Module 4: Energy

Topic 1 Content: Work and Power

Work and Power

Physics uses a specific definition for work, and by looking at several examples, you'll be able to determine just how it is applied.

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Topic 1 Content: Work and Power


The man does work.

The waitress does no work.

The woman does negative work.

The mother does work.

The weight lifter does no work

Friction does negative work

See if work is done or not!

Example one: A man lifts a box from the floor onto a shelf. The man does work as he lifts the box.

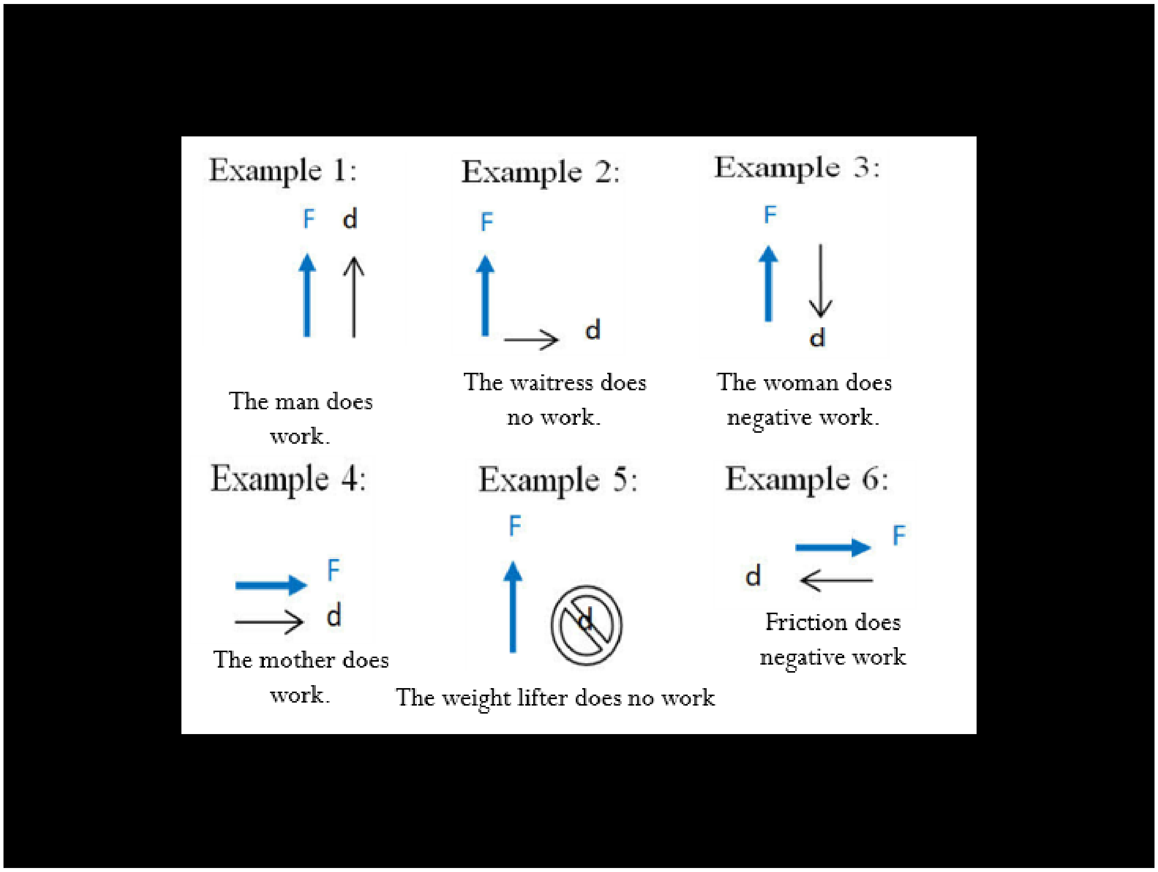
Example two: A waitress carries a tray full of food across the restaurant. The waitress does no work carrying the tray.

Example three: A woman takes a box from a shelf and places it on the floor. The woman does negative work while lowering the box.

Example four: A sled on ice skates is pushed from behind by a mother so that the sled starts moving. The mother does work while pushing the sled.

Example five: A weight lifter holds weight over his head. The weight lifter does no work.

Example six: A baseball player slides into third base. Friction does negative work while acting on the baseball player.



Example 1:

The man does work.

Example 2:

The waitress does no work.

Example 3:

The woman does negative work.

Example 4:

The mother does work.

Example 5:

The weight lifter does no work

Example 6:

Friction does negative work

Do you see a pattern here? Perhaps it will be useful to look at the applied force or forces in each situation.

In example one, the man applies a force to the box in an upward direction. At the same time, the box moves up, in the same direction.

In example two, the waitress must apply a force up on the tray as she carries it, but the direction of motion is forward perpendicular to the direction of the force.

In example three, while the box is being lowered, the woman is still applying an upward force to keep the box from falling. The force points opposite the direction of motion.

In example four, the mother pushes forward, and the sled moves forward in the same direction as the force.
In example five, the weight lifter does not move the weight any distance, so no work is done.

And in example six, friction provides a force opposite the direction of motion, bringing the baseball player to a stop.

