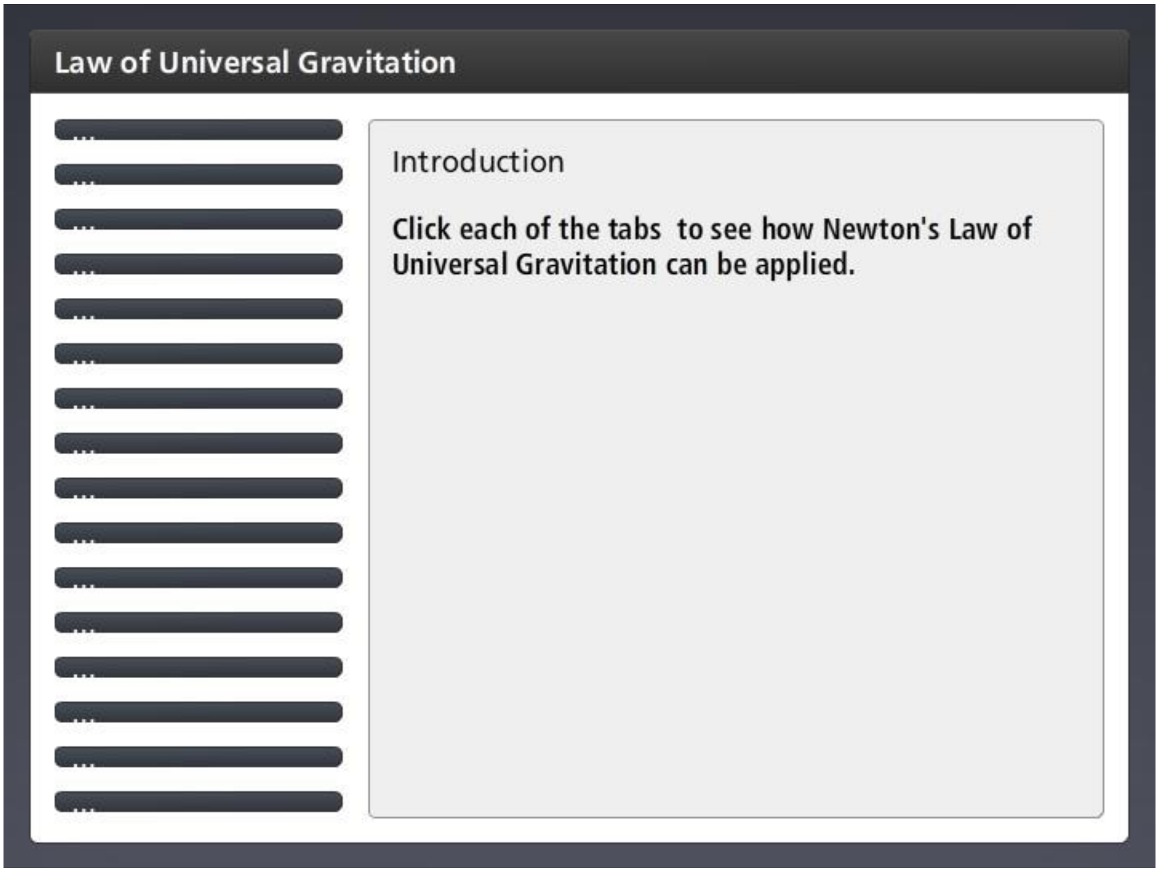
Module 3: Motion In Two Dimensions
Topic 5 Content: Law of Universal Gravitation

Introduction



Click each of the tabs to see how Newton's Law of Universal Gravitation can be applied.

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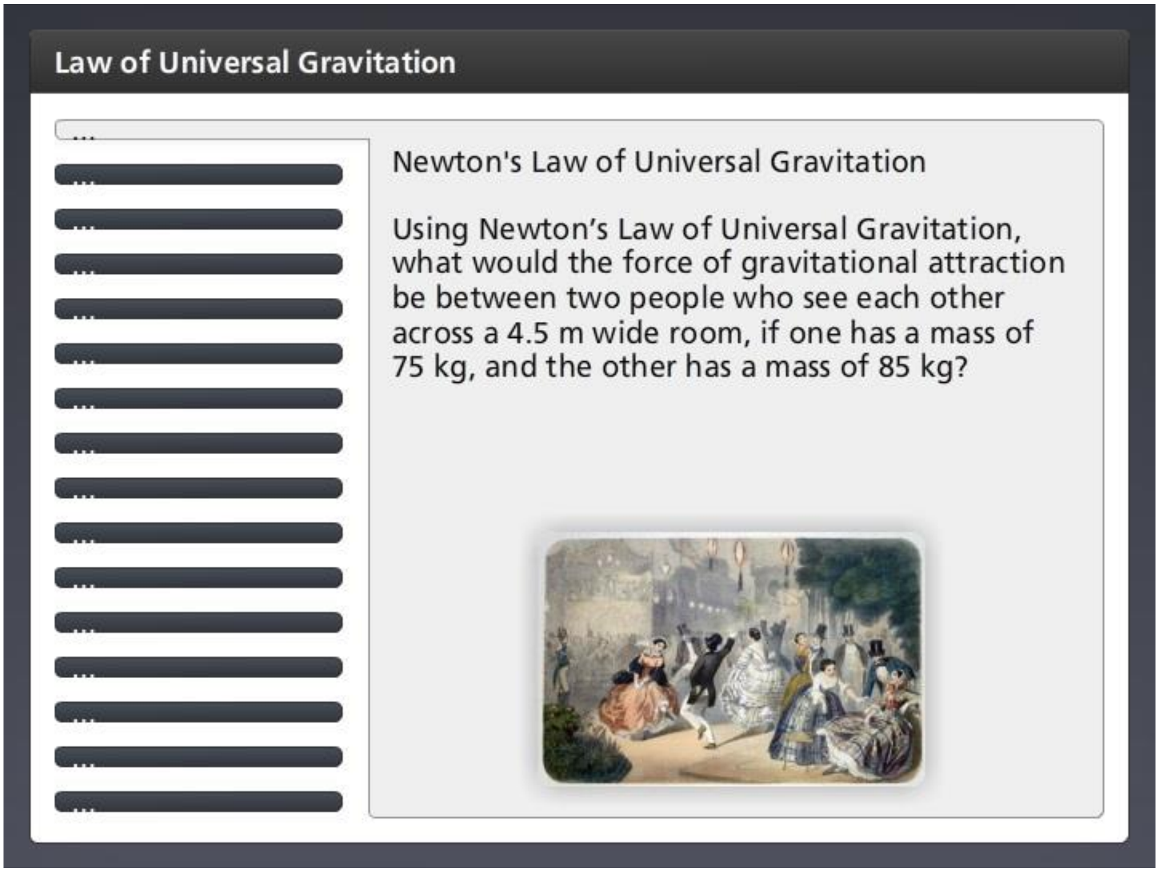
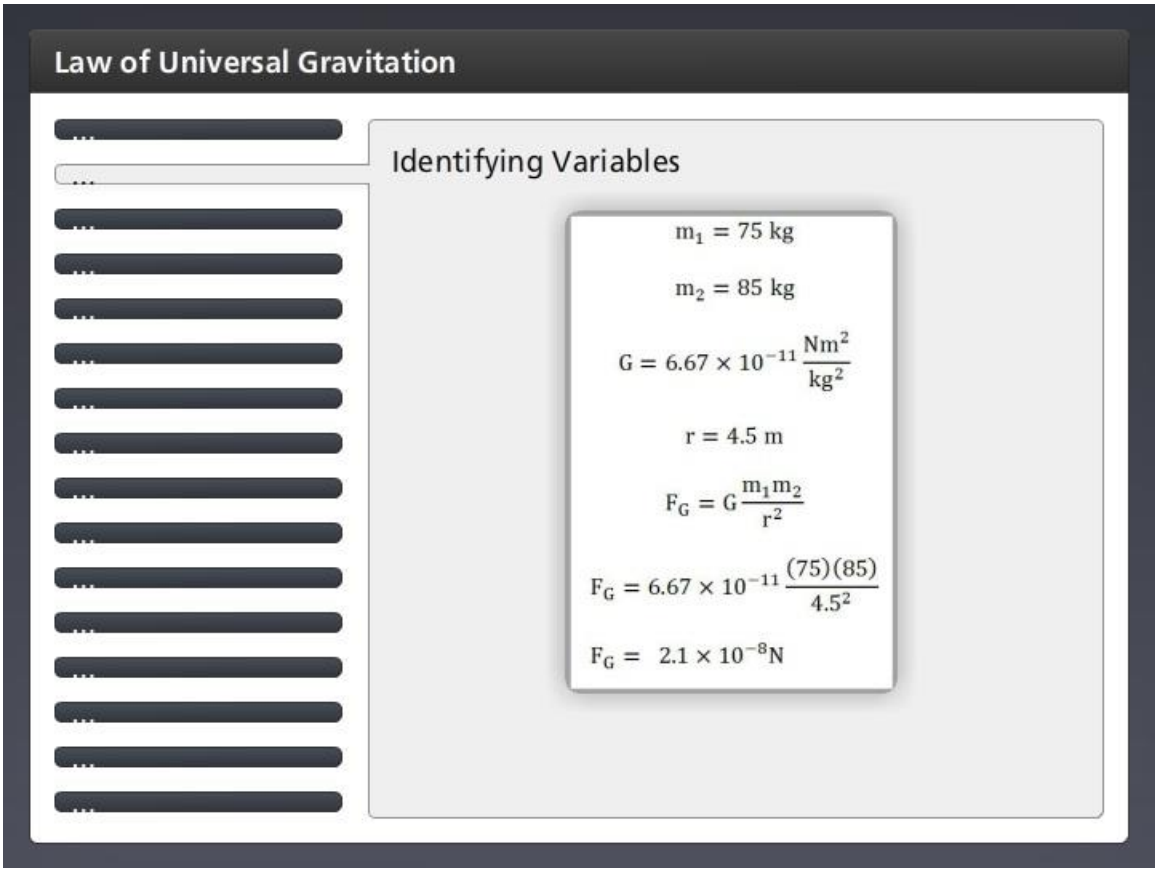
Newton's Law of Universal Gravitation



