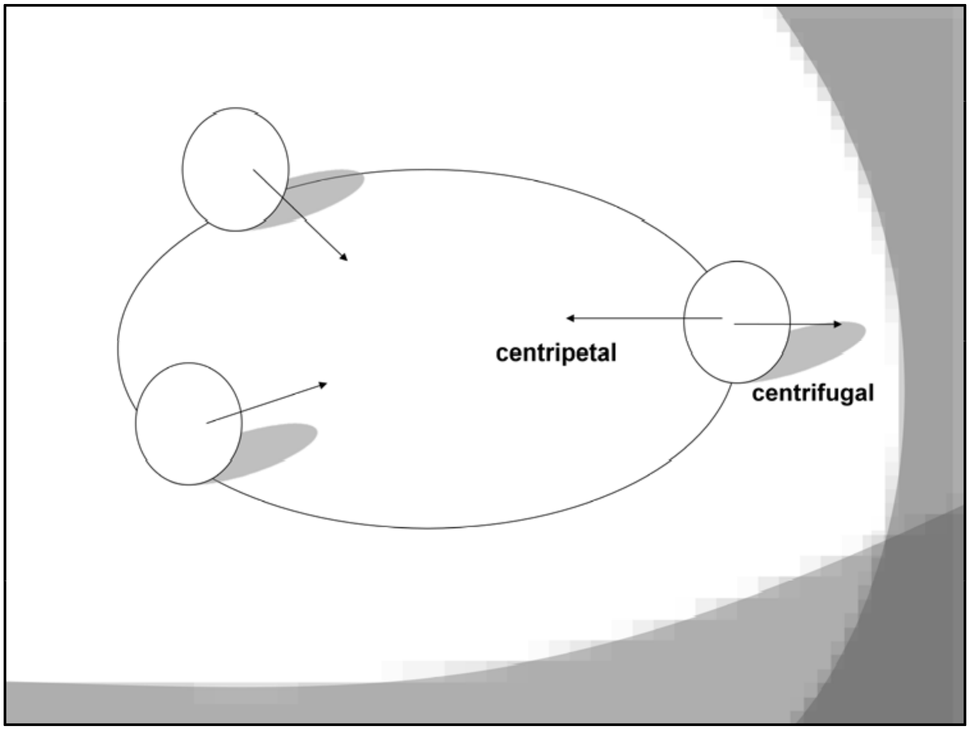
Module 3: Motion in Two Dimensions
Topic 4 Content: Uniform Circular Motion Presentation Notes

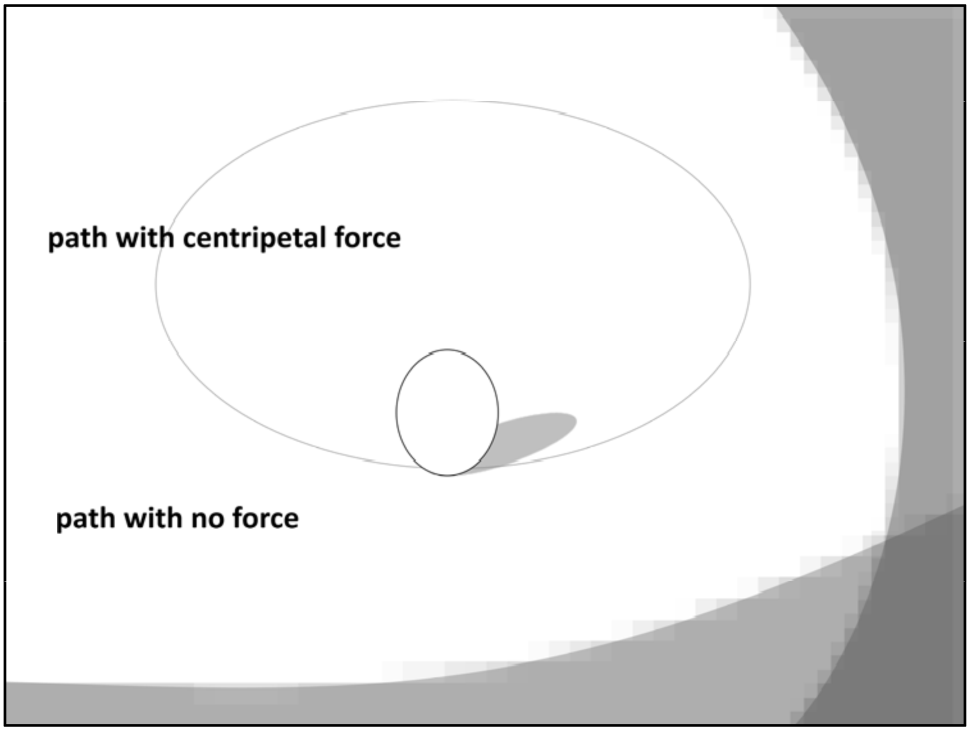
Uniform Circular Motion



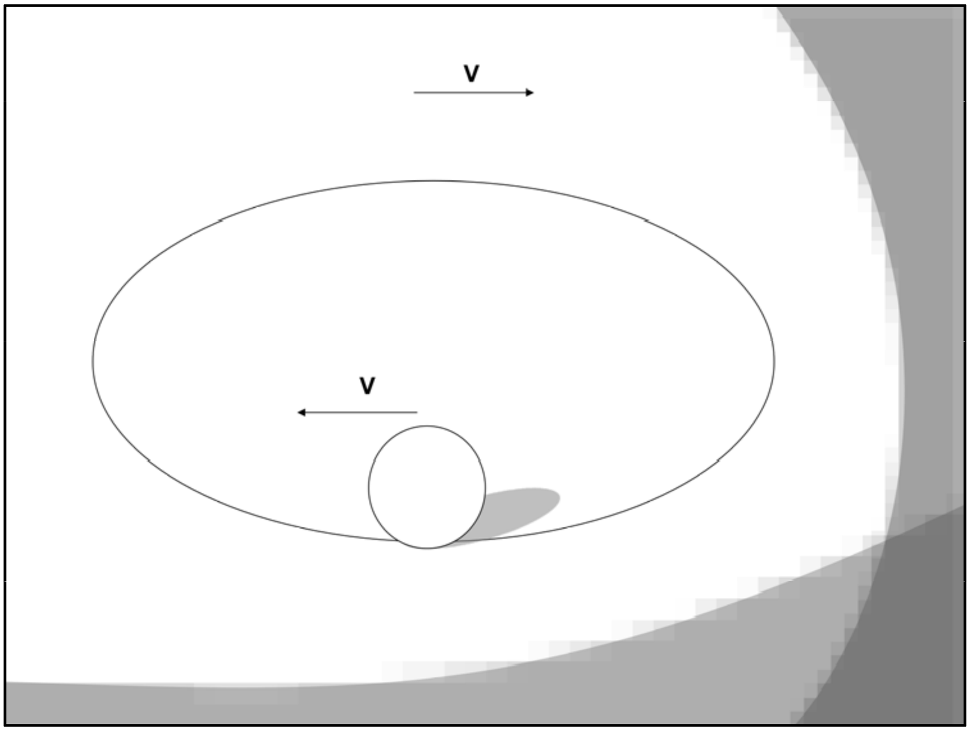
Think carefully about your about the activities you have done thus far. To keep a ball moving in a circle, how did you apply a force to the ball? If you applied it in the direction of the motion of the ball, tangent to the circle, it would speed up and move in a straight line. In order for it to keep turning around the circle without speeding up, you should have noticed that you had to continually tap it from the outside, directly towards the center of the circle.

This sort of directed-to-the-center force is called a centripetal force, and it is what we will be studying.

You may have heard the term centrifugal force. A centrifugal force would point directly away from the center of a circle. Sometimes it may appear that there is such a force, such as when you are driving in your car with a bag on the passenger seat. If you take a turn to the right that is too fast, the bag may appear to move to the left. However, what is really happening is that there was not enough force to make the bag move in the new direction and it merely continues moving forward in the direction it was already moving. It is therefore incorrect .



To see this, you can look at a situation where the center-directed force is completely removed. If this happened, the object would move in a straight line, tangent to the circle according to Newton's first law. It moves in a circle due to the centripetal force acting on it.



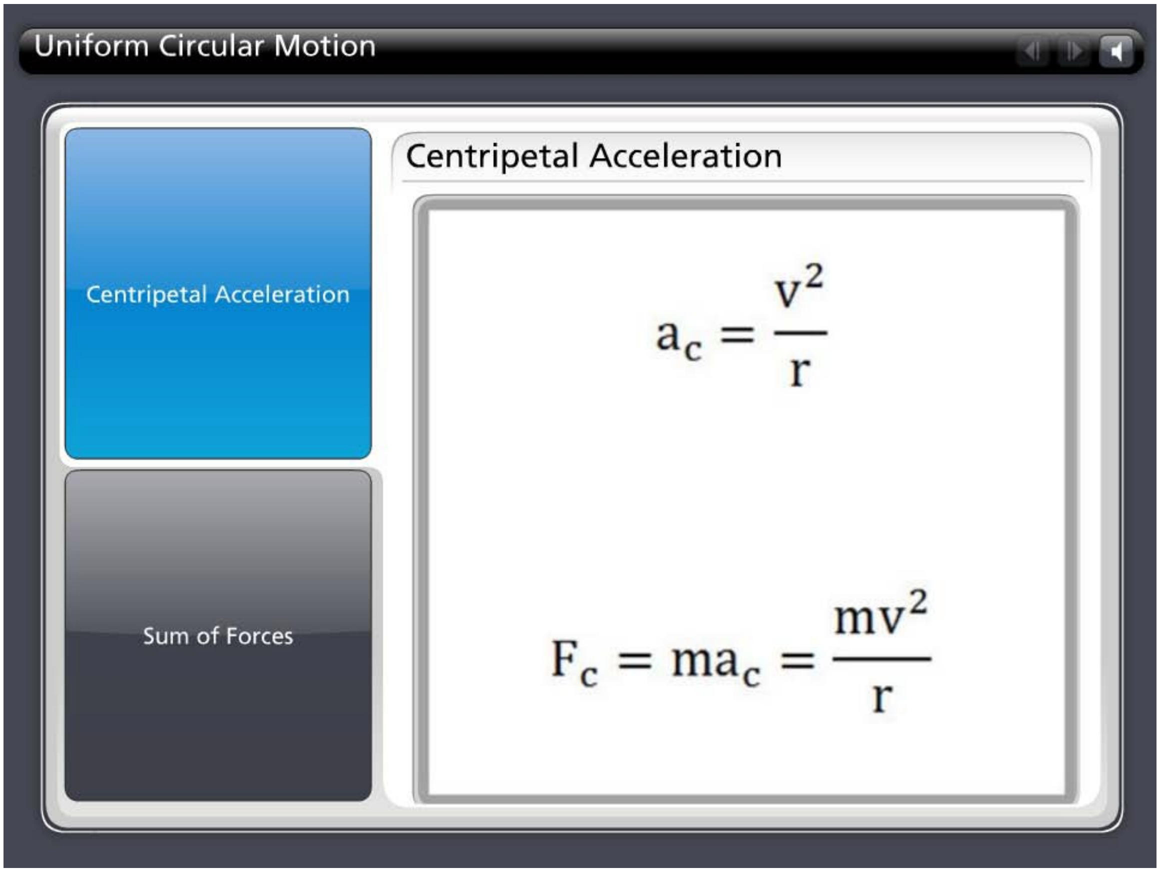
Now think about the ball moving at a constant speed in a circle. Is the ball accelerating?

Your first reaction might be that since it is moving at a constant speed, it is not accelerating. However, we were very specific in defining acceleration as a change in velocity over time, and remember that velocity is a vector quantity that includes direction.

With circular motion, even at a constant speed, the direction is constantly changing, so the object must be accelerating. Also, think back to the activity where you were kicking the ball. The acceleration could only be zero if the net force was zero. What forces acted on the ball? The normal force and gravitational force cancel out, but your kick continued to act as an unbalanced force directed towards the center of the circle that accelerated the ball towards the center as Newton s Second Law tells us it will.

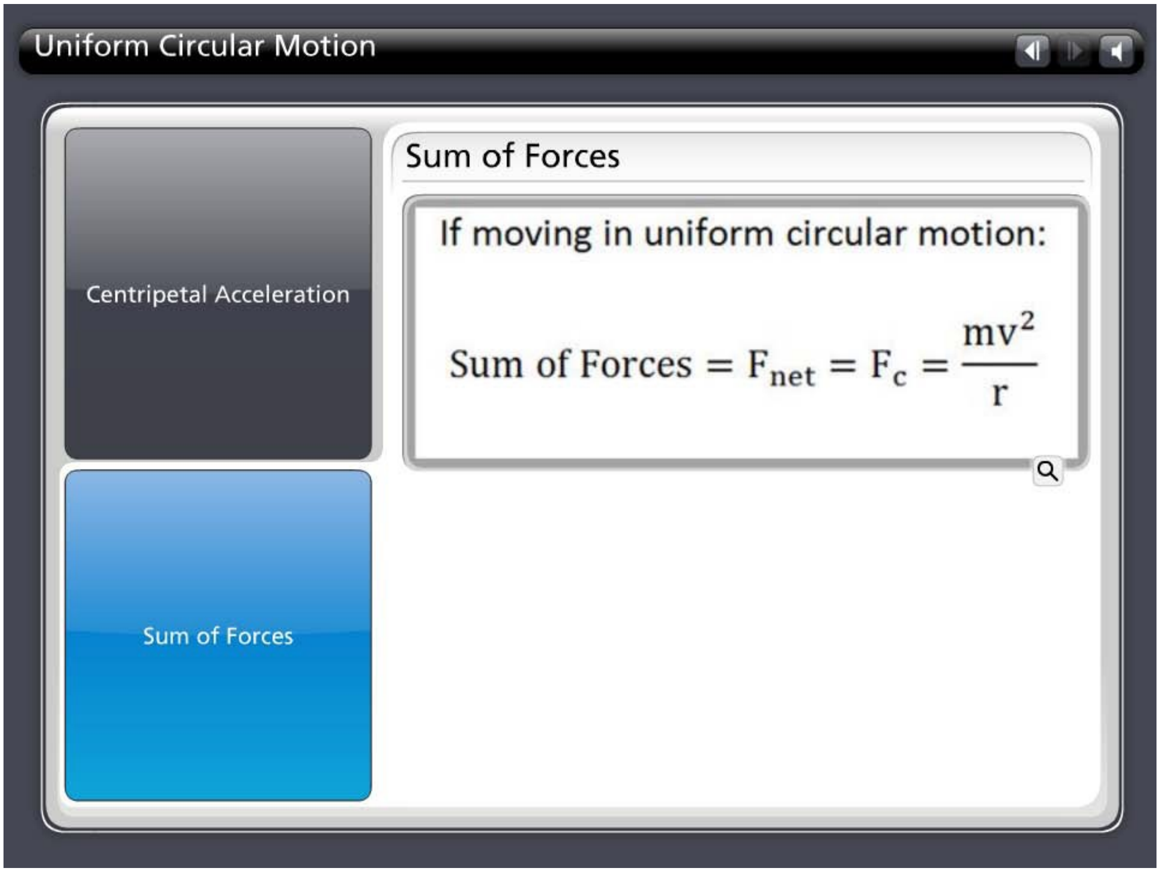
When an object moves in a circle at a constant speed, we call this uniform circular motion. There are many situations that are not quite uniform circular motion, but which we can use the equations of uniform circular motion to analyze. A car driving in a circle, or over a curved hill; a moon in orbit around Saturn; a ball being spun around on a string; even the motion of a person on a swing can be approximated in part as uniform circular motion.