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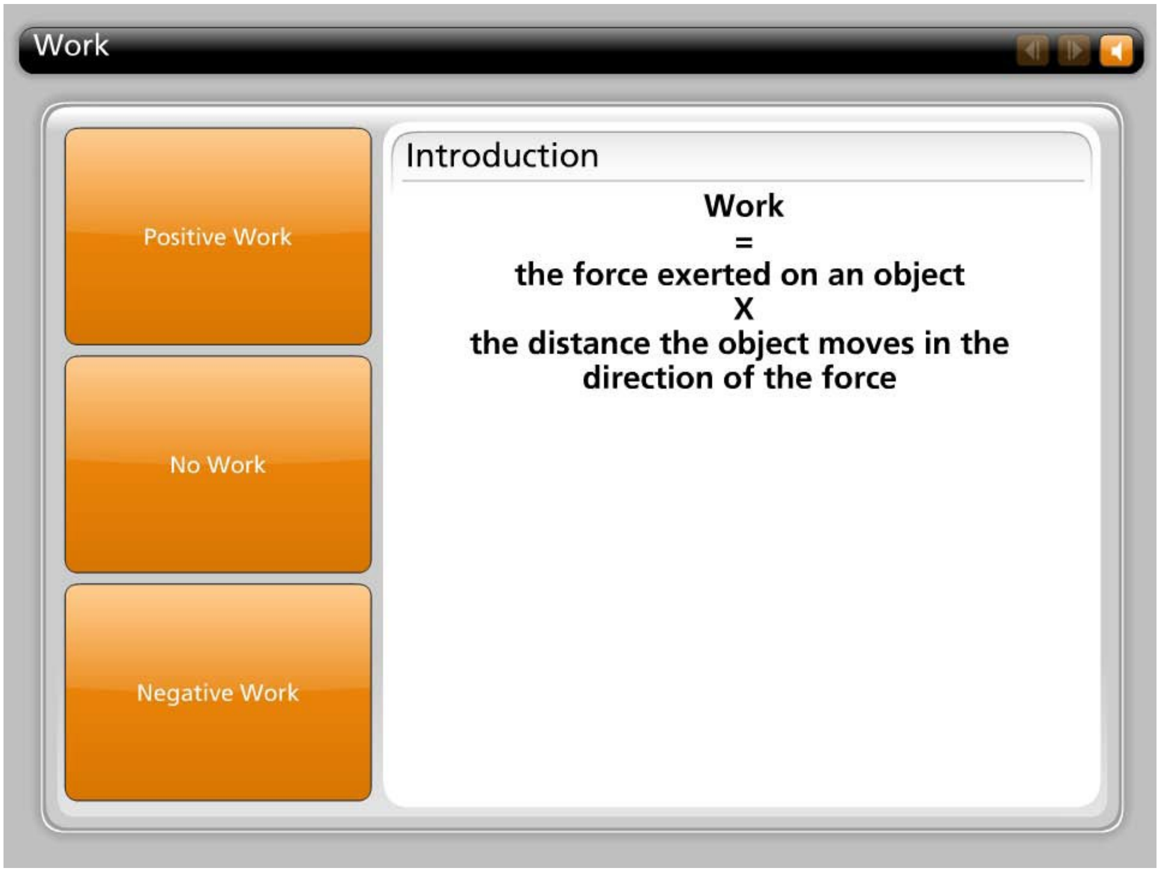
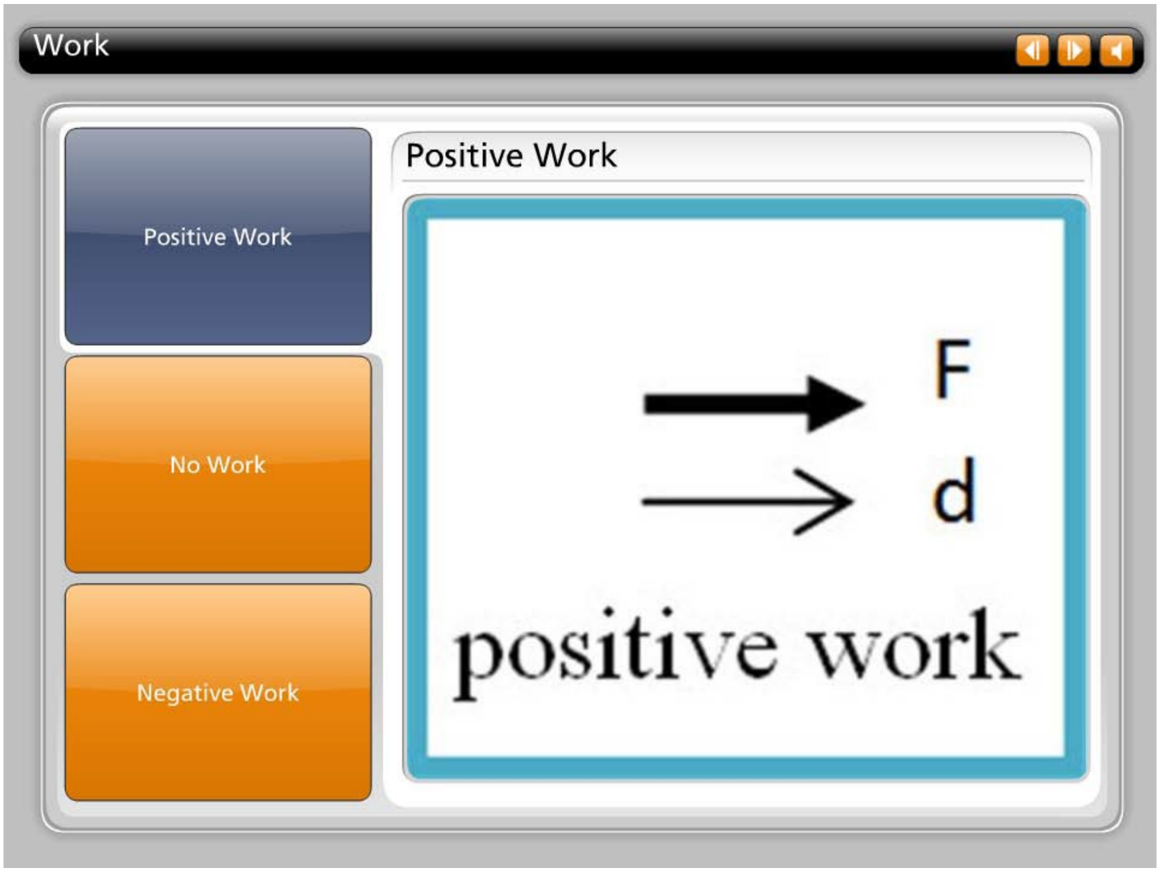