Using Newton's Law of Universal Gravitation, what would the force of gravitational attraction be between two people who see each other across a 4.5 m wide room, if one has a mass of 75 kg, and the other has a mass of 85 kg?

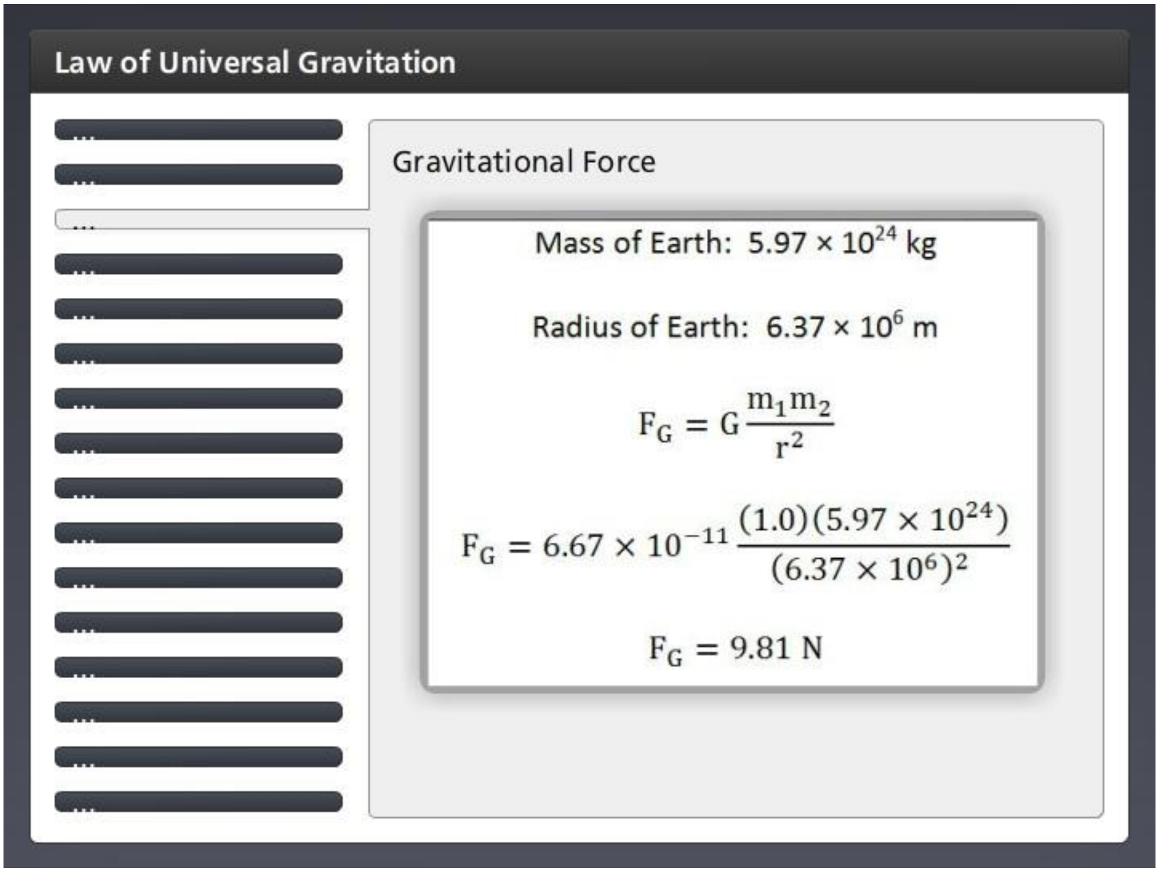
Identifying Variables



Let's first identify our variables. The first mass is seventy five kilograms. The second mass is eighty five kilograms. The gravitational constant is six point six seven times ten to the negative eleven Newtons meters squared per kilograms squared. The radius is the distance between them, or four point five meters.

Substituting and solving, you find that the gravitational force is only two point one times ten to the negative eight Newtons. A remarkably small amount.

Gravitational Force



So with results this small, why do you even care about gravity?

It’s because there are very massive objects that result in much greater forces. The mass of the earth, for example, is five point nine seven times ten to the twenty fourth kilograms. The radius of the earth is six point three seven times ten to the sixth meters.

What would the gravitational force be on a one kilogram object on the surface of the earth? If you substitute our values and solve, you find that the force should be nine point eight one Newtons.