Even when an object is not moving in a complete circle, or if the path is not truly circular, any part of a curved path can be approximated by a circle that fits the curve. When we are talking about the radius of the path, we're talking about the radius of a circle that best fits that portion of the path.

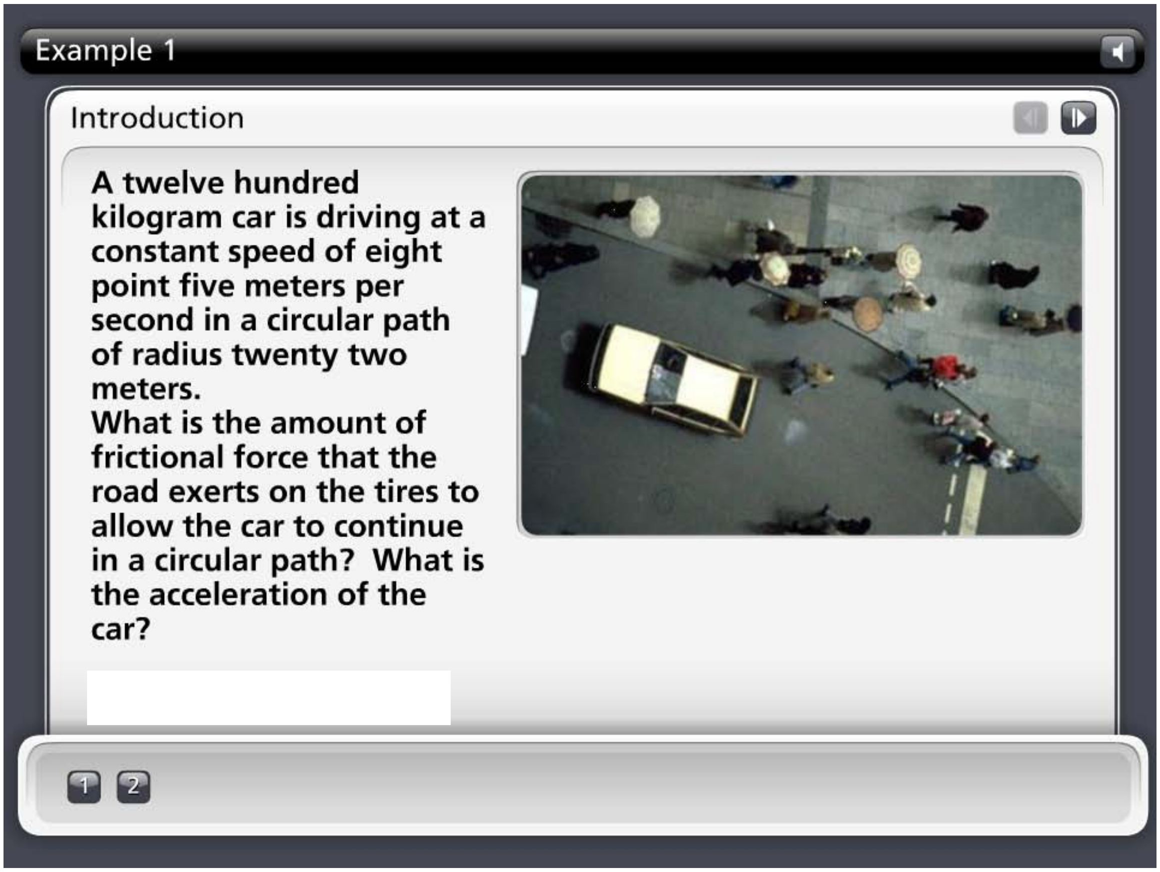


So what is the acceleration of an object moving in a circle at a constant speed? This is a reasonably straightforward and easy to follow derivation, which you can link to in the supplemental materials of this lesson. It turns out that for uniform circular motion, the centripetal acceleration is the velocity squared divided by the radius of the circle, or a c equals v squared over r.

You can use Newton's second law to determine the force required to move an object in a circle at a constant speed, or the centripetal force. Since the force equals the mass times acceleration, you can write that the centripetal force equals the mass times the centripetal acceleration, or F c equals m v squared over r.

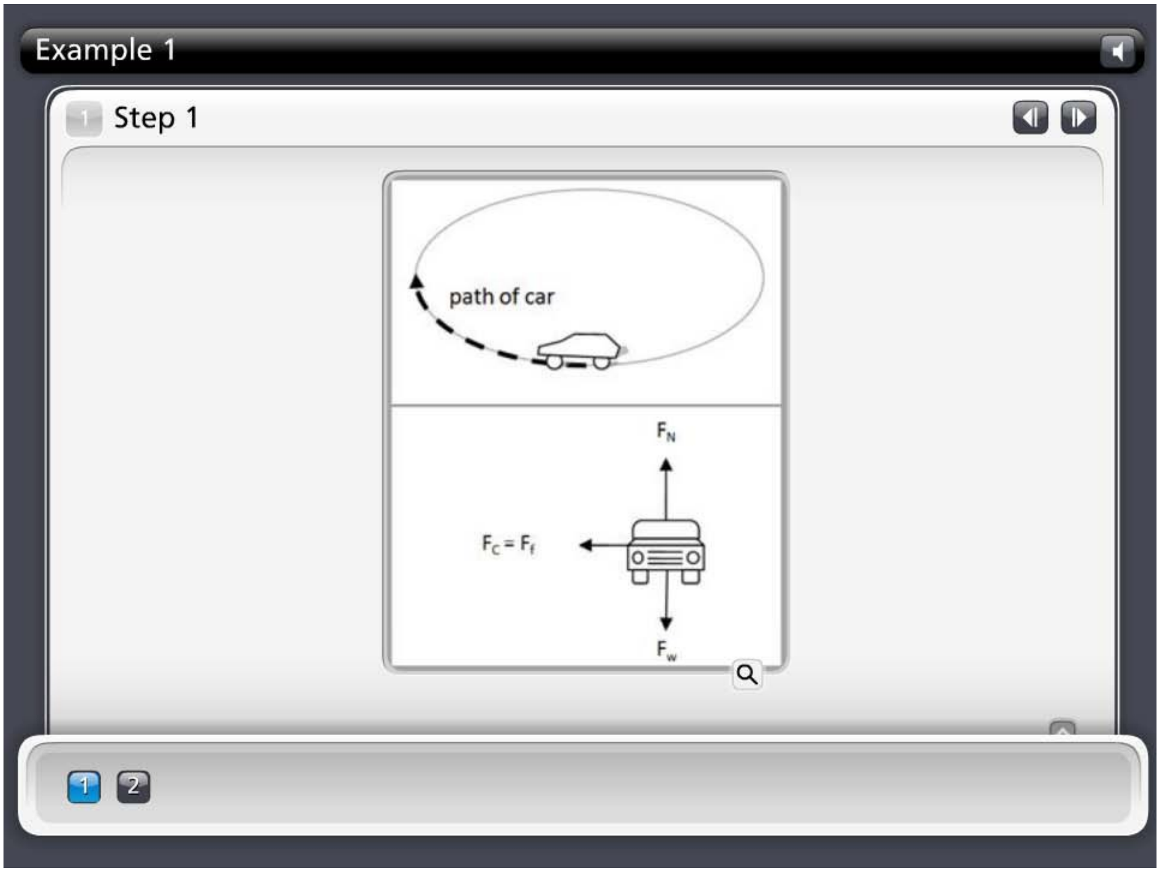


Note that the centripetal force is always the result of real, physical forces or combinations of forces. It may be due to tension in a rope. It may be due to friction. It may be due to the gravitational force. It may be due to a normal force. But each time you have an object moving in a circular path, some combination of real forces results in the net force being directed towards the center of the circular path with a magnitude of m v squared over r.

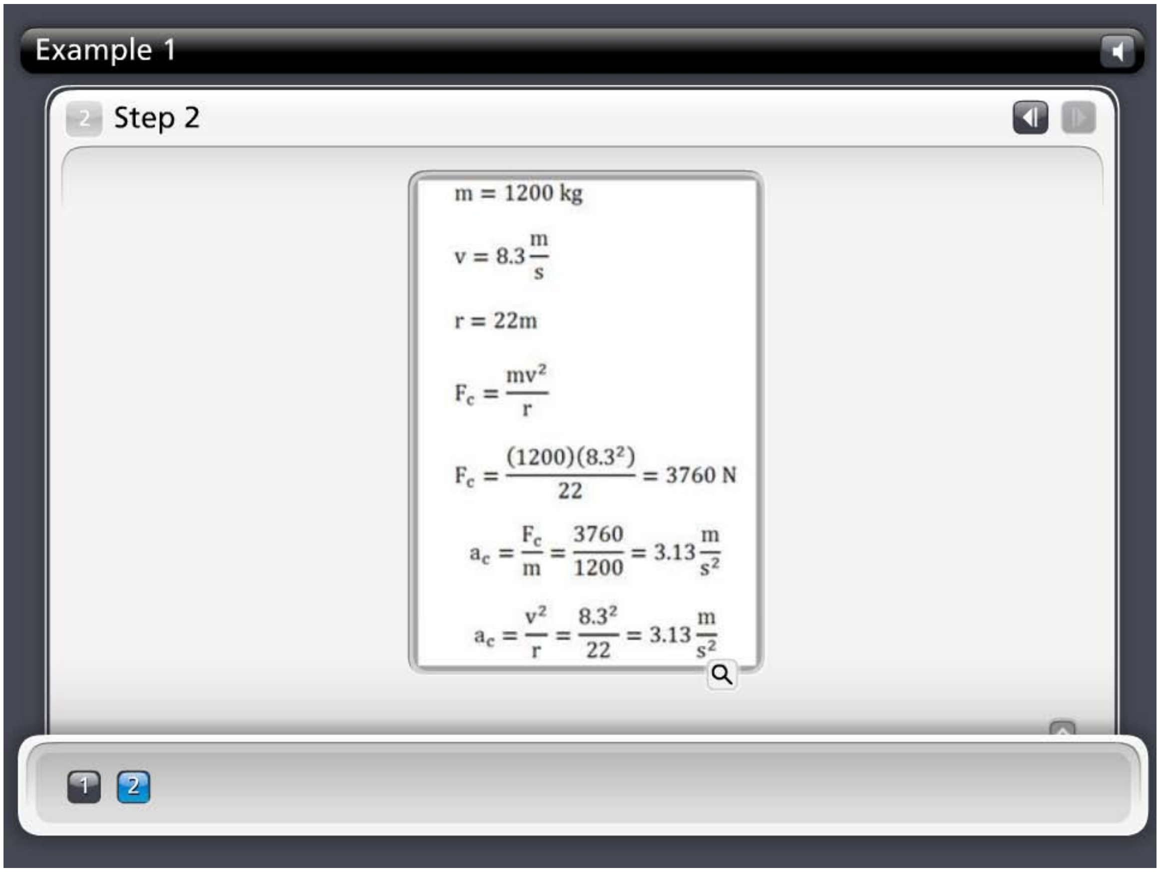


Let's take a look at a twelve hundred kilogram car driving at a constant speed of eight point five meters per second in a circular path of radius twenty two meters.

What is the amount of frictional force that the road exerts on the tires to allow the car to continue in a circular path? What is the acceleration of the car?