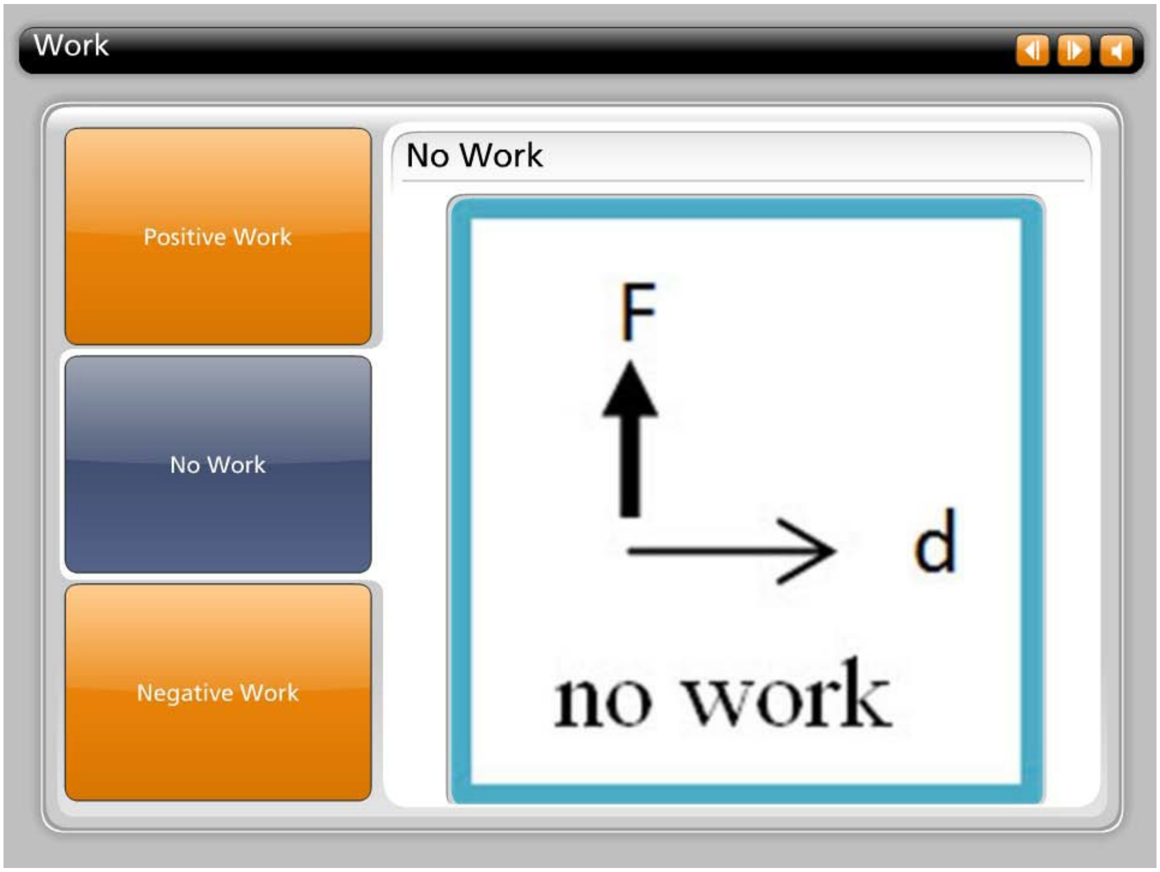
Introduction

Work
=
the force exerted on an object
X
the distance the object moves in the direction of the force