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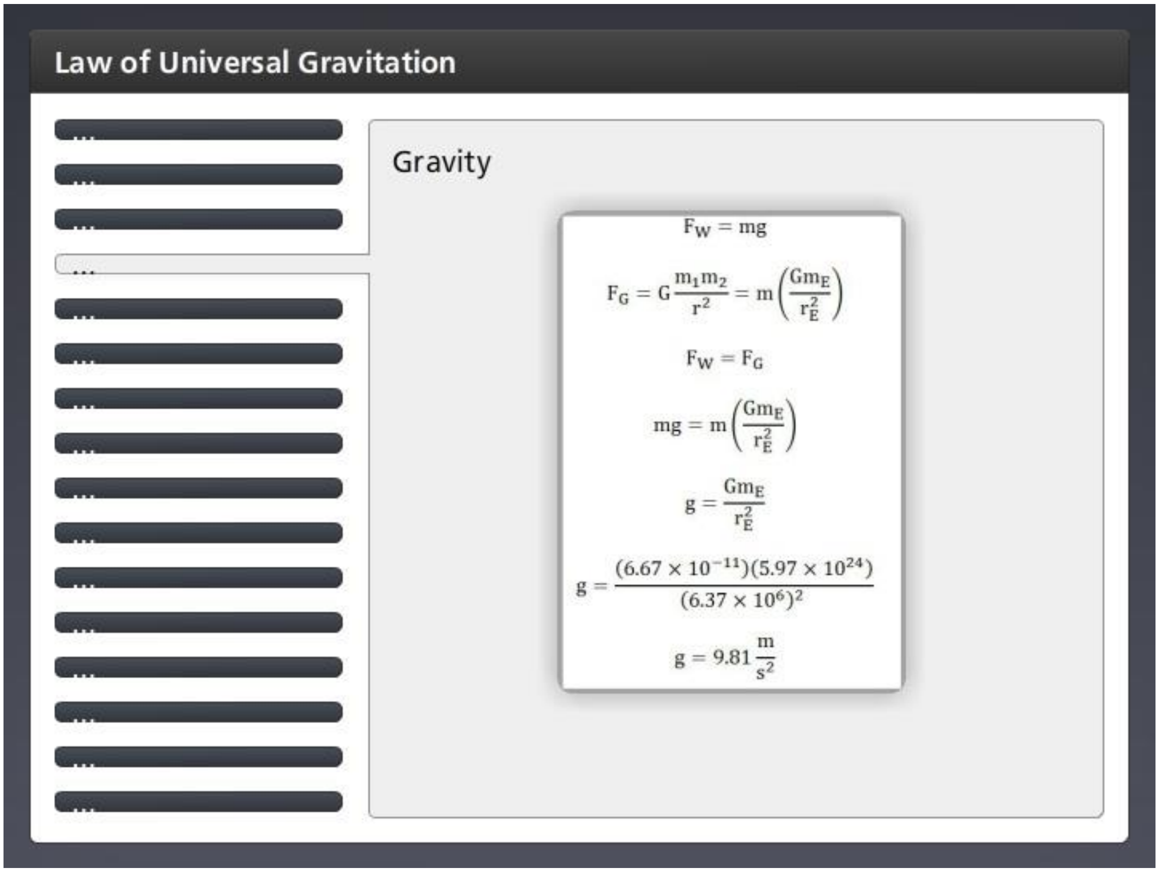
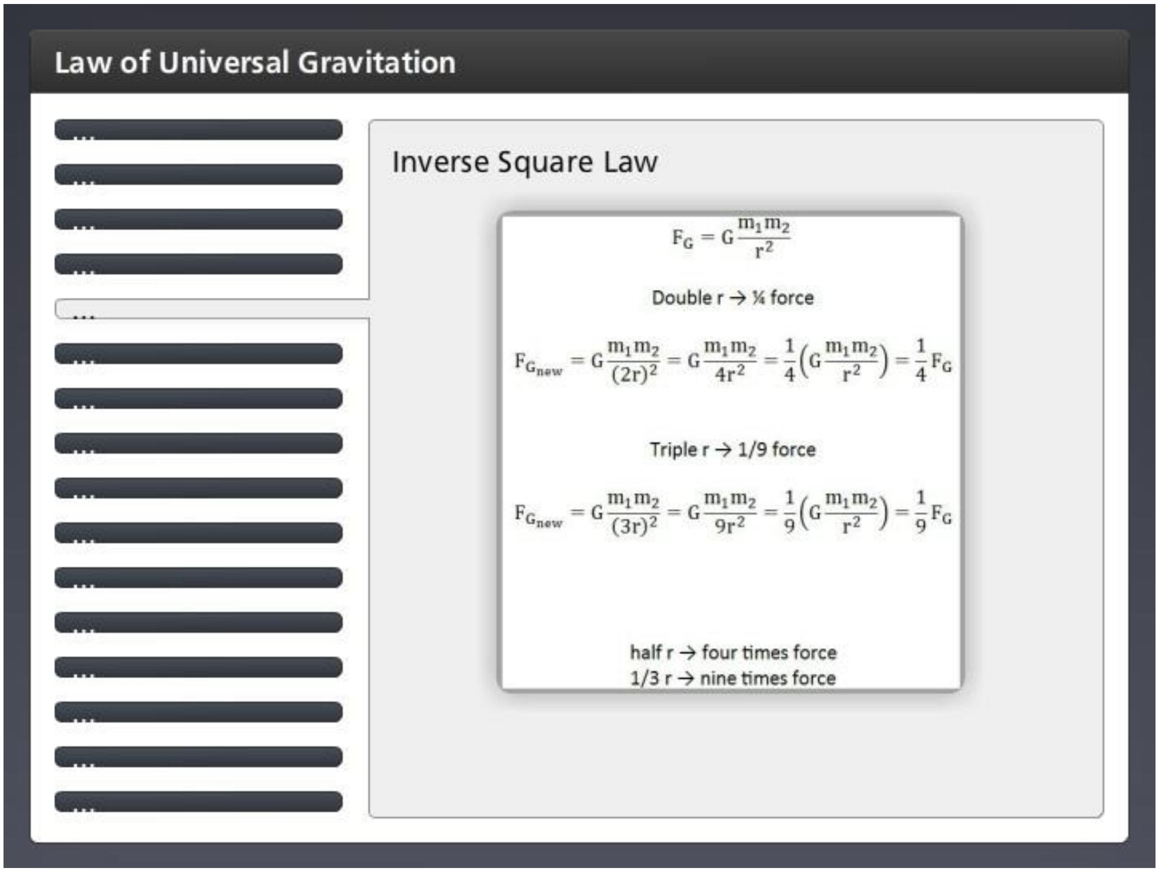
Gravity



Does this sound familiar? It should. It is the same result we would get if you used little g, the gravitational acceleration near Earth's surface.

If you look at Newton's Law of Universal Gravitation and factor out one of the masses, you find that G times the mass of the earth divided by the radius of the earth squared should be equal to little g, which, of course, it is.

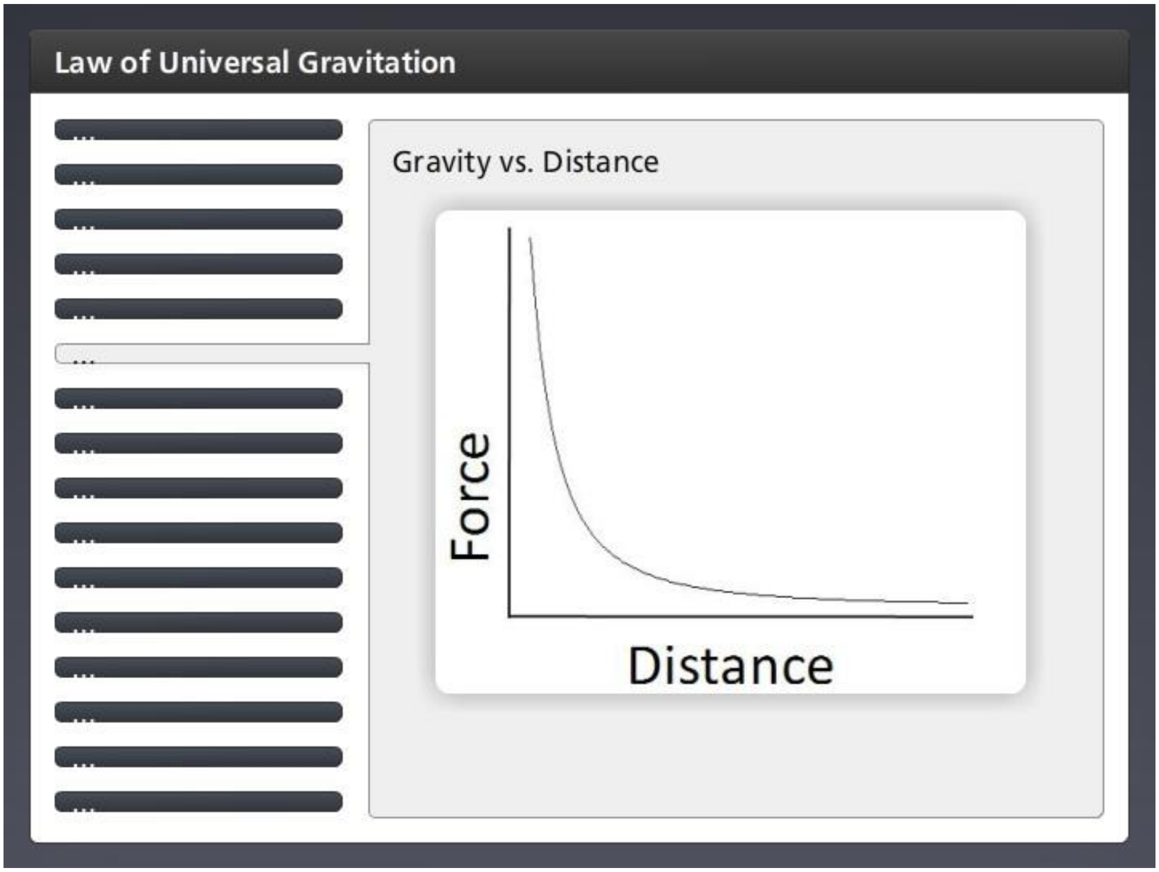
Inverse Square Law



Let's take a closer look at the Law of Universal Gravitation. This is an example of what's known as an inverse square law. The r squared in the denominator means that as you increase the distance between the two objects, the force decreases by the square of the distance. If you double the distance between two objects, the force drops to one fourth of what it was, and if you triple the distance, the force drops to one ninth of what it was.