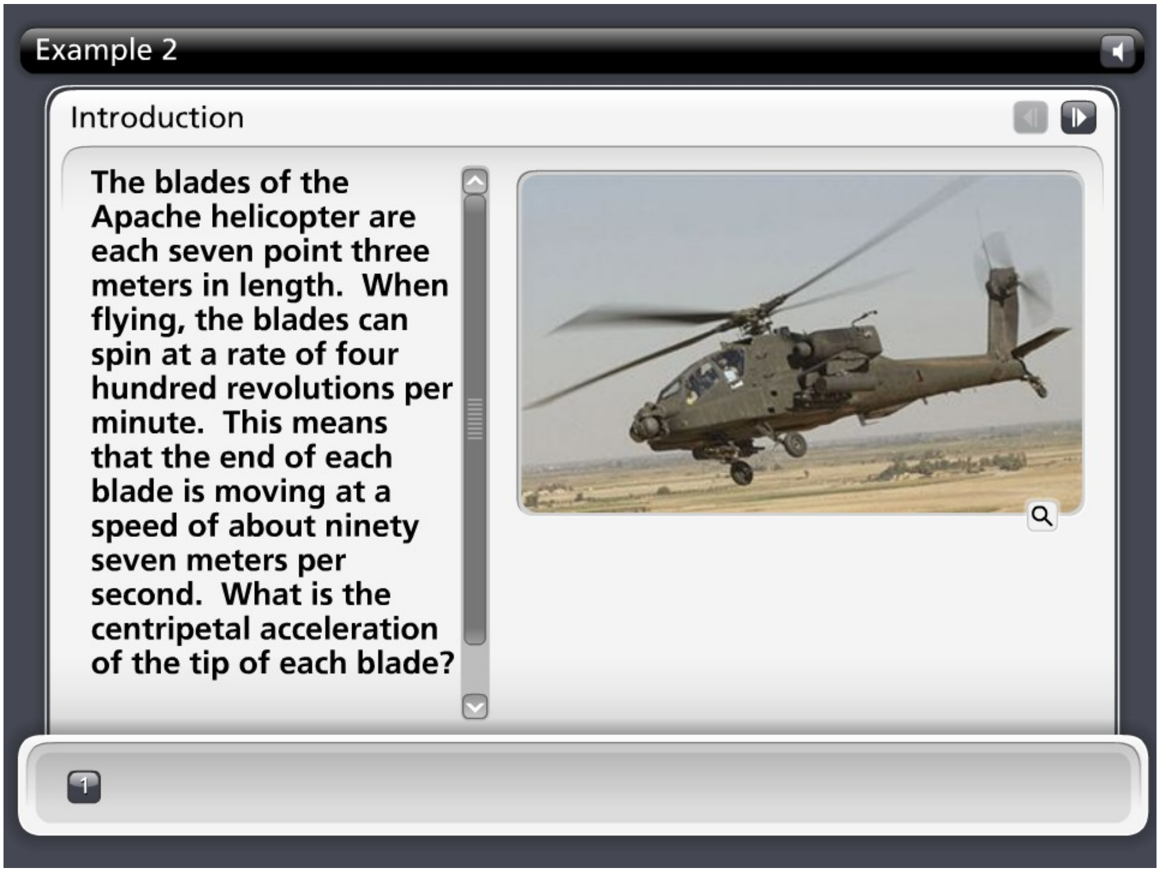
First, you must draw a free body diagram of the car. It is easiest to see the forces if we see the car coming towards us. There is the gravitational force directed straight down, and the normal force directed straight up. There must be an additional force pointed towards the center of the circle, left in this case, which provides the centripetal force necessary to complete the turn. In this case, it is the static friction of the tires on the road that provide this force.



Now, you must write down what we know. The mass is twelve hundred kilograms. The speed is a constant eight point five meters per second. And the radius is twenty two meters.

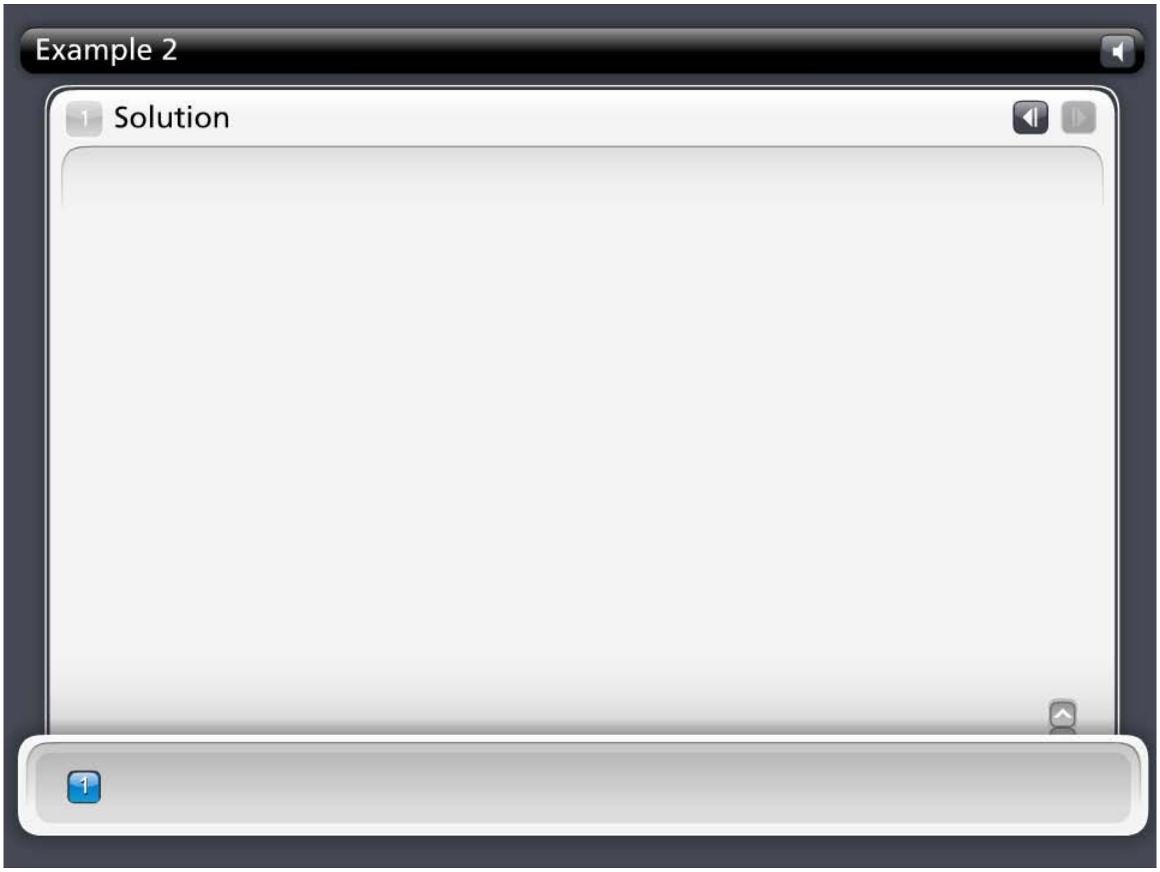
Using the equation F c equals m v squared over r, we see that the necessary force is three thousand seven hundred sixty Newtons. This force is due to the static friction of the tires on the road, and provides the center-directed, or centripetal force necessary to keep the car moving in a circle. If this force were to suddenly disappear, if the car hit a patch of ice, for instance, the car would continue in a straight line at a tangent to the circle due to inertia, instead of continuing in a circular path.

Now, either by dividing the force by the mass, or by using the equation a c equals v squared over r, you see that the acceleration is three point one three meters per second squared.



Let's look at another example.

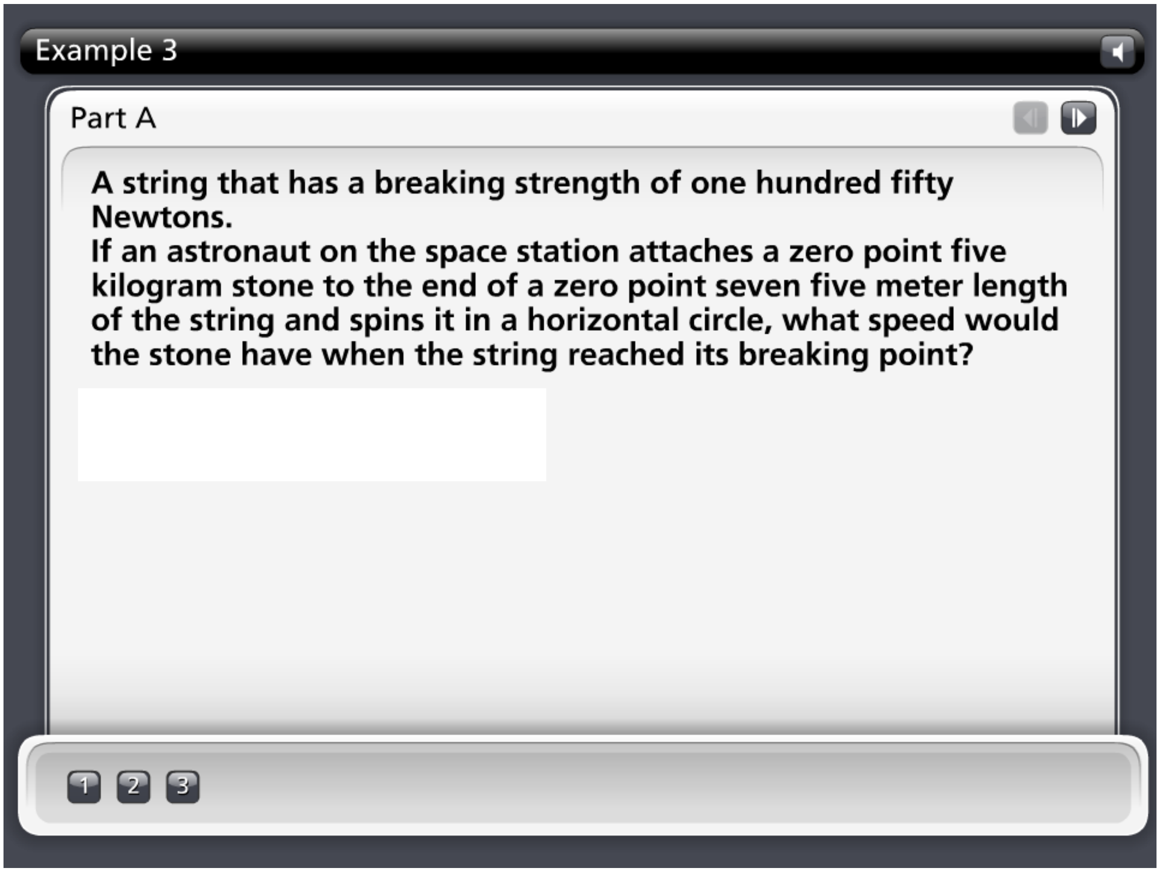
The blades of the Apache helicopter are each seven point three meters in length. When flying, the blades can spin at a rate of four hundred revolutions per minute. This means that the end of each blade is moving at a speed of about ninety seven meters per second. What is the centripetal acceleration of the tip of each blade?



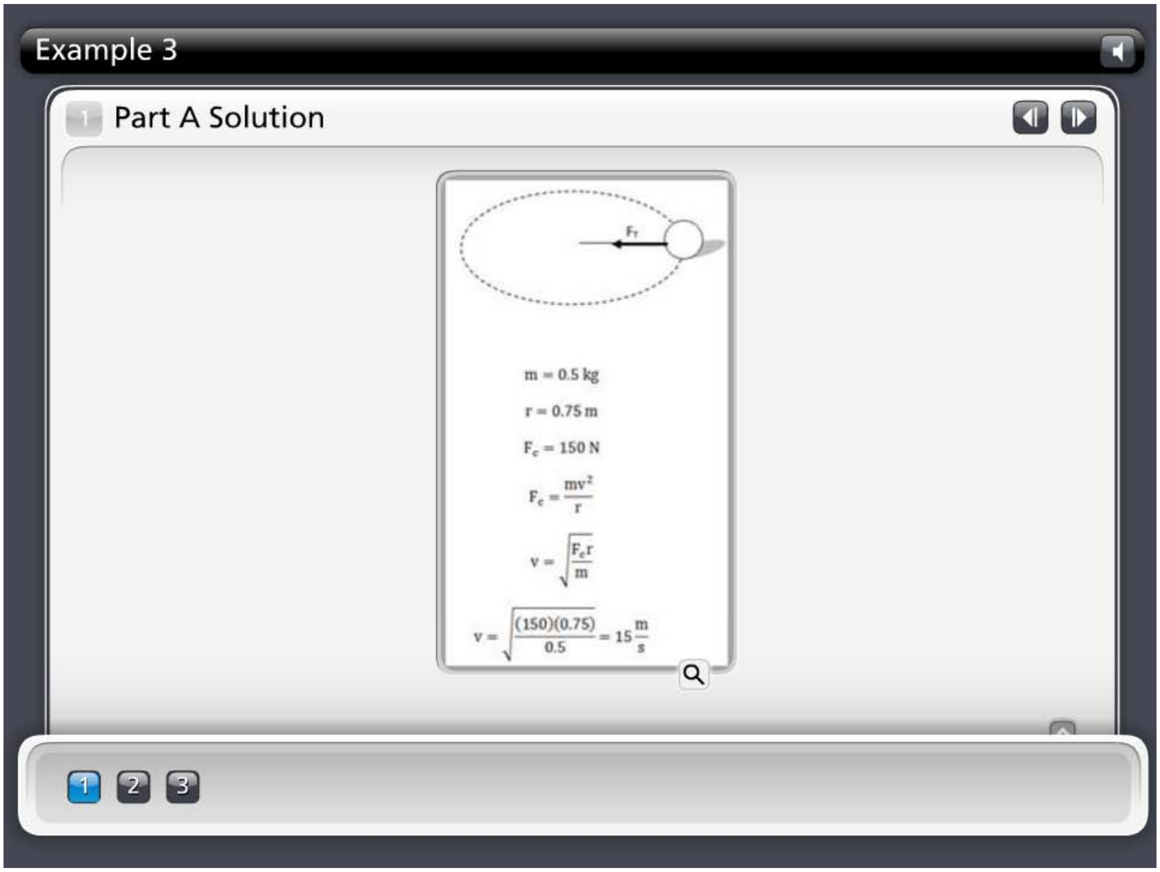
Again, you first write down what we know. The speed of the end of the blade is ninety seven meters per second. The radius is equal to the length of the blade, which is seven point three meters.

Substituting in your equation for centripetal acceleration, you find that the acceleration is thirteen hundred meters per second squared.

What is the force providing this acceleration? It is the tensional strength of the material making up each rotor blade.



What if you had a string that had a breaking strength of one hundred fifty Newtons. If an astronaut on the space station attaches a zero point five kilogram stone to the end of a zero point seven five meter length of the string and spins it in a horizontal circle, what speed would the stone have when the string reached its breaking point?