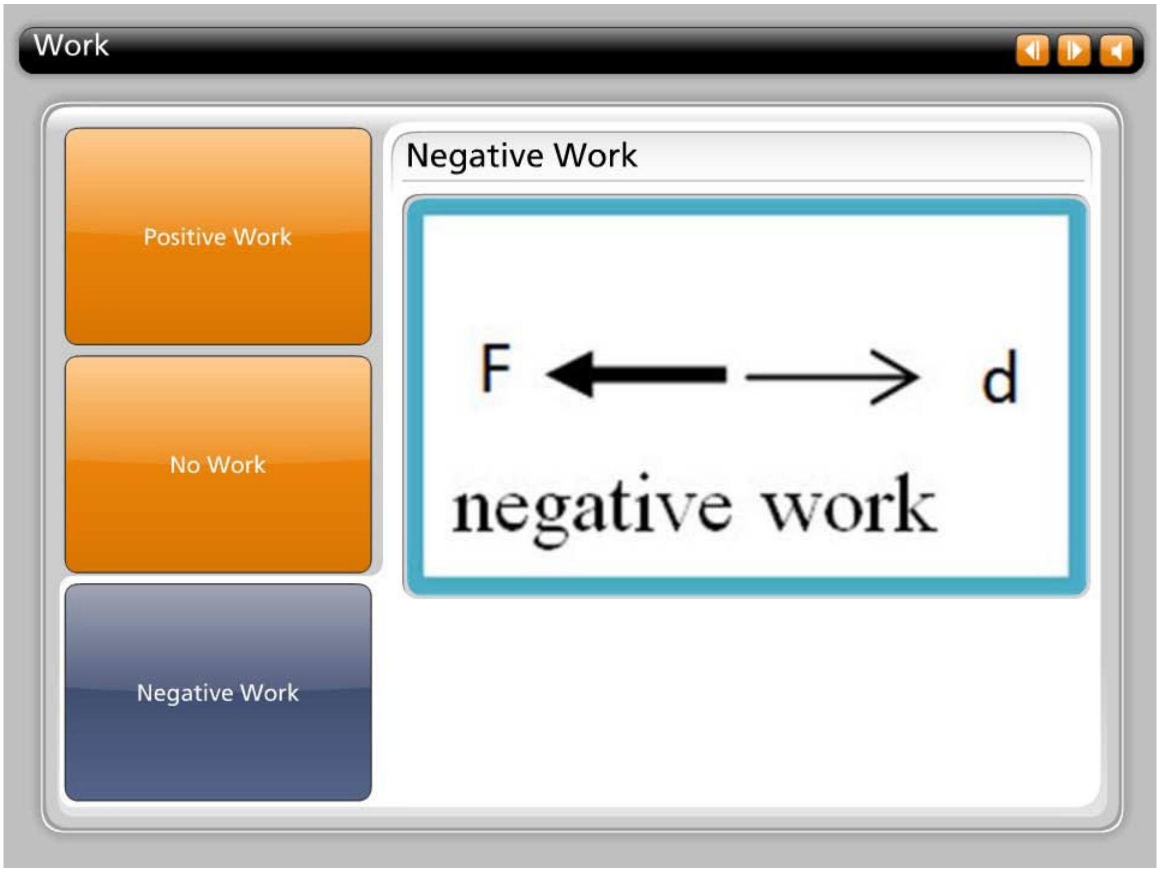
In physics, work is the product of the force exerted on an object and the distance the object moves in the direction of the force.



As you can see, when the force is in the same direction that the object moves, work is done.

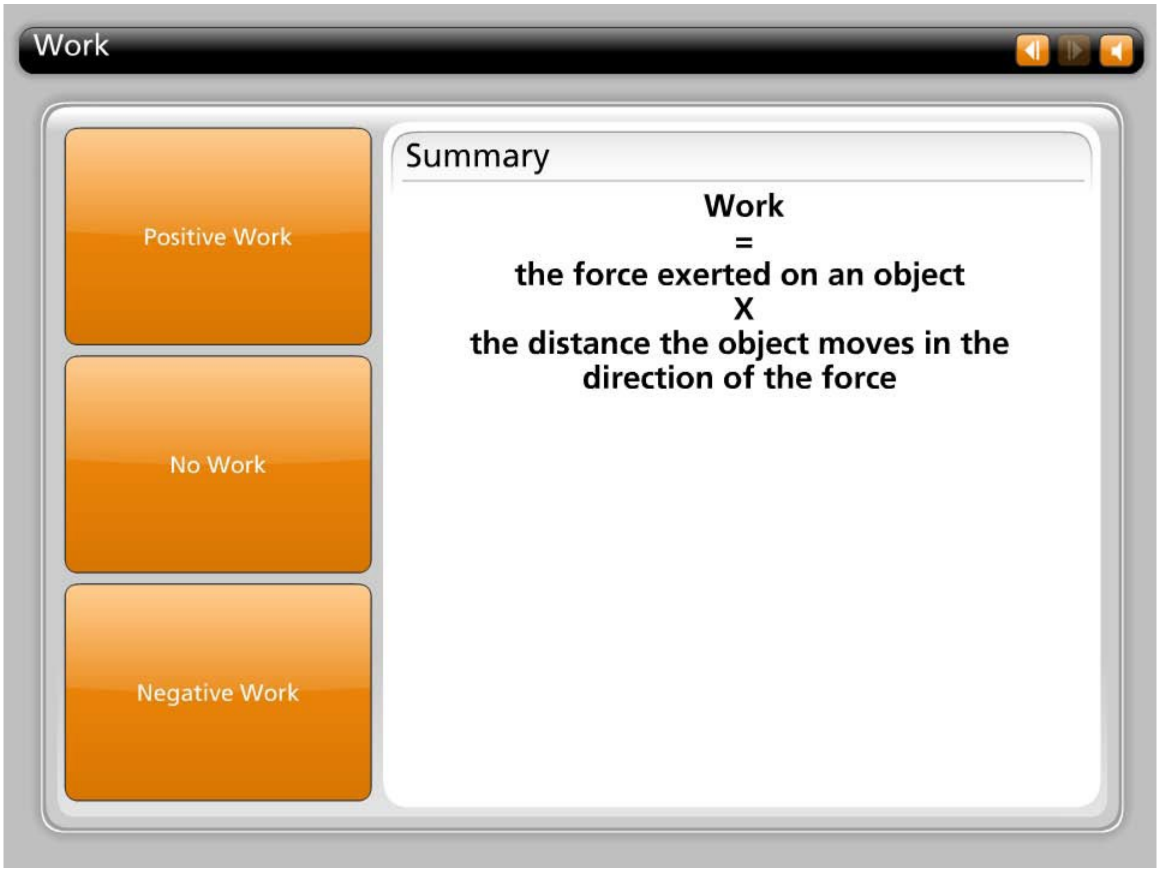


When the force is perpendicular to the displacement, there is no work done. Also, when the object does not move, no work is done.