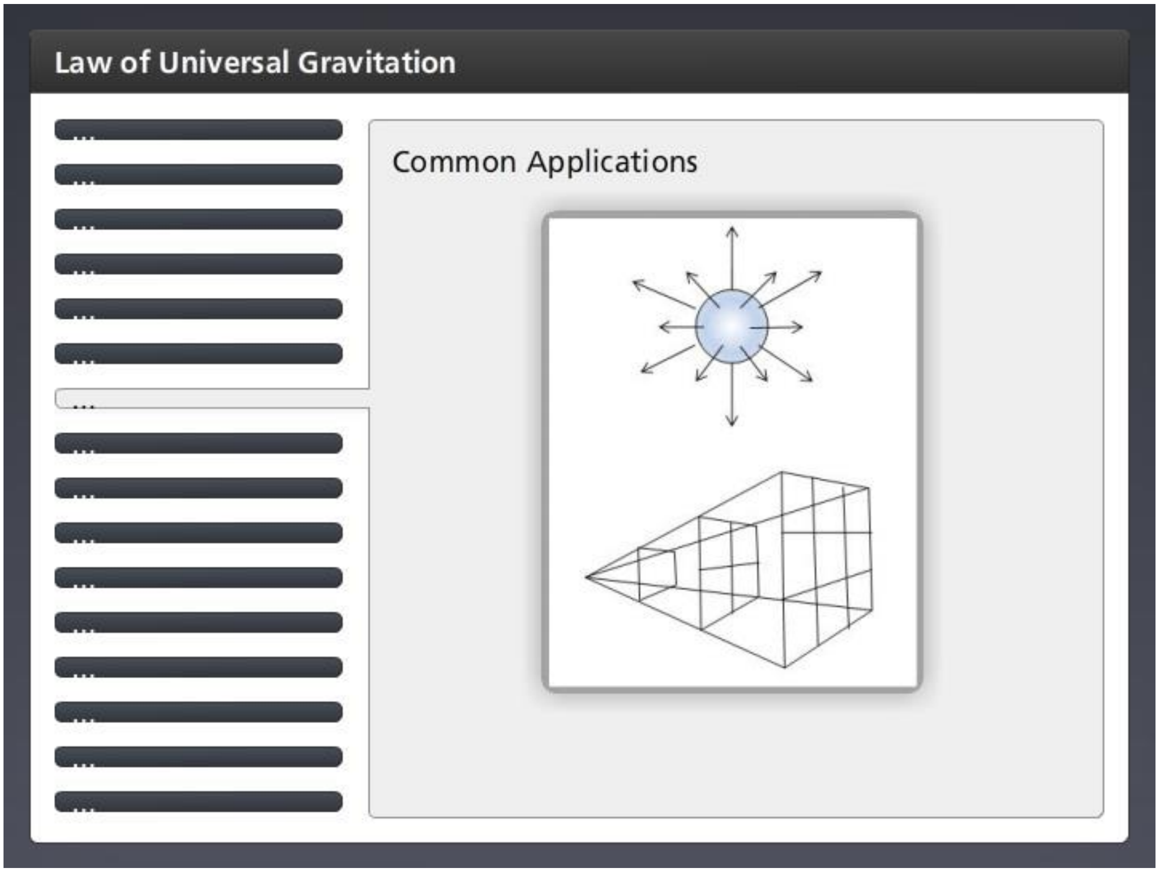
Similarly, if you decrease the distance to a half of what it was, the force grows by four times and if you decrease the distance to one third, the force increases to nine times what it once was.

Gravity vs. Distance



If you were to graph the force of gravity versus distance, the graph would look like this. This is an example of an exponentially decreasing graph, where the force quickly approaches zero as distance increases.

Common Applications

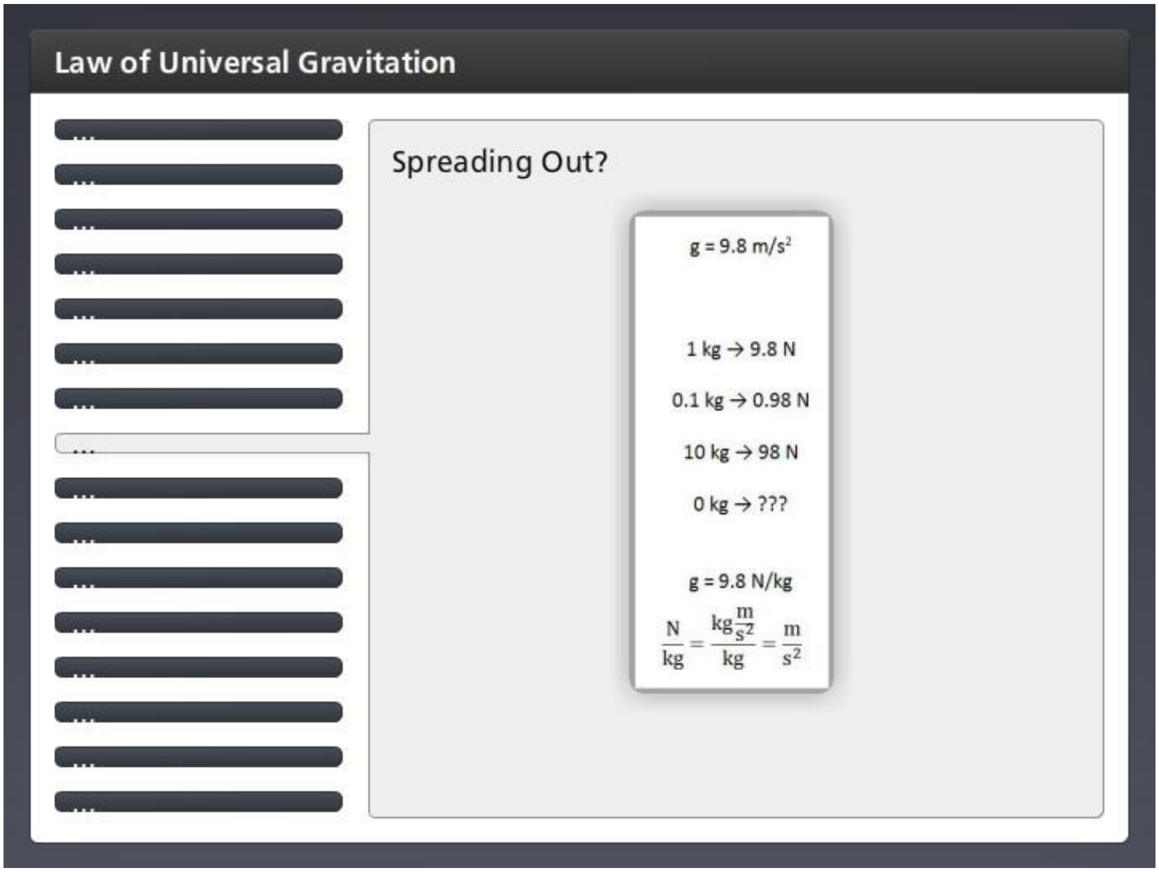


Inverse square laws are not uncommon in physics. Every time something has the opportunity to spread out in three dimensions, there is a likelihood of an inverse square law.

Light spreads out in all directions from a source such as the sun or a light bulb, so if you move twice as far away from the bulb, the intensity of the light will decrease to one quarter of what it was.

You can also think of it this way. If you move a screen twice as far away from the projector, the light will not only spread out left to right, but also up to down, When each side of a square is doubled, the area is four times as large, so the picture will become four times as big, and the light intensity will decrease to one fourth of what it was. If you move the screen to a point three times as far as from its original position, the light will spread out covering an area nine times its original area, and the intensity will decrease to one ninth its original intensity.

Spreading Out?



But what is spreading out when you are talking about gravity?