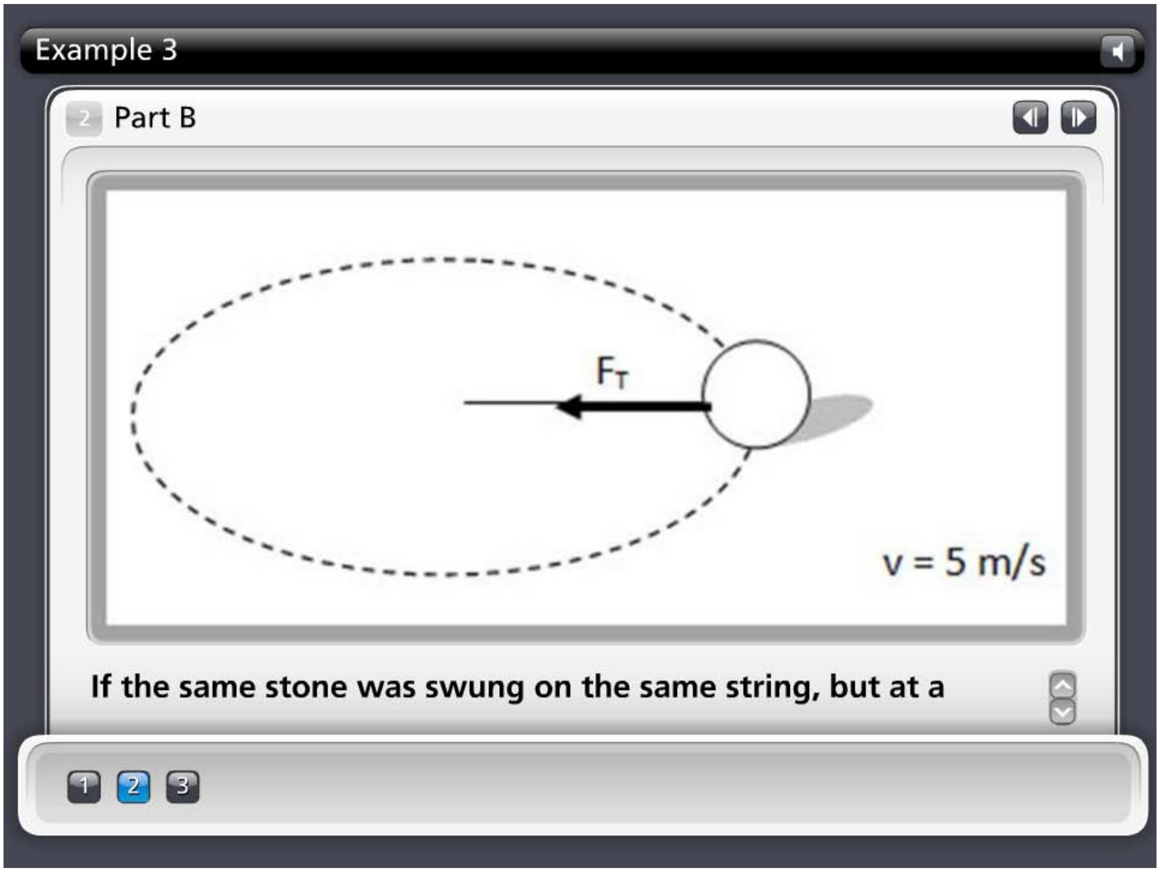


First, you should draw a free body diagram of the stone and string. Since you are on the space station, you don't need to include the gravitational force, so the only force acting on the stone is the tension in the string pointing towards the center of the circle.

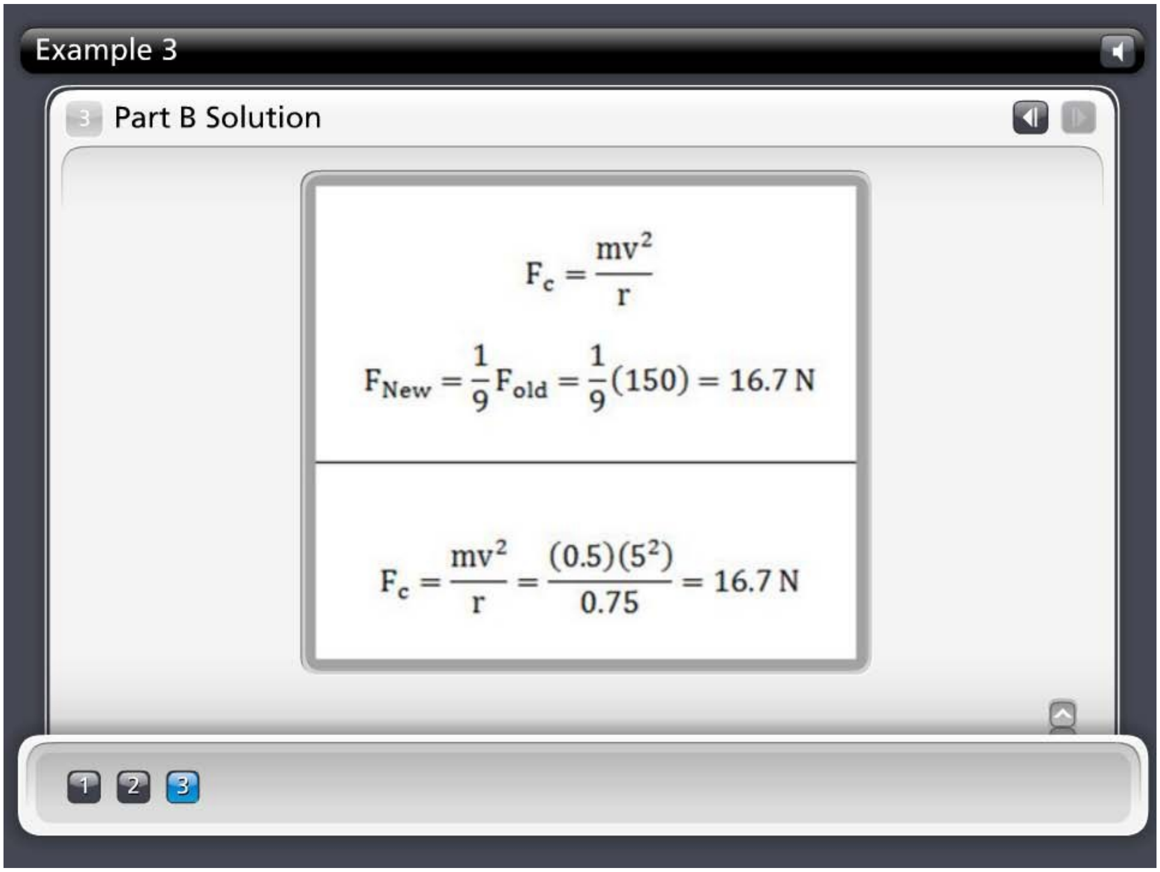
Again, you will write what we know. The mass of the stone is zero point five kilograms. The radius will be the same as the length of the string, or zero point seven five meters. The breaking strength of the string is one hundred fifty Newtons, so this will be the centripetal force that we will use at the breaking point.

you can now rearrange our centripetal force equation to solve for v.

When you substitute our values and solve, we see that the stone will have a speed of fifteen meters per second when the string breaks.



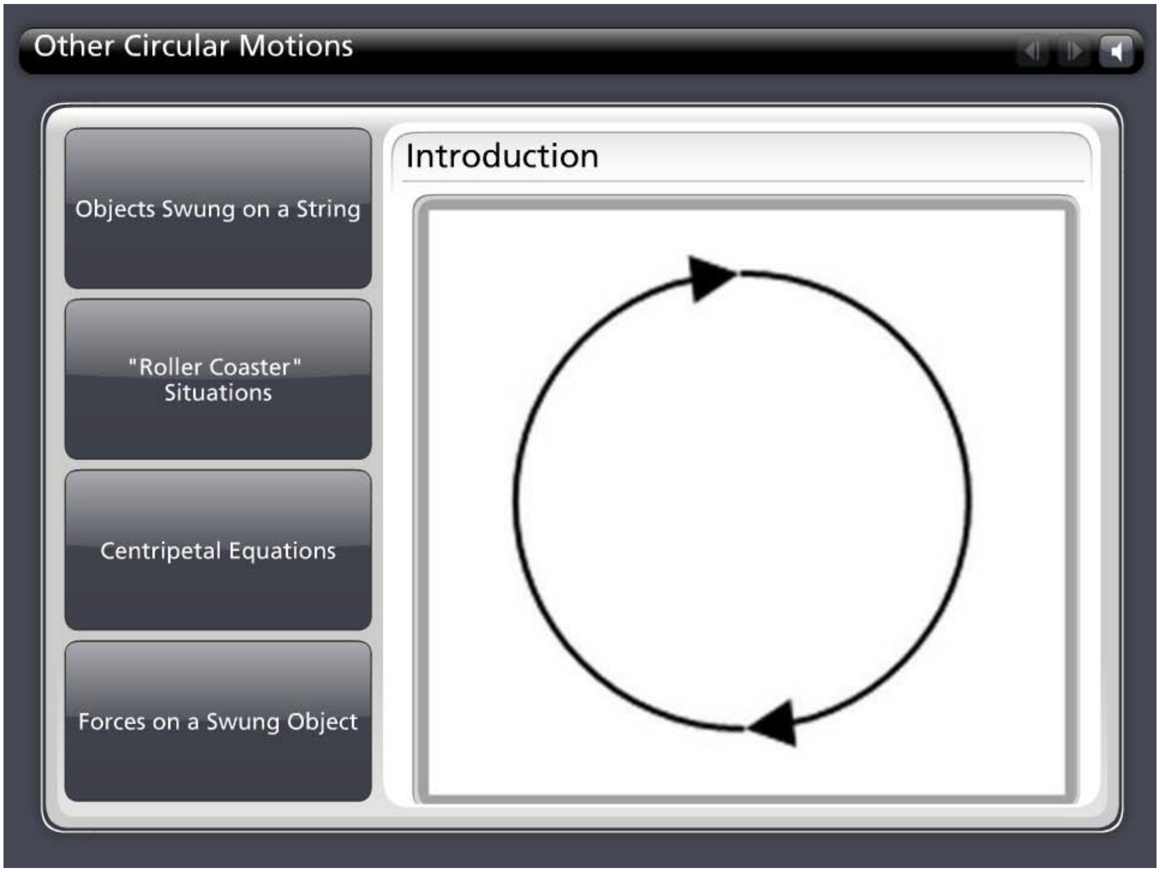
In the previous problem, when the stone was swung at fifteen meters per second, there was a tension of one hundred fifty Newtons. If the same stone was swung on the same string, but at a velocity of only five meters per second, what would be the tension in the string?



Looking at the centripetal force equation, you see that the force is dependent on the square of the velocity. This means that a doubling of velocity would result in an force that is four times as much. Cutting the velocity to one third of its original value would decrease the force to one ninth its original value. So the new force will be one hundred fifty divided by nine, or sixteen point seven Newtons.

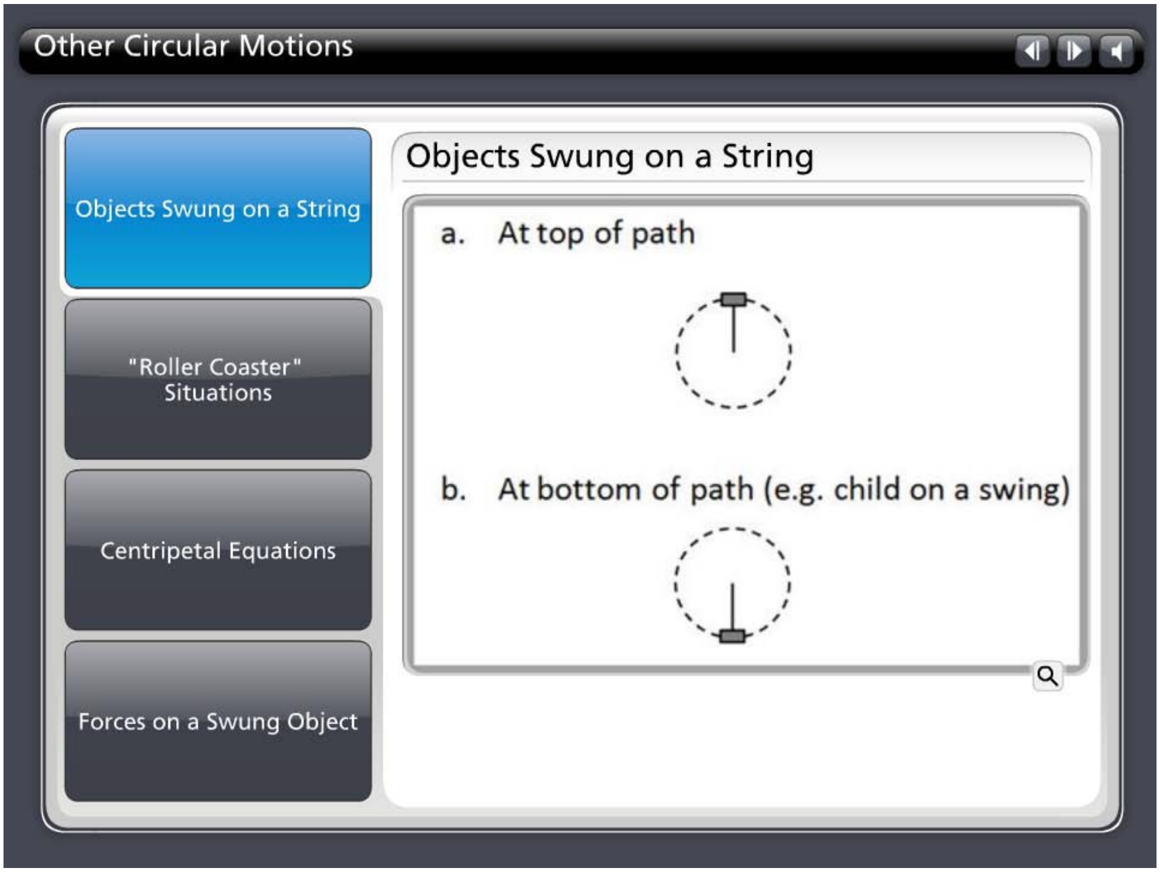
Alternately, you could answer the problem by plugging in new values to the centripetal force equation, and reach the same conclusion.