Negative Work

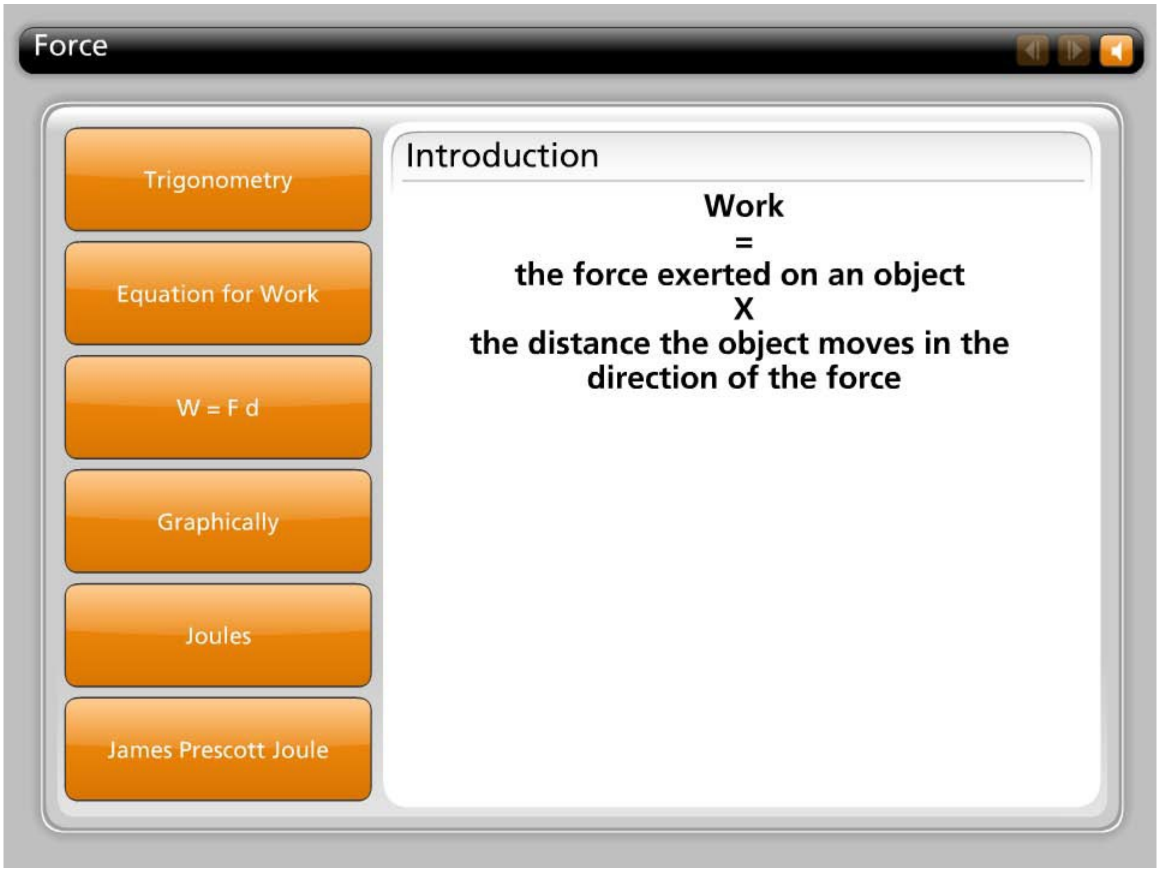
Finally, when the force is applied opposite the direction of motion, this results in negative work.



Summary

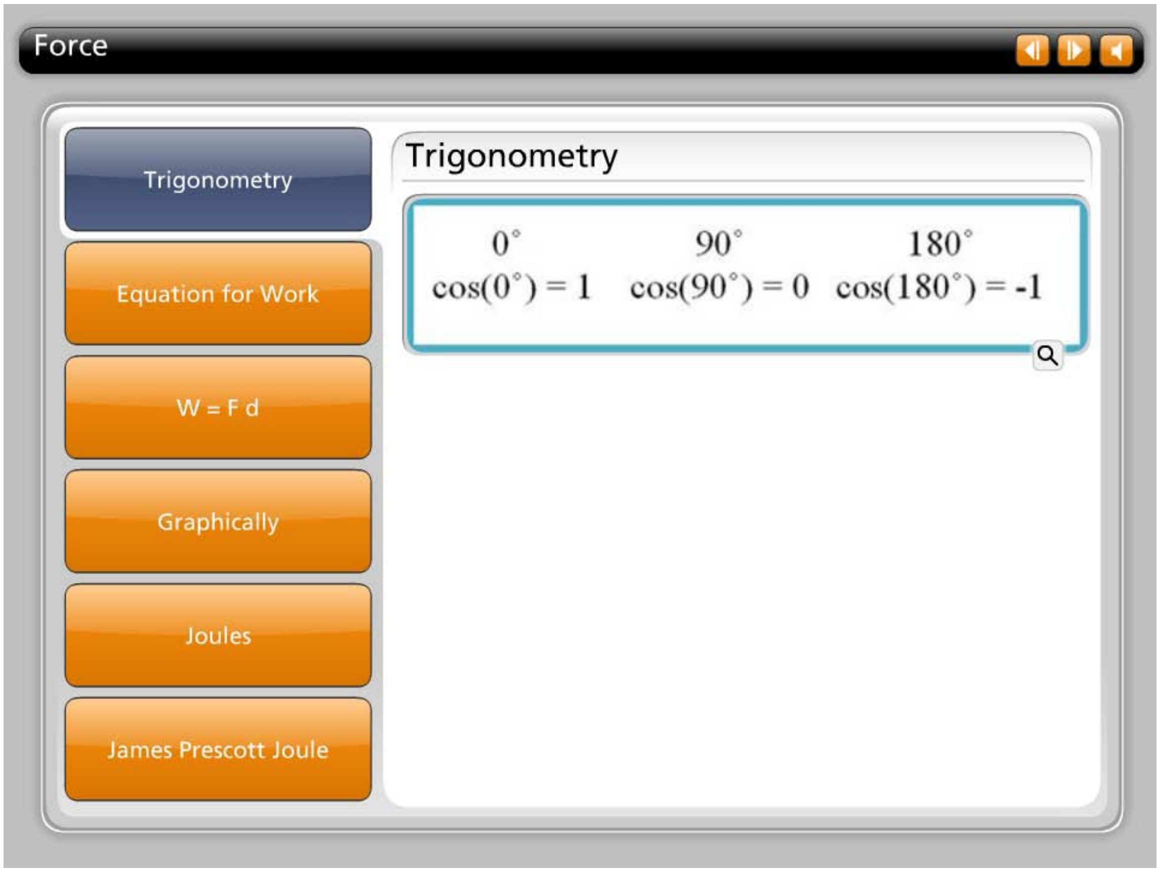
Work
=
the force exerted on an object
X
the distance the object moves in the direction of the force