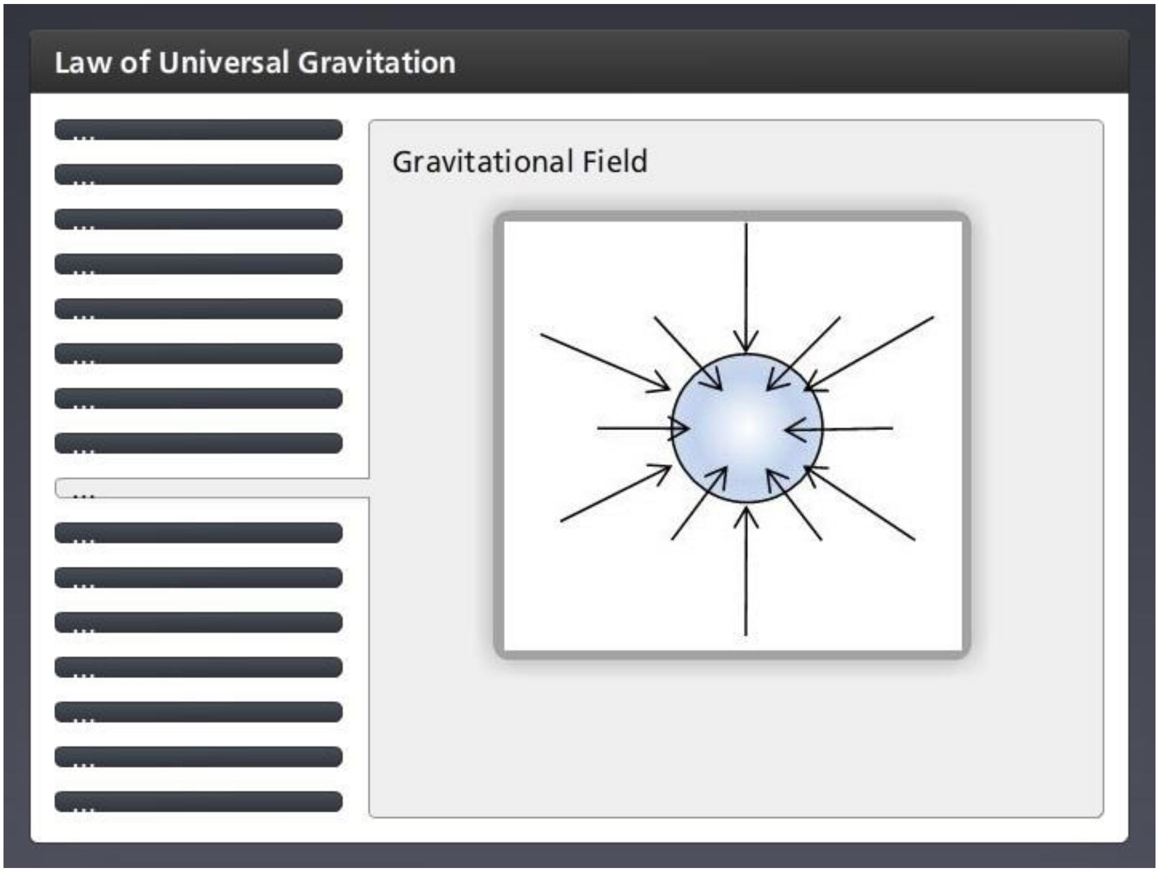
You've considered g to be the gravitational acceleration in meters per second squared. But another way that physicists think about gravity is in terms of a gravitational field, where g is a measure of the gravitational field strength.

If you hold a kilogram one meter above the center of the table, it feels a force of nine point eight Newtons due to the gravitational pull of the earth. If you hold one hundred grams in the same spot, it will feel a force of zero point nine eight Newtons. If you hold ten kilograms there, it will feel a force of ninety eight Newtons due to the gravitational pull of the earth.

But what if you leave that position empty? Is there anything there? You could say that that position has the potential to deliver a force of nine point eight Newtons per kilogram that is placed there, and the force will be directed straight down. This is the gravitational field strength in Newtons per kilogram.

How can g be both Newtons per kilogram and meters per second squared? Recall that a Newton is a kilogram meter per second squared. So if you divide Newtons by kilograms, we get meters per second squared.

Gravitational Field

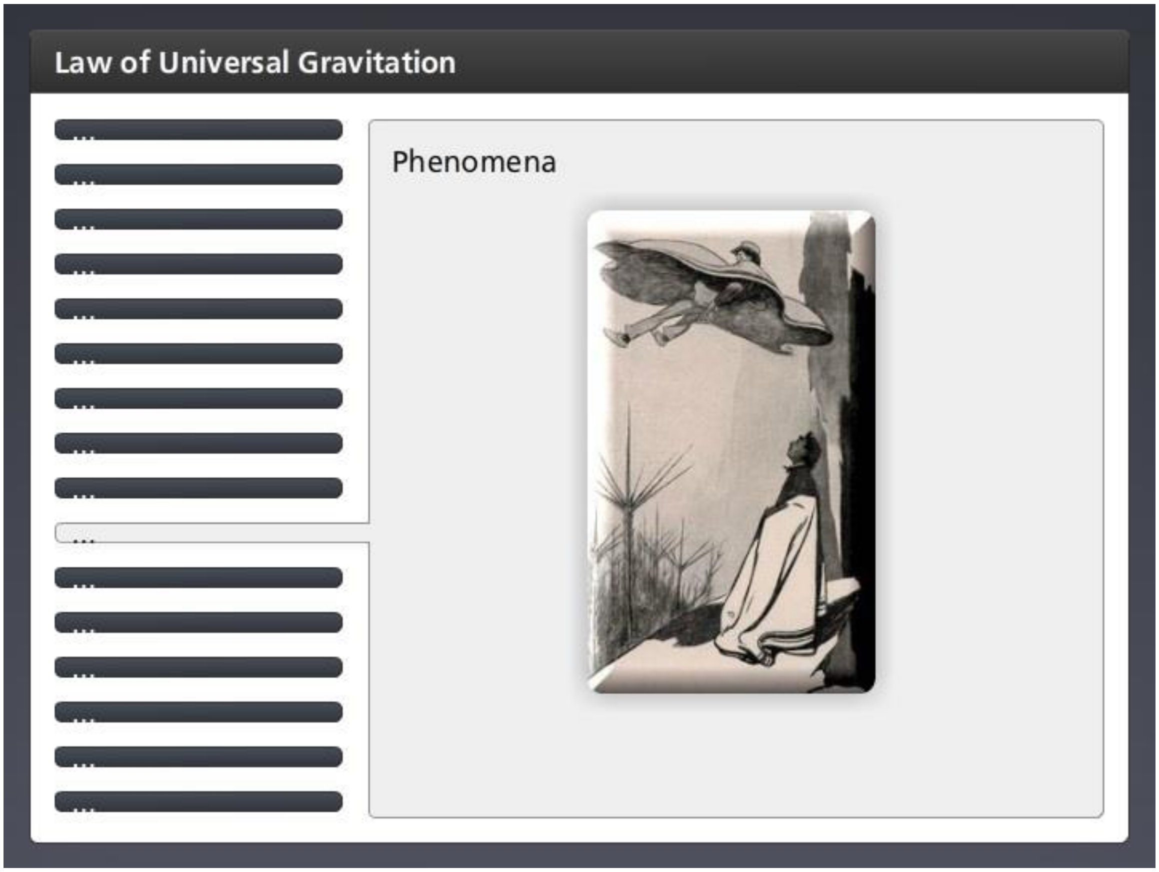


So you can say that near the surface of Earth, there is a gravitational field strength of nine point eight Newtons per kilogram directed down towards the center of the earth.

If you imagine tracking the gravitational field around the earth both nearby and farther out in space, you'd wind up with a set of arrows all pointing towards the center of the earth.

