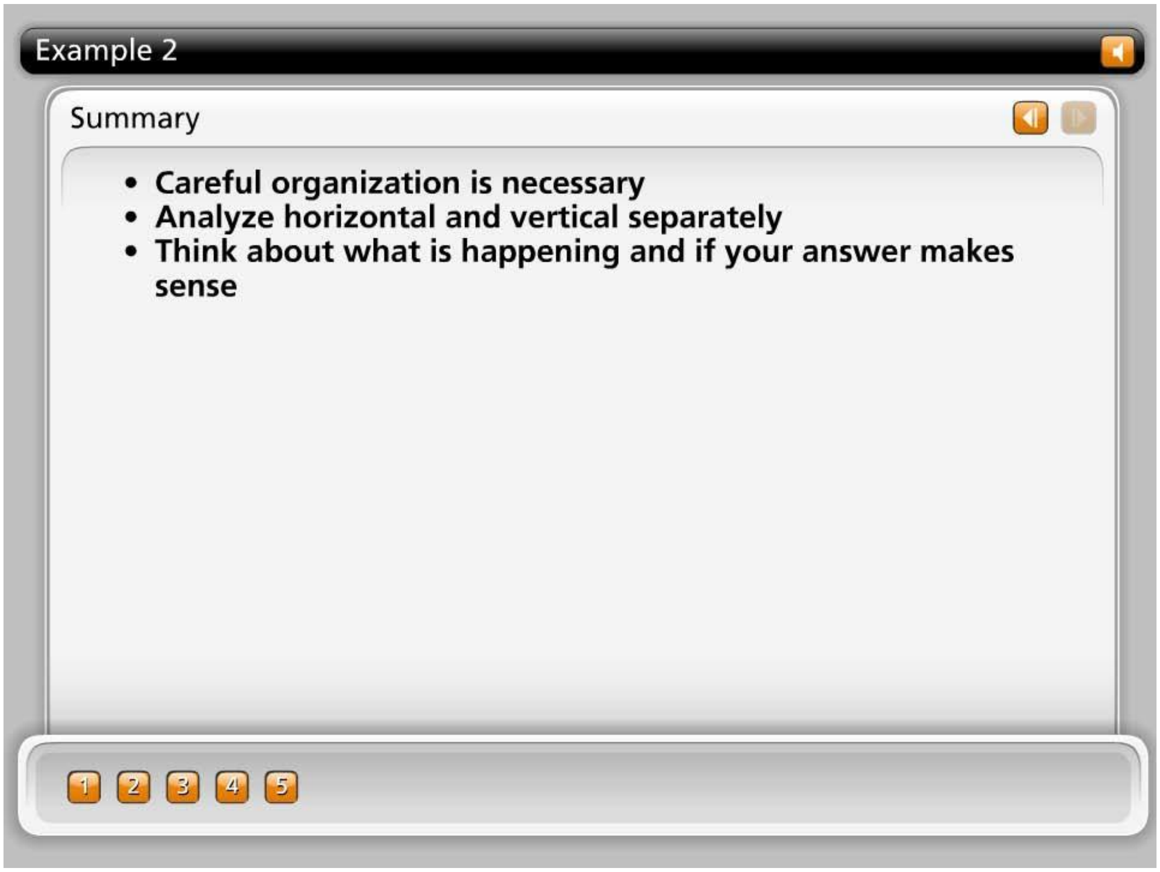


To determine the angle, we need to use an inverse trig function. We can see that the vertical component is opposite the angle and the horizontal component is adjacent to the angle.

Therefore, the inverse tangent function should give us the angle.

Since the tangent of the angle is the opposite divided by the adjacent, the angle will be the inverse tangent of the opposite divided by the adjacent. Substituting and solving, we see that the angle is equal to forty eight point one degrees.

So the ball was thrown with a speed of twenty one point one meters per second at an angle of forty eight point one degrees above the horizontal.



We've now seen a couple of situations involving projectiles launched at an angle. These problems may seem complex, but they become quite manageable if you are willing to work with discipline, always organizing what you know and carefully analyzing horizontal and vertical motion independently.

Also, you must always think about what is happening in the problem before you jump to equations. This will give you a sense of what the answer should look like before you solve for it.