As you can see, this very specific definition is likely different from what you thought before studying physics. Most of the time, if you have a good work out, or work hard on your homework or at your job, you don't really accomplish any work as defined in physics.



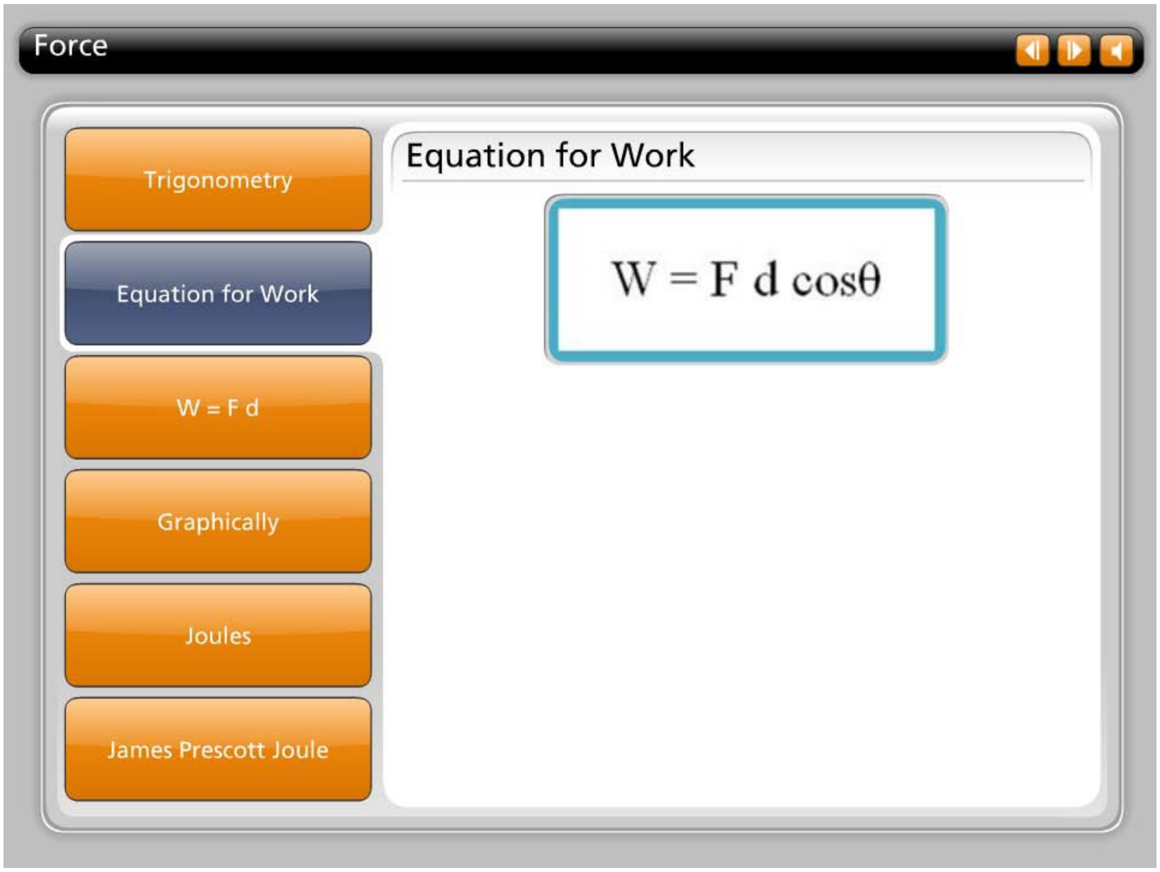
Work
=
the force exerted on an object
X
the distance the object moves in the direction of the force

Given that work is the product of the force exerted on an object and the distance the object moves in the direction of the force, you need a method to deal with forces and displacements at any angle.