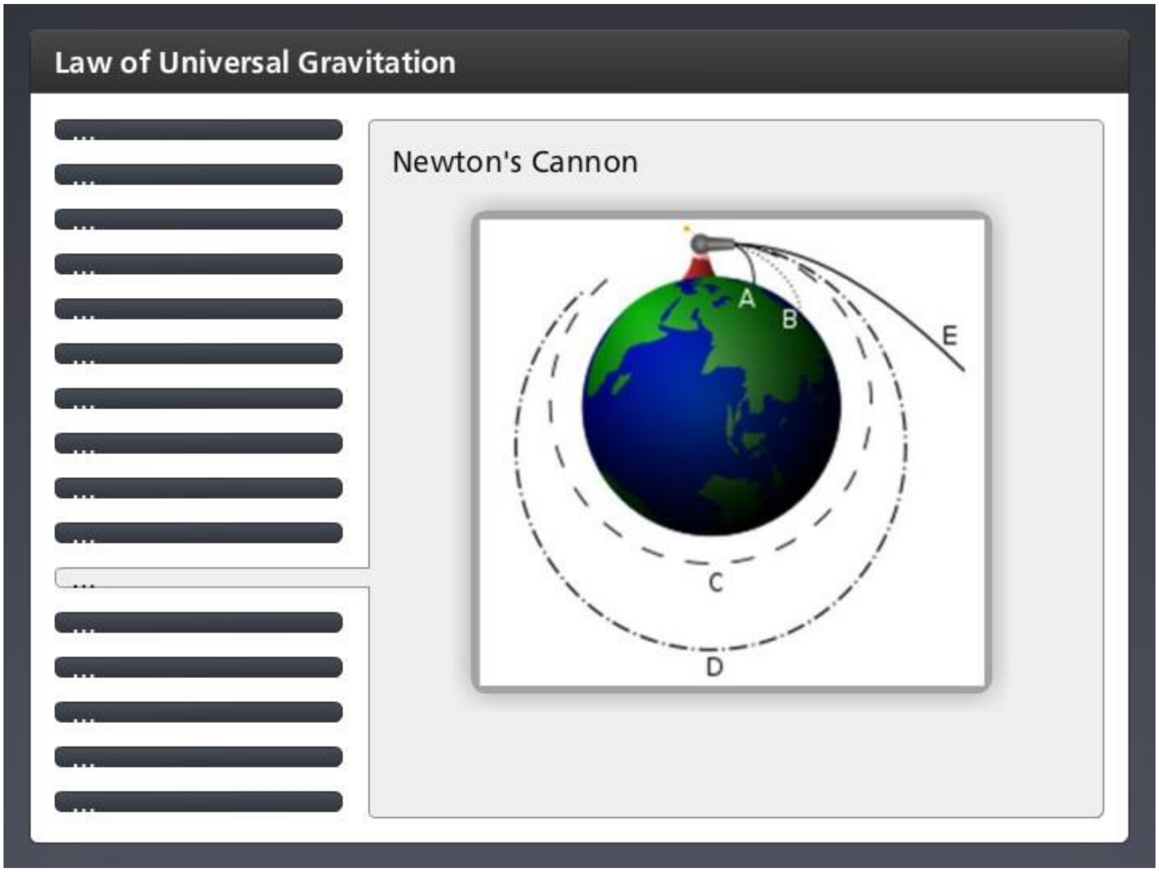
It is the gravitational field lines that are spreading out as you move farther from the earth, and when the lines spread out, the field is weaker by the inverse square law.

Phenomena



With the proposal of the Law of Universal Gravitation, Newton was able to equate the motion of objects near earth, such as a falling apple, with objects in the heavens, such as the orbiting moon, and show that these motions that had previously been considered independent phenomena were actually quite the same thing.

Newton's Cannon



Part of Newton's thought experiment went like this. Suppose you placed a cannon at the top of a tall mountain. As you horizontally launch a cannonball, it falls towards the earth and lands at point A. However, if you increase the initial speed, the cannonball will travel farther, and perhaps land at point B. Notice, however, that with sufficient speed, there is no longer a horizontal surface to land on, but the earth's surface begins curving away from the falling projectile, so it has to fall farther in order to reach the surface of the earth, resulting in a longer time of flight. With this additional time of flight comes additional range.

If you could fire a cannon ball with sufficient velocity, as it fell one meter, the earth would curve away from it one meter, so it would remain the same height above the earth. As it continued to move and fall, the earth would continue to curve away so that the cannonball would never actually hit the earth. It would continue to miss, and would then be considered to be in orbit. This is shown as path C.

Newton calculated that if you shot the cannonball faster still, it would enter an elliptical orbit as shown in path D, or if shot with sufficient velocity, it could fly along path E such that it would never fall back to earth.

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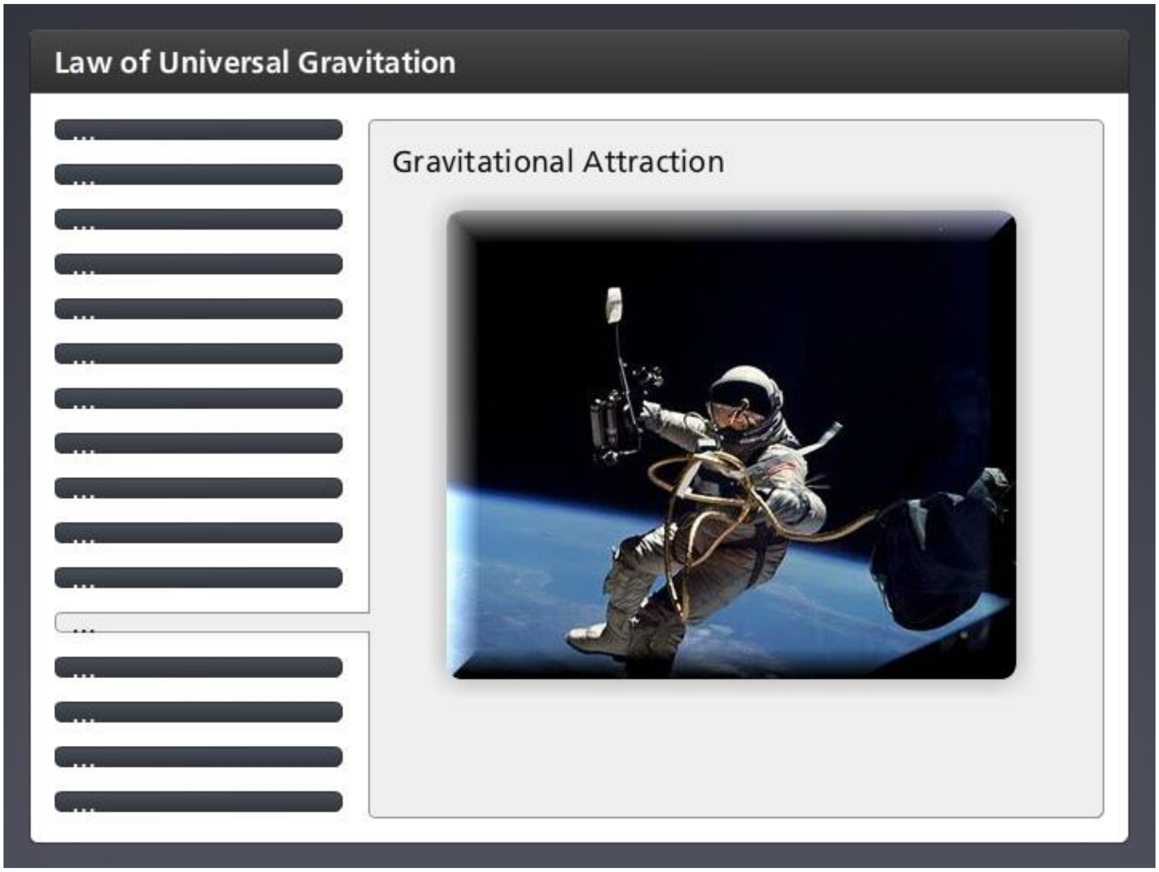
Gravitational Attraction