You should, however, be able to see how changes in each variable would change the centripetal force.



Since the speed will be faster at the bottom and slower up top, the situations we will analyze are not precisely uniform circular motion. However, the points exactly at the top and at the bottom can be approximated by our equations.

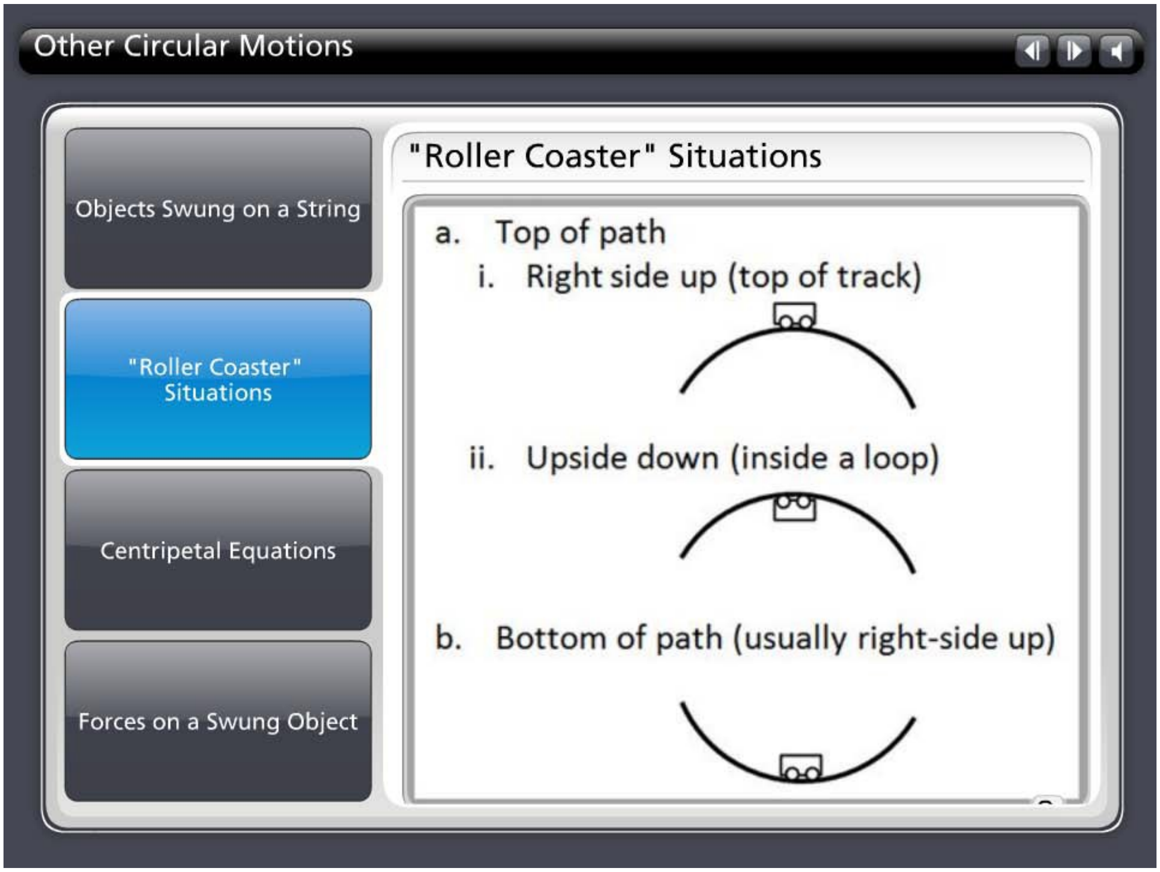
Let's take an overview of the types of situations you will be expected to analyze.



The first typical category of motion involves objects being swung around on a string.

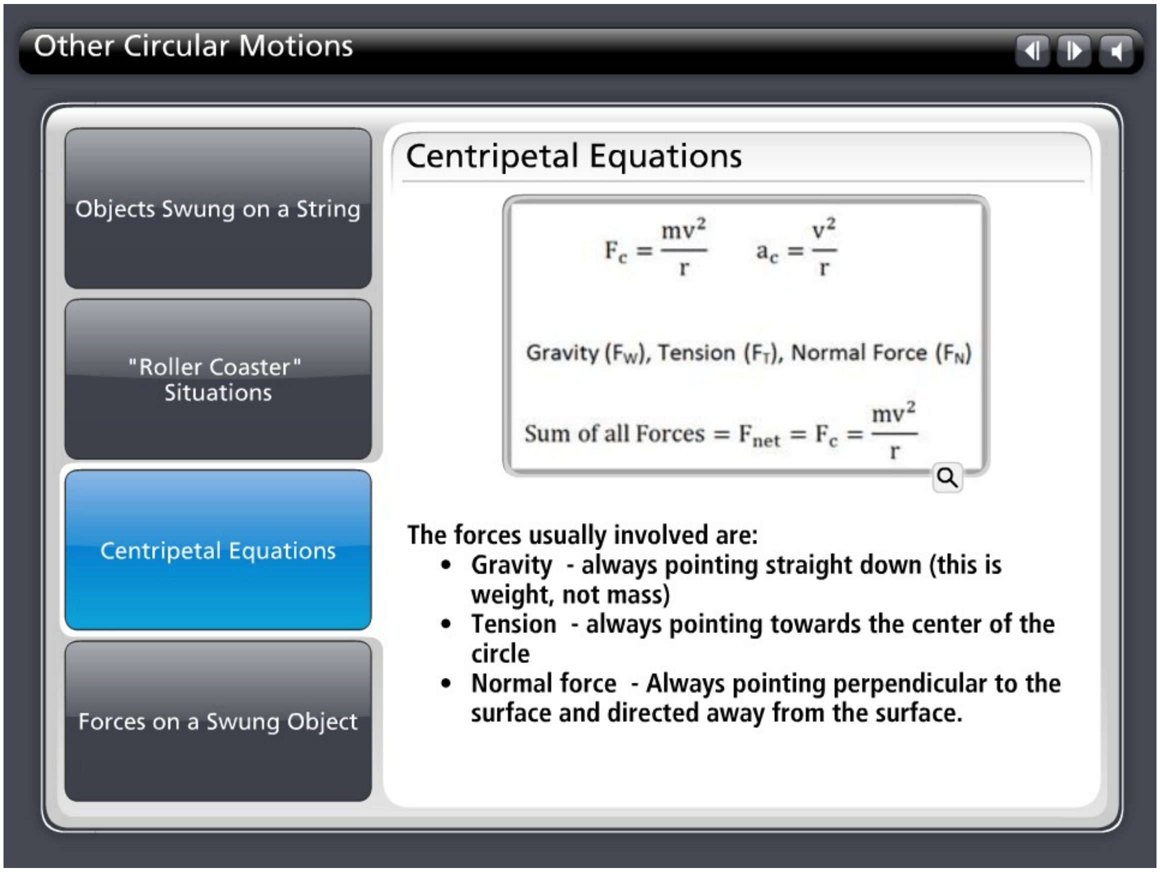
You will be expected to recognize the forces and analyze the motion both at the top of the path and at the bottom.

A child on a swing can be analyzed the same way as the bottom situation even though swing does not continue in a complete circle. While swinging, the swing traces part of a circular path, so you'll be able to look at the bottom of the swing and make some calculations about the forces and motion.



The second category can be generally considered as roller coaster situations, although the upright situations work well for a car on a road either going over a hill or through a dip in the road

The car can be right-side up on the top of the track, upside down on the top of the track, inside a loop, or right-side up on the bottom of the path. There are roller coasters that hang below the tracks and the analysis you would use is quite similar to the last situation.

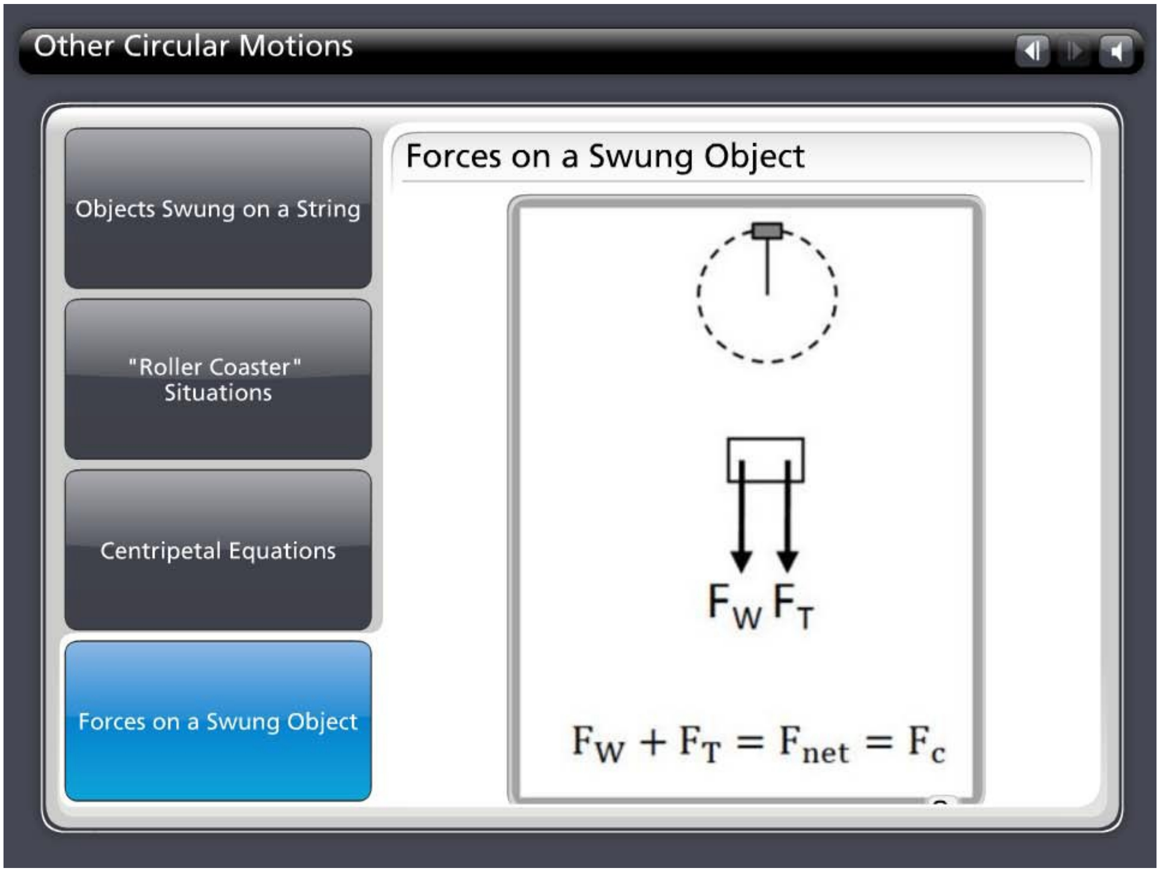


If you look at our centripetal force and centripetal acceleration equations, you'll see that the variables of interest will be the mass of the object, the speed of the object, the radius of the path, and the centripetal force.

However remember that the centripetal force always comes from one or more real forces. in these situations, the forces we will most likely encounter are the gravitational force, which always points straight down, a tension force when a string is involved, always pointing directly to the center of the circle, and a normal force when the object is in contact with a track, always pointing perpendicular away from the surface.

The normal force often takes a little thinking to determine its direction. If a roller coaster is right- side up at the top of a hill, the normal force will point up. If a roller coaster is upside down in a loop, the normal force will point down towards the center of the circle.

Whatever forces are involved, if the object is on a portion of a circular path, then the sum of all these real forces will be the centripetal force, and the magnitude will be m v squared over r.



Let's look at the first of these situations, an object at the top of its path as it is swung around on a string.

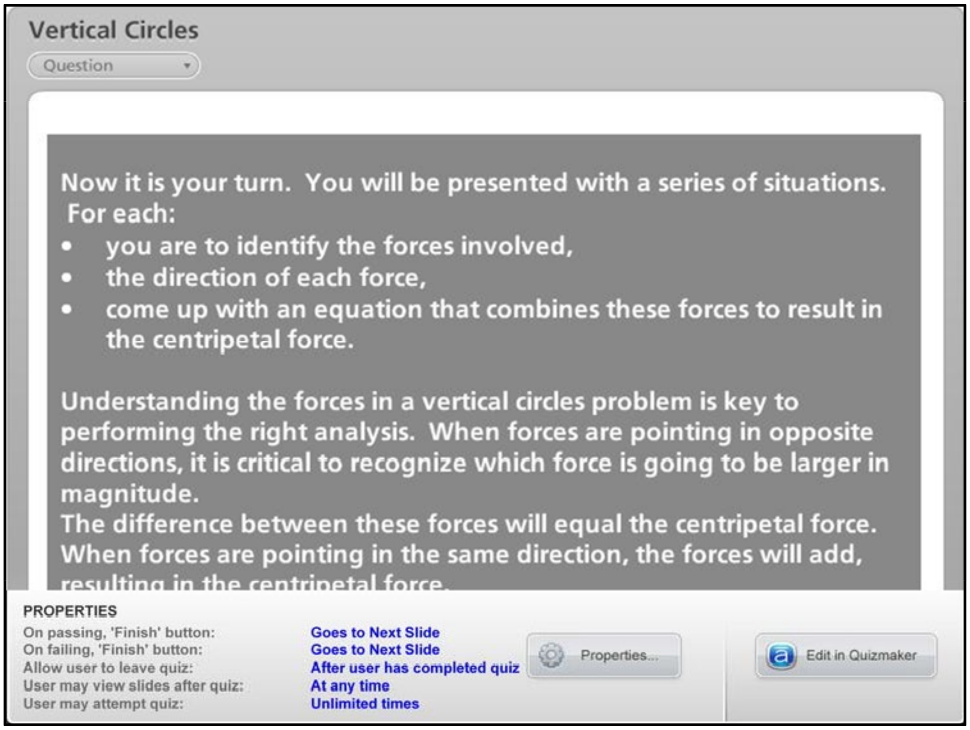
Which forces are involved?