0°	90°	180°
$\cos(0^\circ) = 1$	$\cos(90^\circ) = 0$	$\cos(180^\circ) = -1$

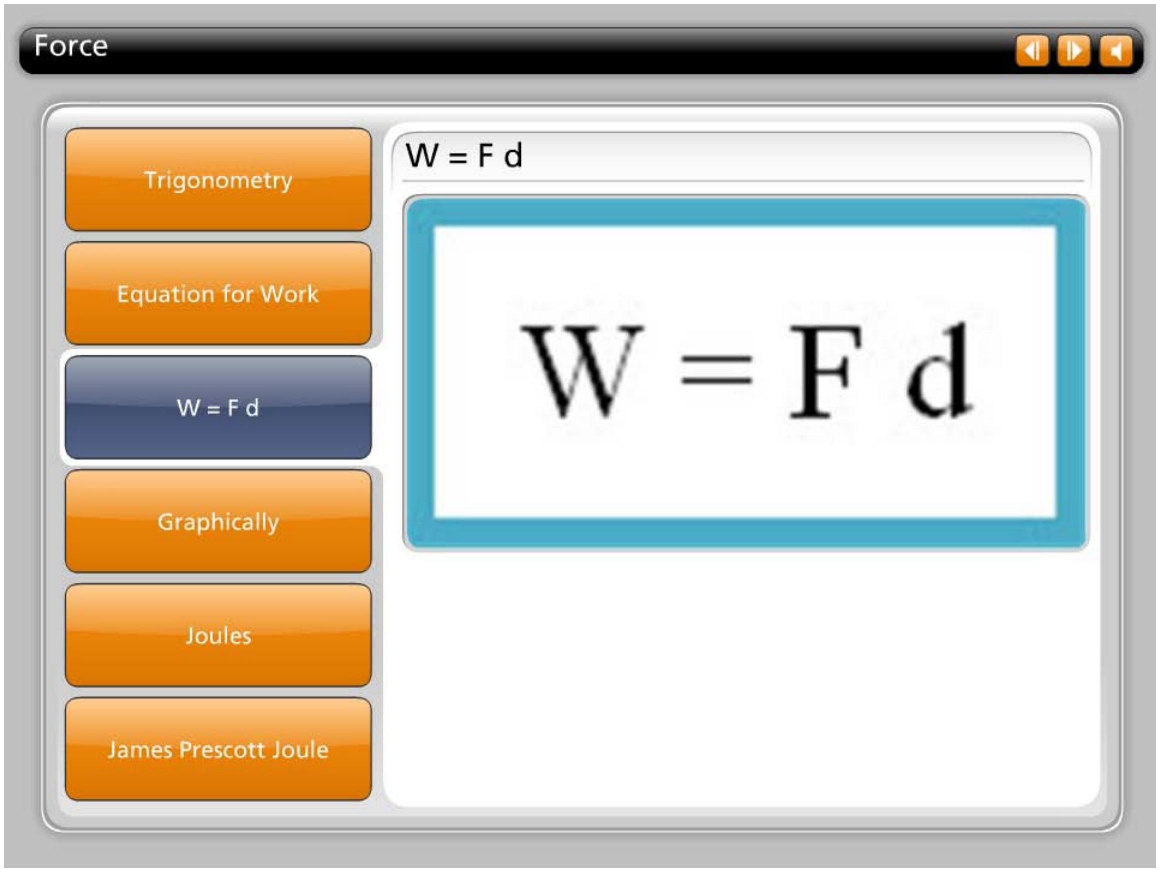
If you consider the angle between the force vector and the displacement vector, when the angle is zero, you get positive work, when the angle is ninety degrees, you get no work, and when the angle is one hundred eighty degrees, you get negative work. If you remember your trigonometry, you should recognize that the cosine function is equal to one when the angle is zero. It is zero when the angle is ninety degrees, and it is negative one when the angle is one hundred eighty degrees.