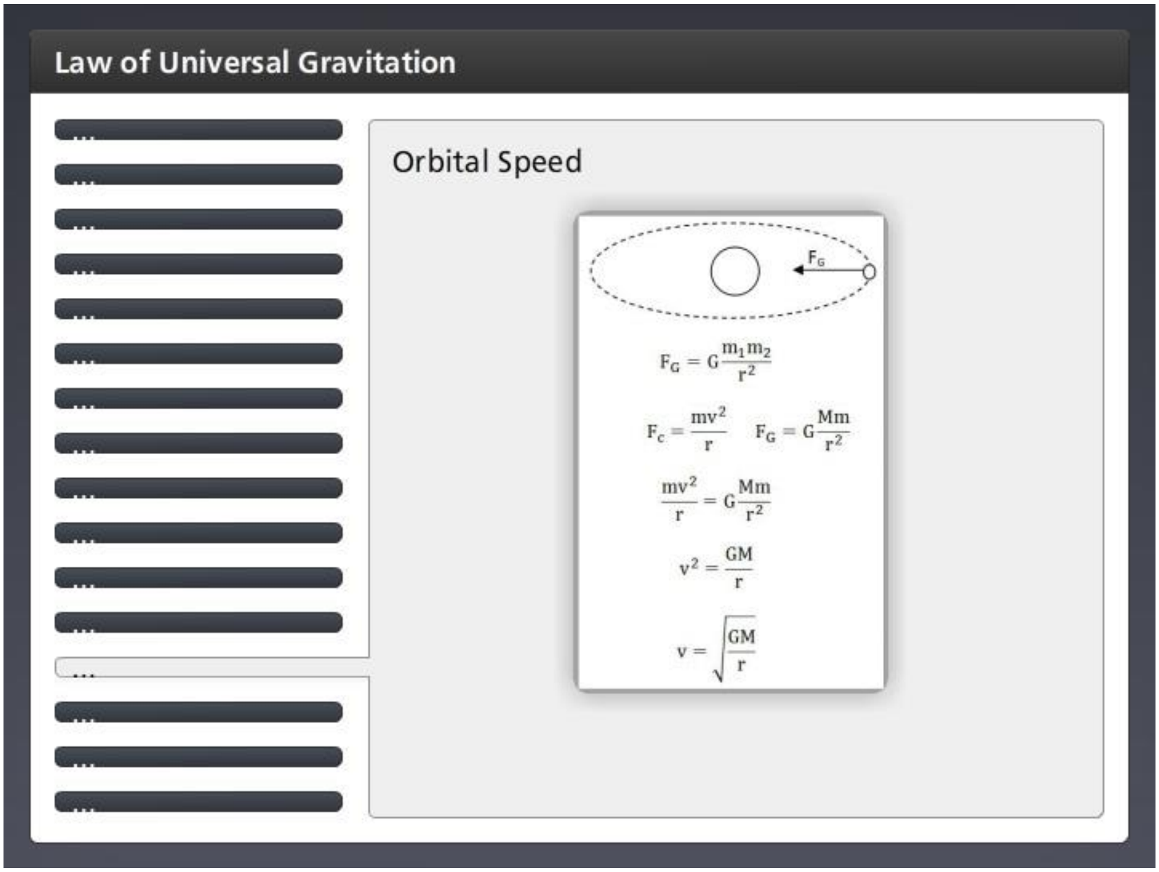


Earlier, you learned that the gravitational acceleration was nine point eight meters per second squared. However, this is only true near the surface of the earth.

Of course, for most of us, this is where we spend our lives, and it is just over five hundred lucky humans that have gone as far as one hundred kilometers above the earth. Only twenty four humans have ever ventured into Lunar orbit. So it is generally safe to say that the gravitational acceleration is nine point eight meters per second squared.

However, the gravitational attraction decreases with the square of the distance from the center of the earth. Even on the international space station, at just below two hundred miles above the surface of the earth, objects are subject to about ninety percent the gravitational force they would feel on earth.

Orbital Speed



And it is precisely this gravitational force that provides the centripetal force to keep objects moving in orbit.

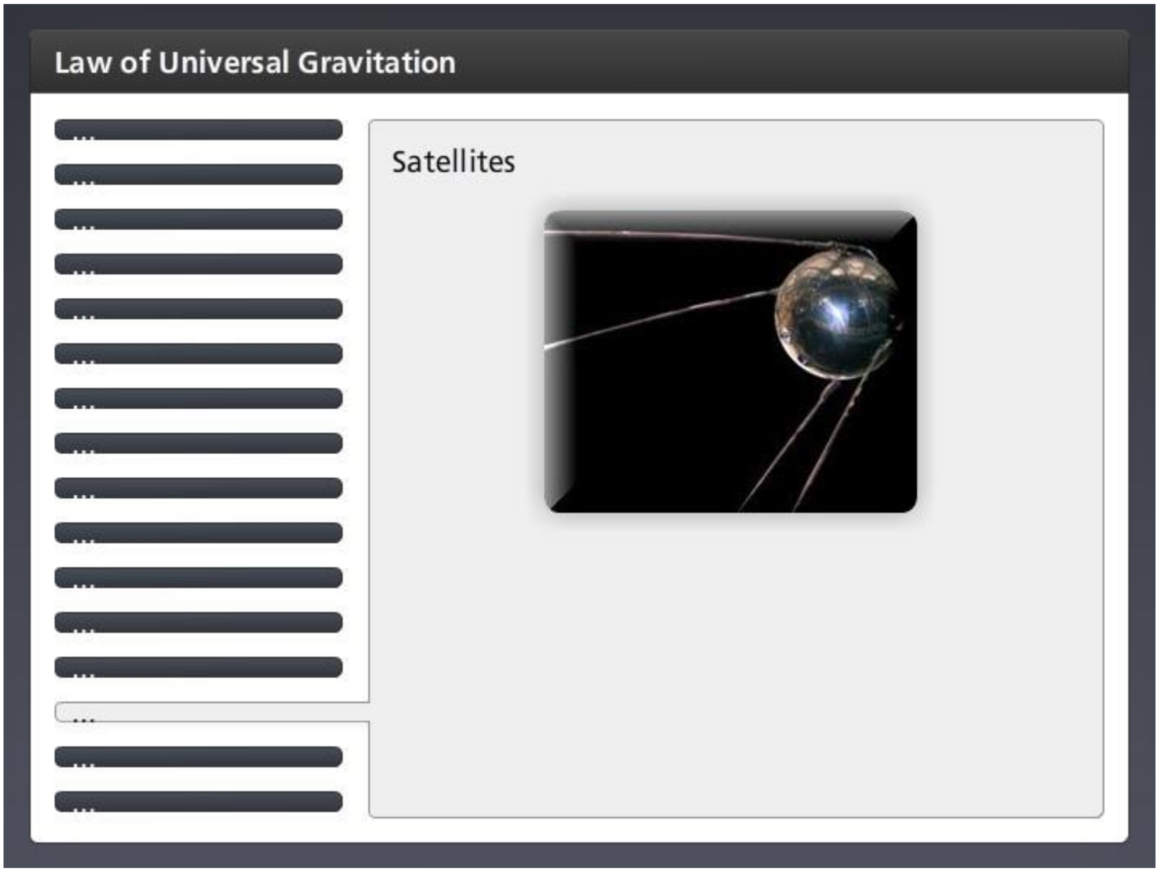
When talking about an object in orbit, you'll say that m one is the mass of the earth, and replace it with a capital M. you'll let m two be the mass of the orbiting object and leave this as a simply a lower case m. For the centripetal force, m is also the mass of the orbiting object.

If you set the formula for universal gravitation equal to our formula for centripetal force, you can see that the mass of the object cancels out, as does one of the r's

What is left is the universal gravitational constant, the mass of the earth and the distance from the center of the earth that determine the orbital speed. You will not need to work with this equation, but you can see that since the mass of the orbiting object cancels out, astronauts can be assured that they don't have to worry about a less massive tool orbiting at a different velocity than the more massive space station! Instead, all objects at a certain height above the earth will orbit at the same speed.