First, the gravitational force is involved, as it always is, and it points straight down.

Next, you can see that there is a string attached to the object, so there is a tension force pointing along the string, in this case, also straight down.

There is no normal force, since the object is not in contact with any surface.

What can you say about the tension and the normal force, as far as the centripetal force goes? you know that the real forces must combine to a net force pointed towards the center of the circle. This is the centripetal force. So in this case, you can say that the gravitational force plus the tension force equals the net force, and this is equal to the centripetal force.

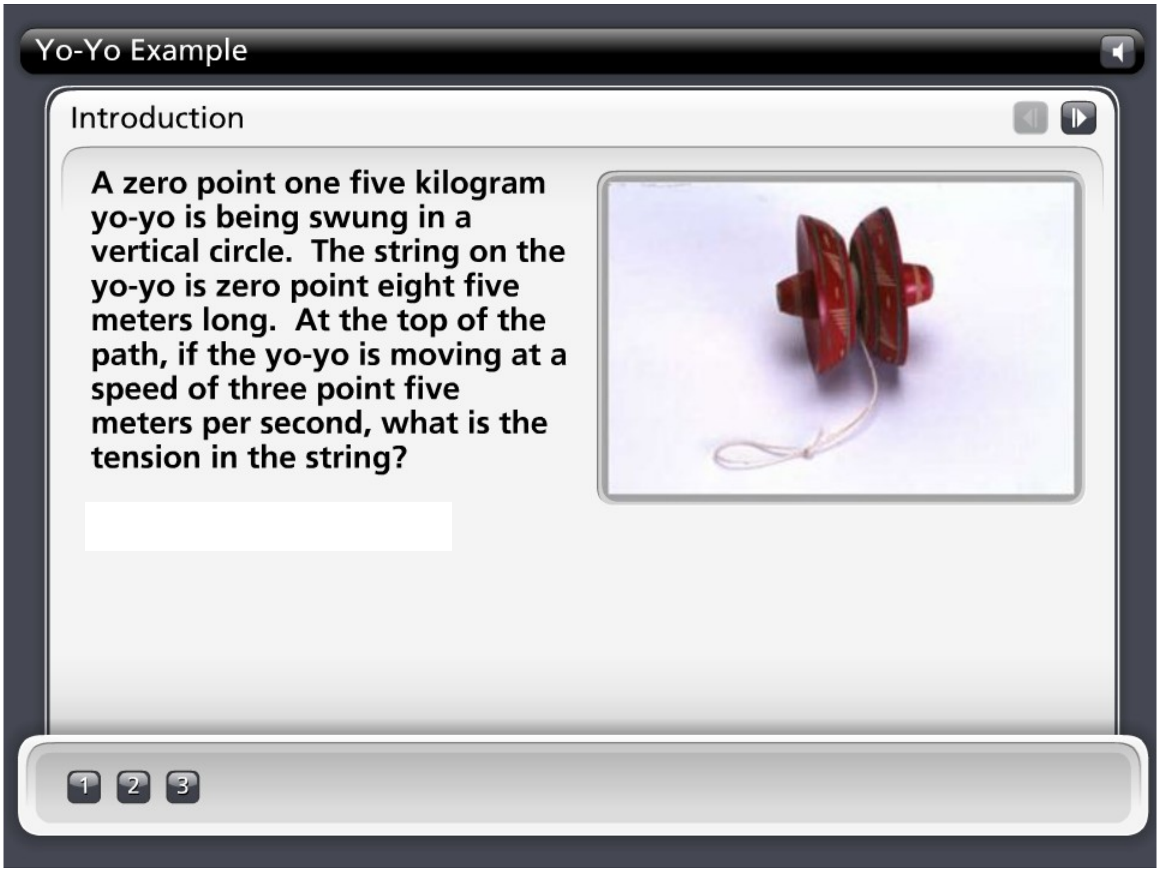


Now it is your turn. You will be presented with a series of situations. For each you are to identify the forces involved, the direction of each force, and come up with an equation that combines these forces to result in the centripetal force.

Understanding the forces in a vertical circles problem is key to performing the right analysis. When forces are pointing in opposite directions, it is critical to recognize which force is going to be larger in magnitude.

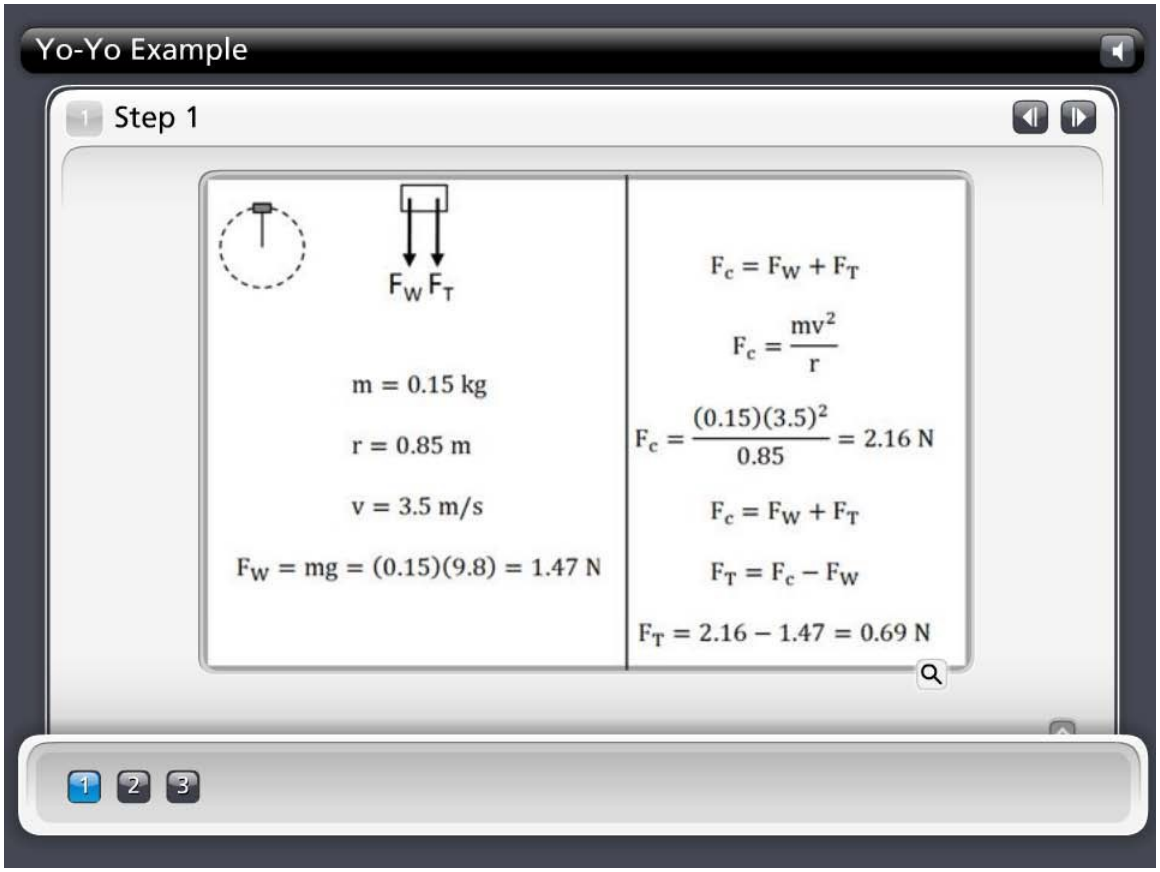
The difference between these forces will equal the centripetal force.

When forces are pointing in the same direction, the forces will add, resulting in the centripetal force.



Let's look at an example problem.

Say a zero point one five kilogram yo-yo is being swung in a vertical circle. The string on the yo- yo is zero point eight five meters long. At the top of the path, if the yo-yo is moving at a speed of three point five meters per second, what is the tension in the string?



Since understanding the forces is so critical, your first step is to draw a free body diagram of the yo-yo. You see that there are two forces acting on the yo-yo while it is at the top of its path: The gravitational force and the tension force. Both forces are pointing down, which is towards the center of the circular path.

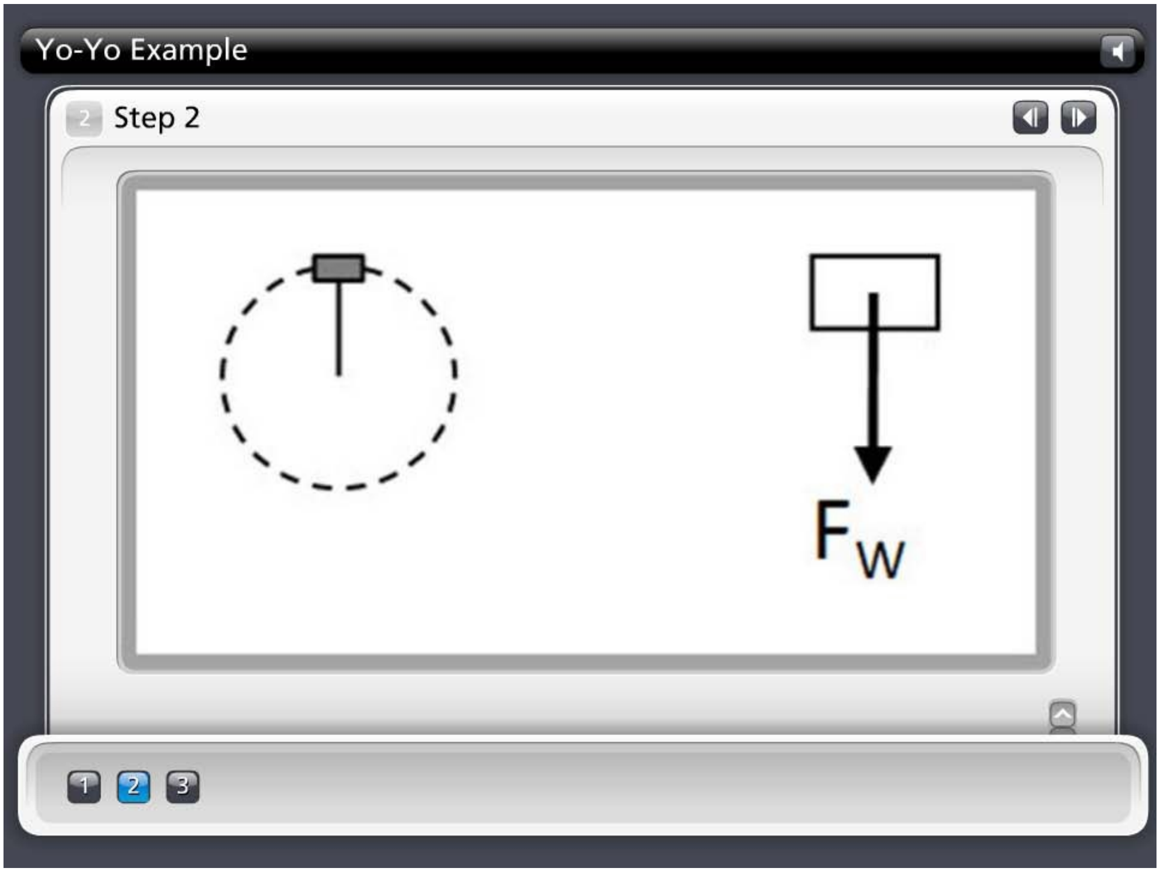
You can now identify the variables and write down the values that we know. The mass is zero point one five kilograms. The radius is equal to the length of the string, so is zero point eight five meters. The speed is given as three point five meters per second.

The gravitational force will be equal to the mass times the acceleration of gravity. Substituting, we see that the gravitational force is one point four seven Newtons.

Now how do you find the tension from this information? You also know that the centripetal force is going to be the result of the sum of the gravitational force and the tension. So if you can calculate the centripetal force, we can then calculate the tension.