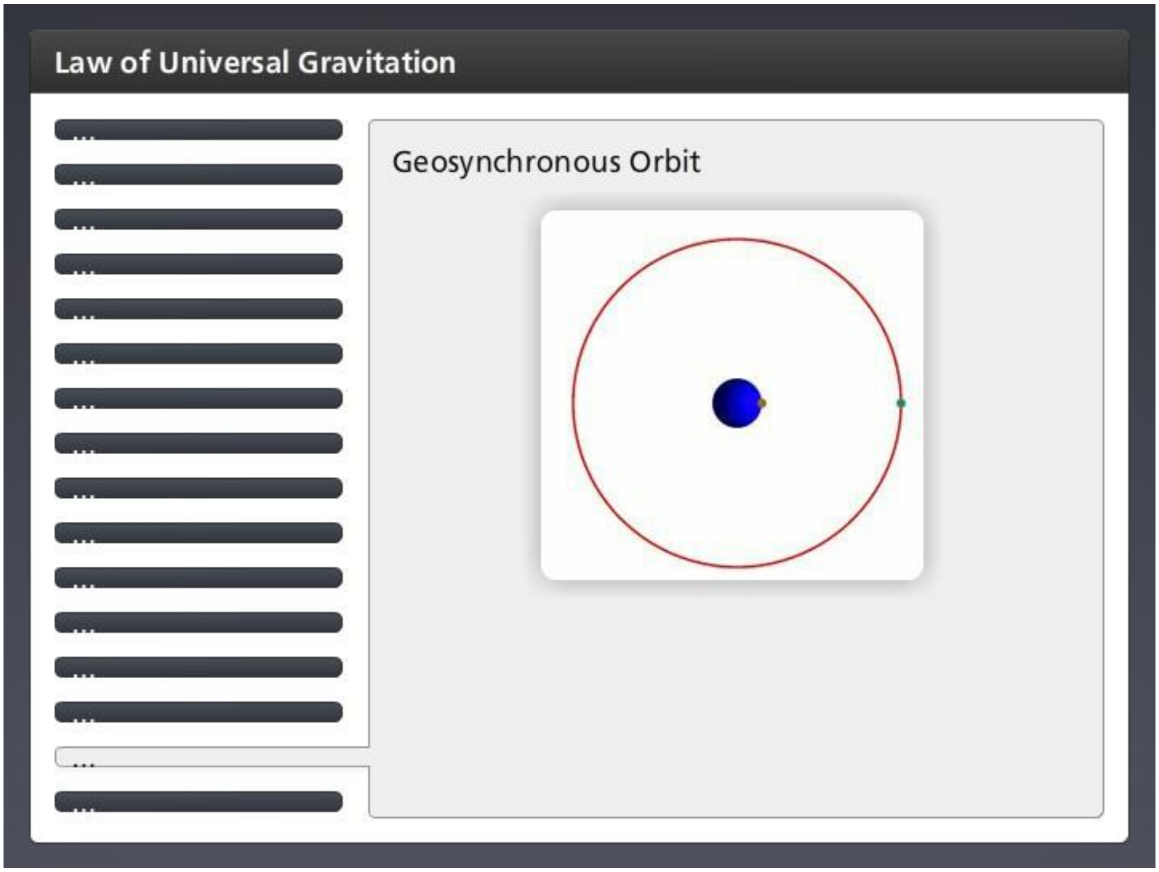
Since r is in the denominator, you can also see that the higher the orbit, the slower the orbital speed. If the speed decreases and the circumference of your circular path increases, it is clear that the time to complete an orbit will increase.

Satellites



One interesting application of this fact is the concept of the geosynchronous satellite. If you look up in the sky at night, there are several objects that you can see with the naked eye that are orbiting the earth. The most obvious of these is the moon. The moon takes nearly a month to complete an orbit of the earth. If you look at a clear evening sky, away from city lights where lots of stars are visible, you'll occasionally see what appears to be a dim, drifting star moving slowly across the background. If it is an airplane, it will blink, but if it is not blinking, you are likely seeing one of the many man-made satellites that have been sent into orbit for commercial, scientific or military purposes. Also, if you know when to look, you can see the international space station pass overhead, which is a remarkable sight, as it can be brighter than the planet Venus, which is often the brightest object in the morning or evening sky.

Geosynchronous Orbit



All of these seem to drift across the sky, so they would be difficult to track. But there is one particular orbital height at which it takes satellites precisely twenty four hours to orbit the earth. This is known as the Geosynchronous orbit. Satellites placed in this orbit will appear fixed in the sky as the earth turns below the orbiting satellite. This makes it quite easy to point a fixed receiver at the satellite. Science fiction author Arthur C. Clarke first proposed the idea of using such satellites for communications in 1945, twelve years before the first man-made satellite, the Soviet's Sputnik, was placed in orbit.

Today, you'll see many homes with small satellite dishes pointed directly at one of these geosynchronous communications satellites to pick up their television signals.

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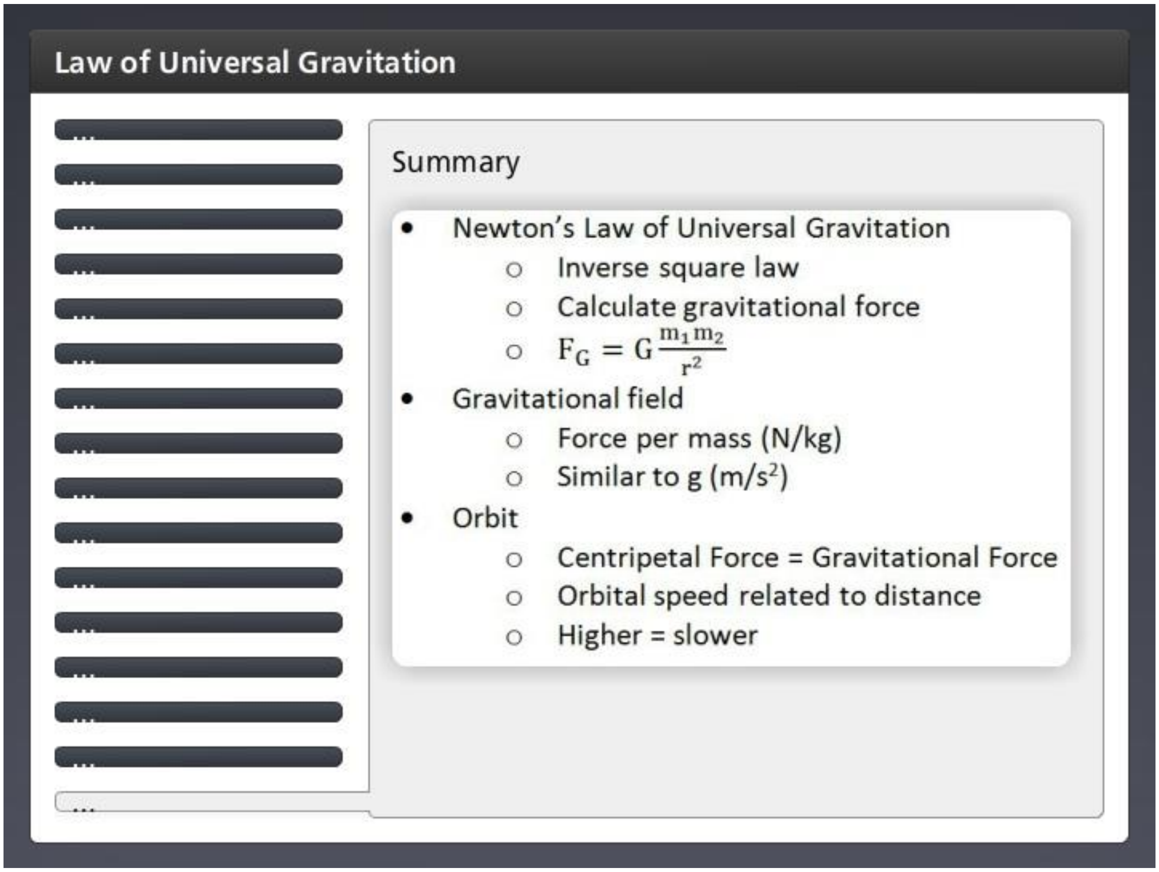
Summary



In summary, Newton's law of universal gravitation is one of several examples of an inverse square law - one in which a quantity is inversely proportional to the square of the distance. It allows you to calculate the gravitational force felt by any two objects so long as you know their masses and their distances from one another.

Objects with mass, such as the earth, create a gravitational field. The interaction of the mass of another object with that field determines the force that another object will have when placed in its vicinity. Gravitational field strength, measured in Newtons per kilogram is a different way of expressing the acceleration of gravity, measured in meters per second squared.

Objects in orbit always feel a force directed towards the center of the earth, and by combining our understanding of centripetal force with universal gravitation, we can understand the motions of planets and satellites.

All objects orbiting at a specific distance from the earth orbit at the same speed, and the higher the orbit, the slower the speed.