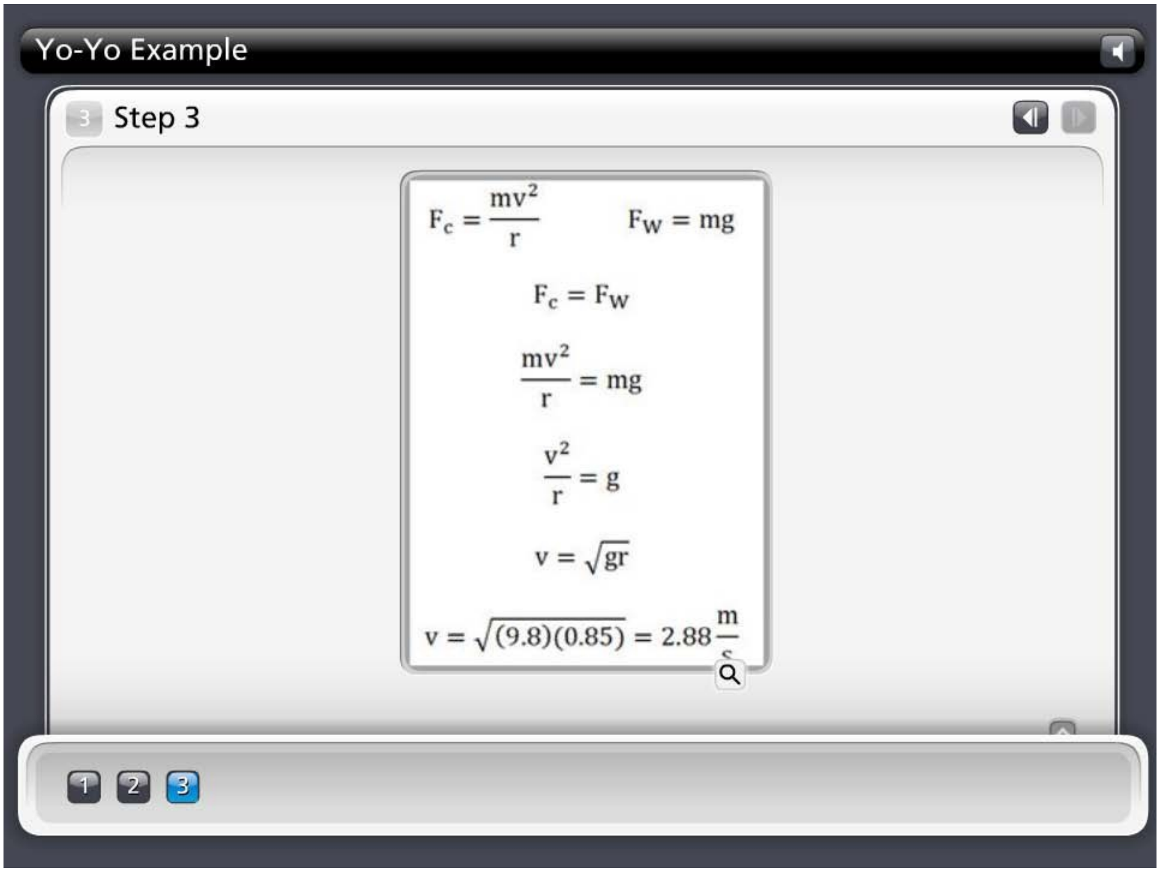
Substituting in our centripetal force equation, you see that the centripetal force equals two point one six Newtons.

Rearranging your equation that combines the gravitational force and the tension, you see that the tension equals the centripetal force minus the gravitational force. Substituting and solving, you find that the tension in the string equals zero point six nine Newtons.



If you take the same yo-yo and swing it slower and slower, the tension at the top will become less and less. You should try this for yourself At what speed will the tension become zero?

What is interesting about this speed is if tension is zero, then the only force available to provide a centripetal force is the gravitational force. So you can simply set the gravitational force to the centripetal force and solve for the speed.

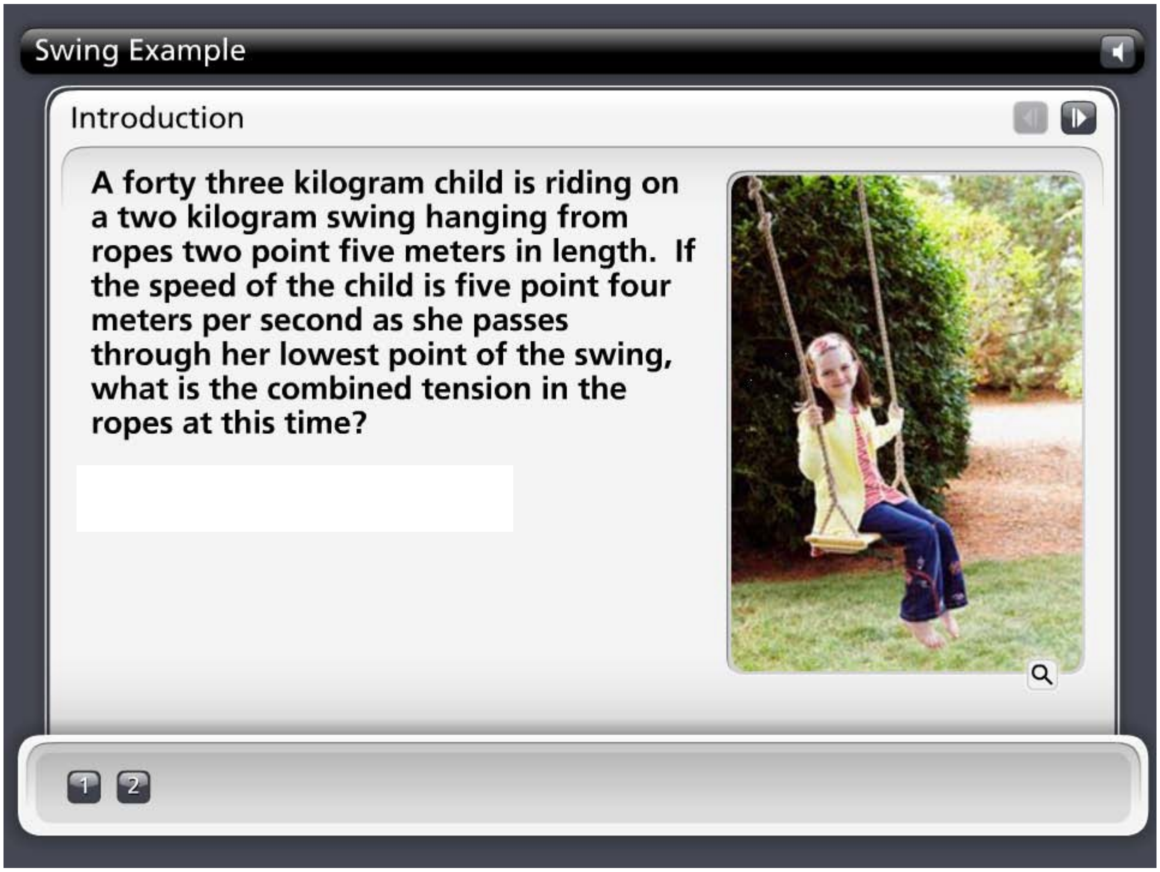


The centripetal force is mass times velocity squared divided by the radius The gravitational force is the mass times the gravitational acceleration.

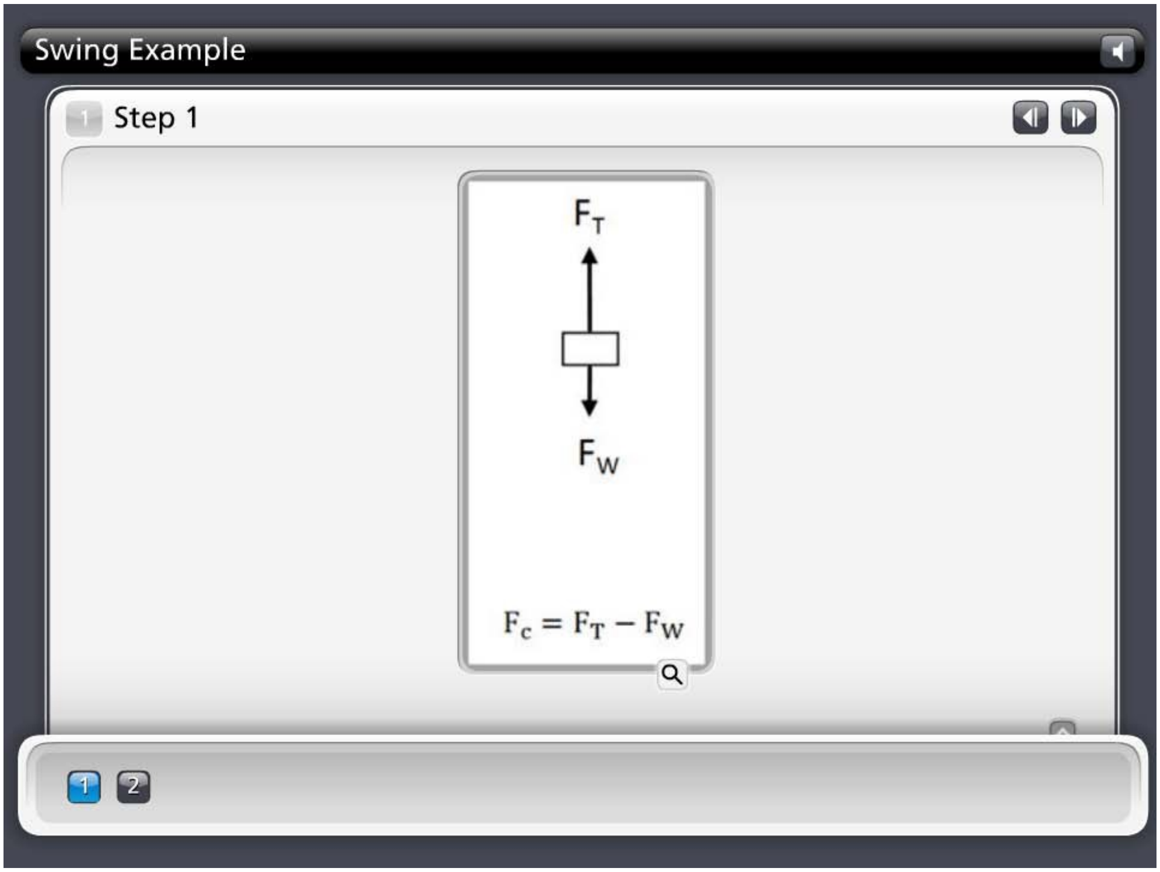
Setting them equal to one another, we see that the mass cancels out, and at this point, the gravitational acceleration is also the centripetal acceleration

Substituting and solving, you find that the speed at which the tension drops to zero, and gravity alone provides the centripetal force is two point eight eight meters per second.

You will see other examples where gravity alone provided the centripetal acceleration.



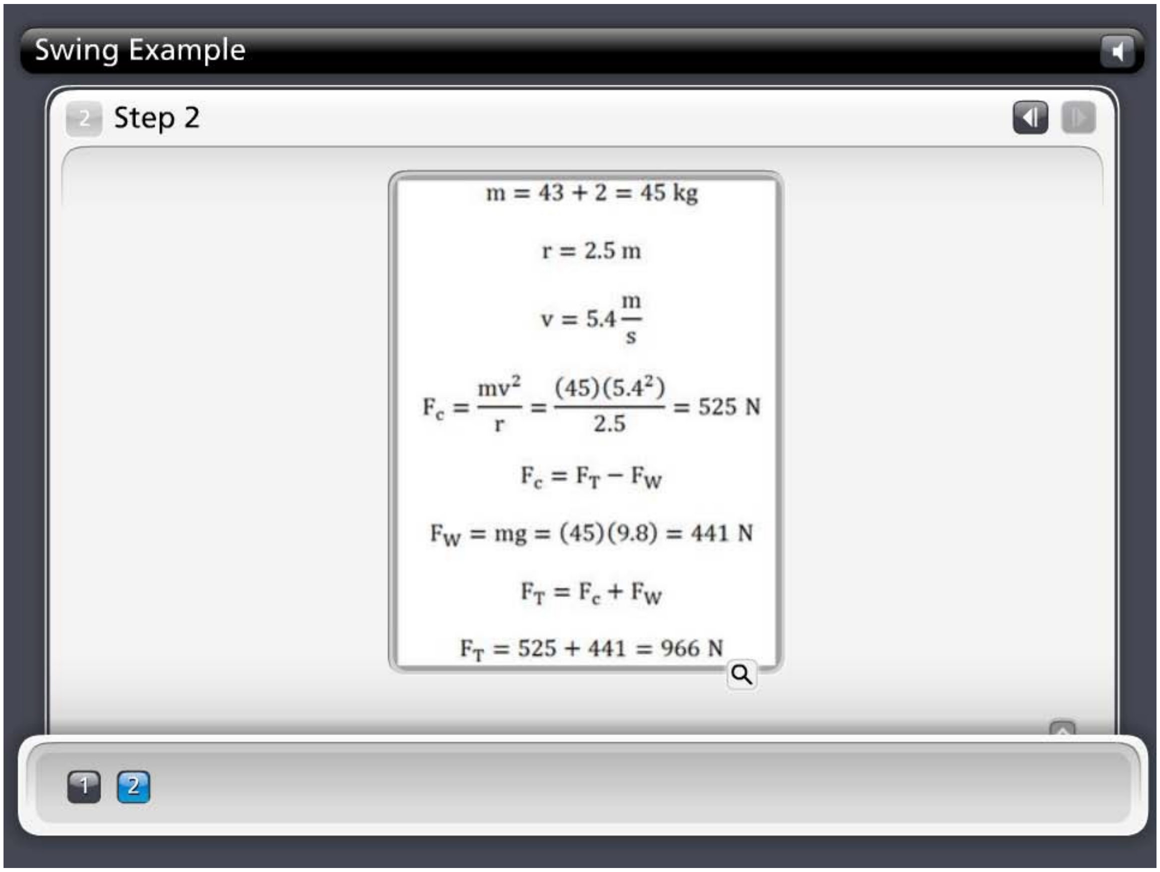
Now, let's look at a forty three kilogram child riding on a two kilogram swing hanging from ropes two point five meters in length. If the speed of the child is five point four meters per second as she passes through her lowest point of the swing, what is the combined tension in the ropes at this time?



Even though the child does not complete a circle, she is on a circular path, so you can analyze this using circular motion.

You should first draw a free body diagram. You'll consider the girl and swing together and look at the forces acting on them. The ropes are pulling straight up at this point, and the gravitational force is pulling straight down.

You know that the force of tension must be greater than the weight of the girl and swing so that they can maintain an upward-curving path as they swing. If the forces were the same, the girl would continue ahead in a straight line, which you know is not the case. So the tension must be greater than the gravitational force by enough to maintain a circular path. This means that the centripetal force must equal the tension minus the gravitational force.



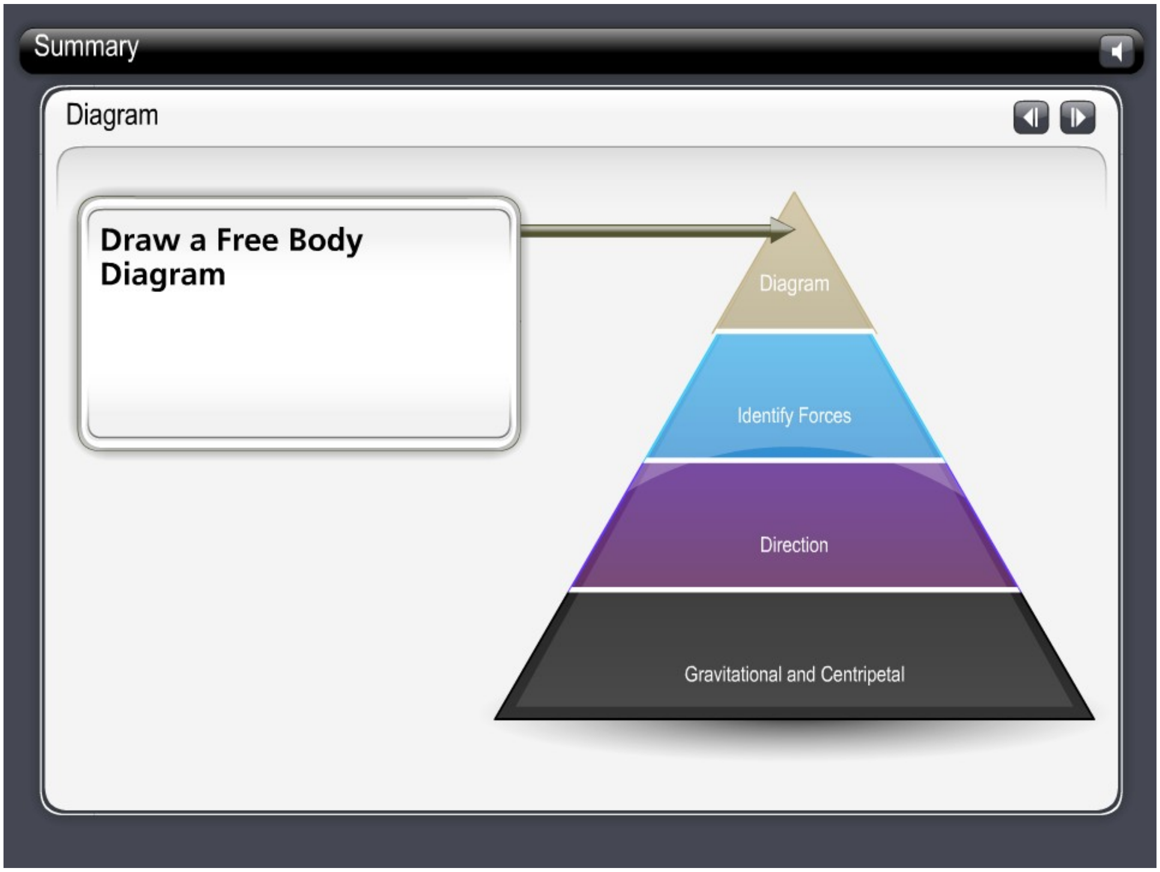
Now let's identify the variables and values.

The mass of the girl plus the swing is forty three plus two equals forty five kilograms. The length of the strings is two point five meters, so this is also the radius of the path. The speed has been given as five point four meters per second.

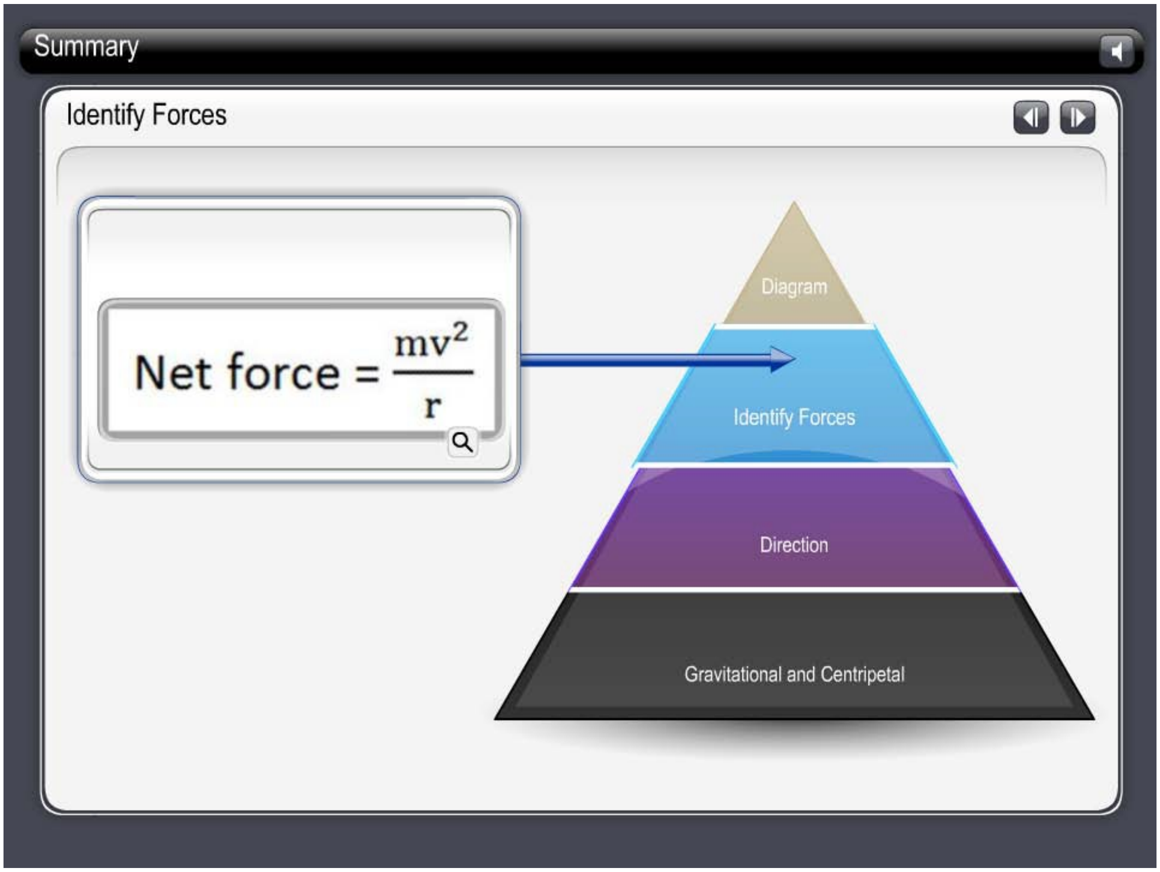
From these, you can determine the centripetal force by substituting into our centripetal force equation. You find that the centripetal force is equal to five hundred twenty five Newtons.

But earlier you determined that the centripetal force is equal to the tension minus the gravitational force. The gravitational force is the mass times the acceleration of gravity, or four hundred forty one Newtons.

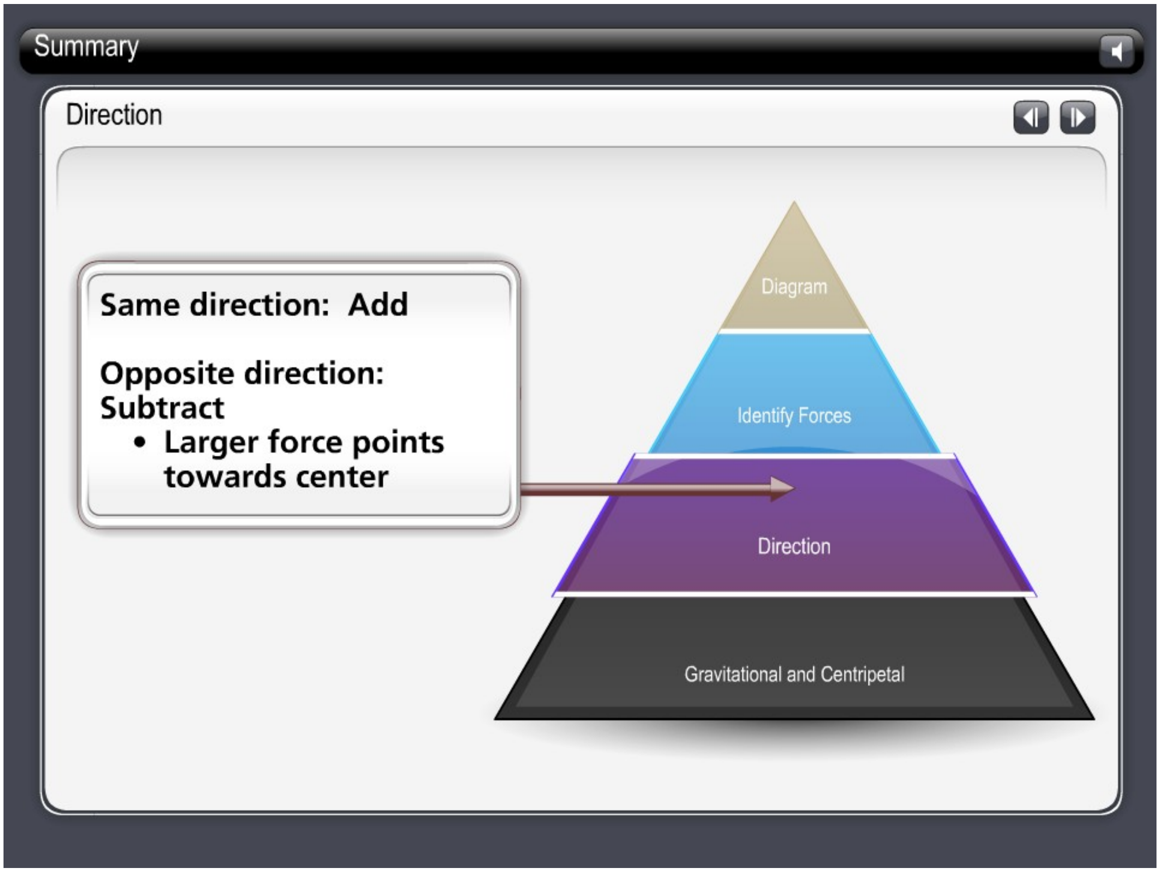
You can now rearrange our equation that includes the centripetal force, tension and gravitational force to solve for the tension. You find that the tension equals the centripetal force plus the gravitational force. Or, the tension force equals nine hundred sixty six Newtons.



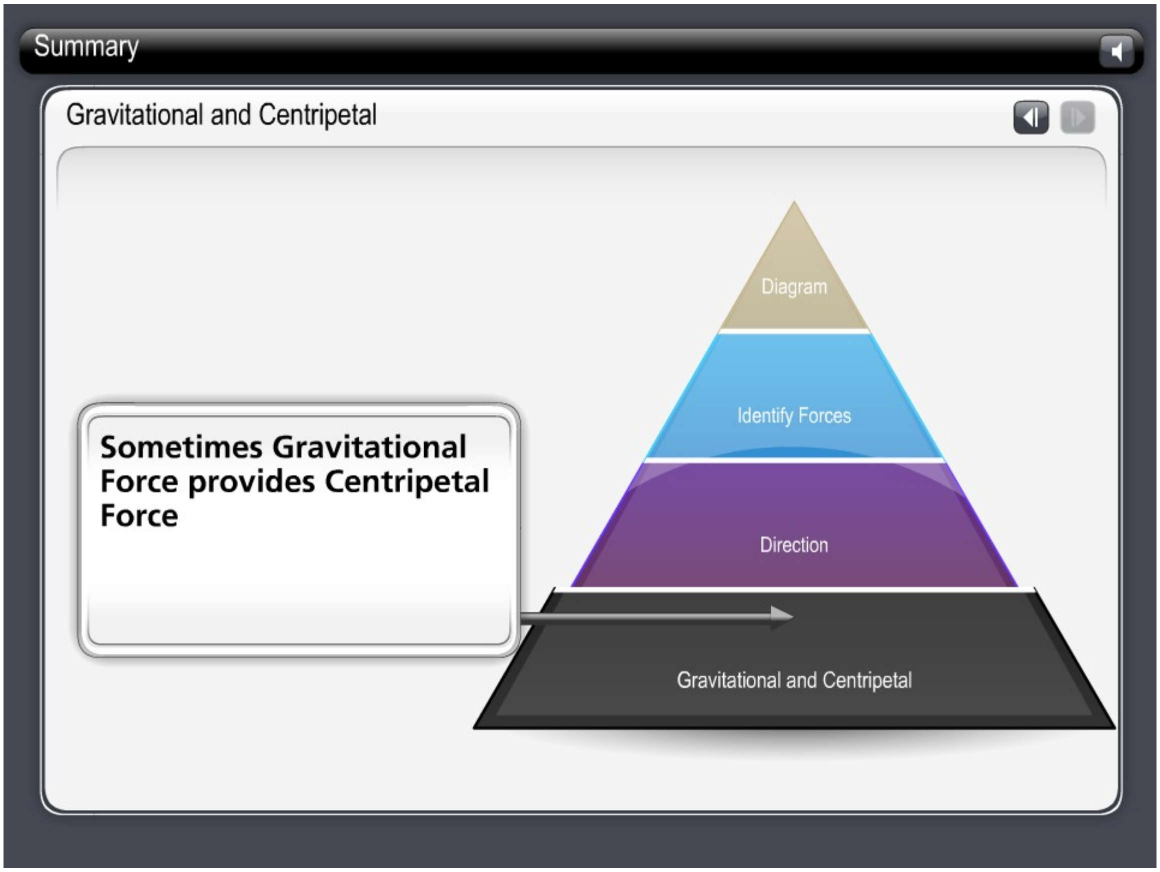
In summary, you've seen that with vertical circles, it is very important to draw a free body diagram and identify forces.



The net force will be pointing towards the center of the circle with magnitude m v squared over r. If the forces are pointing in the same direction, they will add to the centripetal force, and if they are in opposite directions, they will subtract so that the difference is the centripetal force. In both cases the net force is the centripetal force.



To know what order to subtract the forces, remember that the force that is pointed toward the center of the circle must be greater that the force that points away from the center of the circle. So you should always subtract the force that points away from the center from the force that points toward the center. You could also look to see which direction the object is moving next. If it is curving up, then the upward force is greater, and if it is curving down, then the downwards force is greater.



In some interesting situations, at the top of a circular path, gravity alone will provide the necessary centripetal force.