Equation for Work

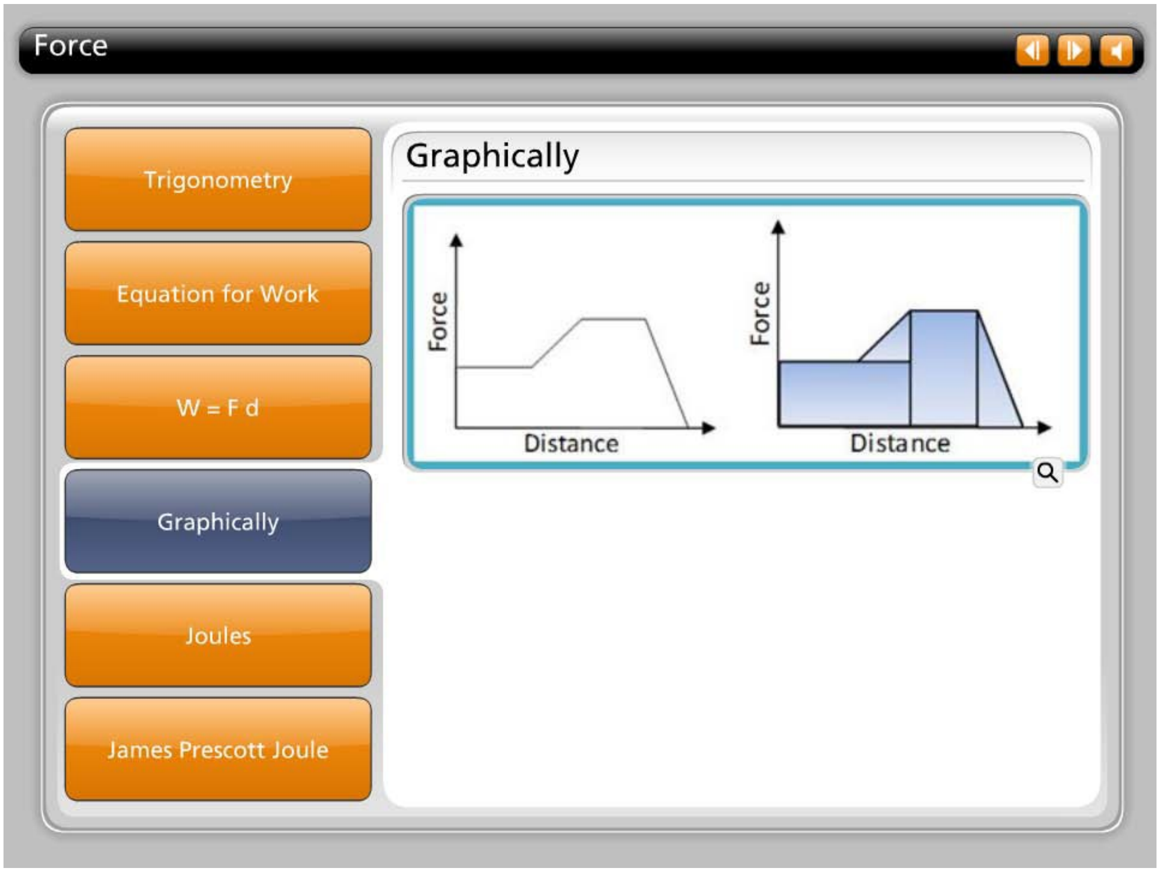
$$W = F\,d\,\cos\theta$$

This lets us see that, in order to handle forces at any angle, our equation for work will be force times distance times the cosine of the angle between the two vectors, or work equals f d cosine theta.



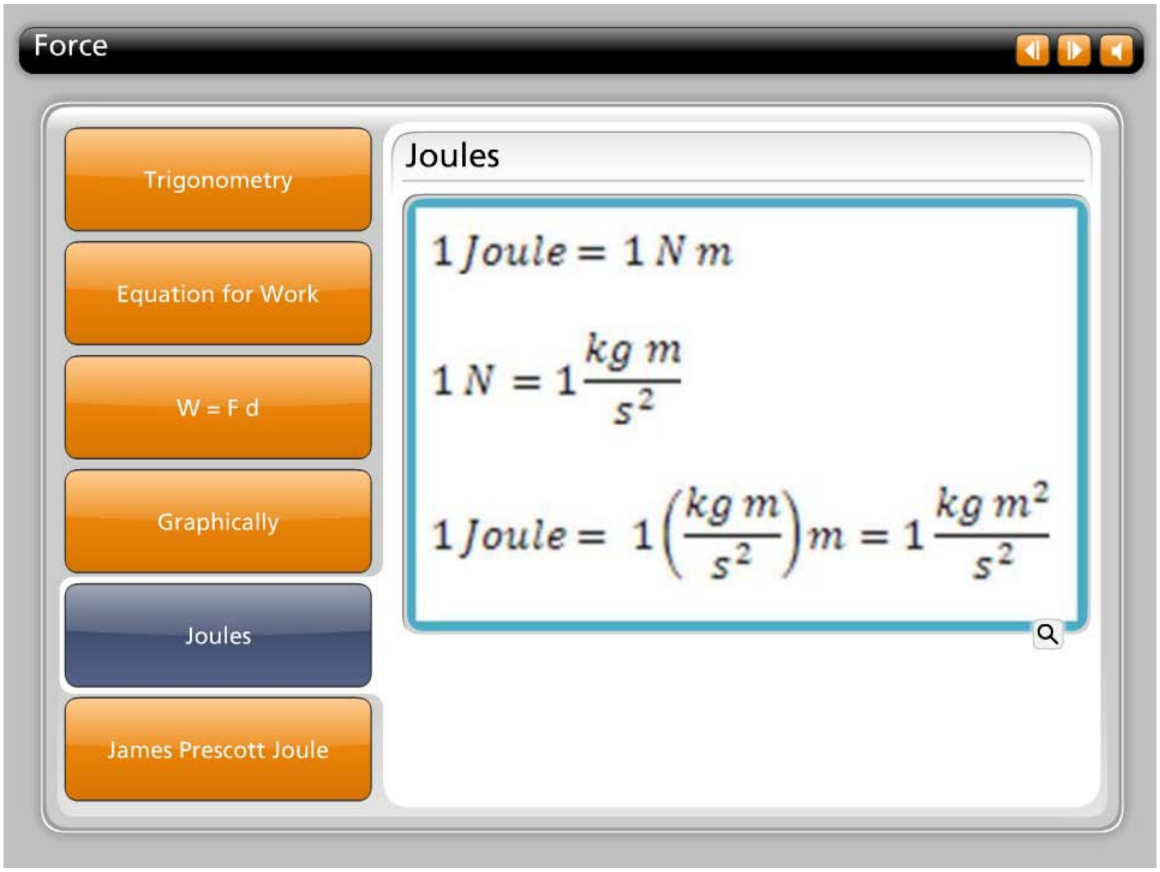
$$W = F\,d$$

In the simplest case, when the force and the displacement are in the same direction, this simplifies to work equals force times distance.



You could also look at this relationship graphically.

This graph shows a non constant force being applied over a distance. Since work is force times distance, the area under the graph will equal the work done.



$$1\,Joule = 1\,N\,m$$

$$1\,N = 1\frac{kg\,m}{s^2}$$

$$1\,Joule = 1\left(\frac{kg\,m}{s^2}\right)m = 1\frac{kg\,m^2}{s^2}$$

The units for work are Joules. Work is equal to force in Newtons times distance in meters time cosine theta, which has no units. A Newton times a meter is defined to be a Joule.

Since a Newton is equal to a kilogram meter per second squared, you can also see that a Joule is equal to a kilogram meter squared per second squared.

Work is also a scalar quantity. Remember that vectors have magnitude and direction, while scalars only have magnitude.

There are two methods of vector multiplication, the dot product and the cross product. The dot product gives a scalar result and the cross product gives a vector result.

When you calculate work you are multiplying the magnitude of the force times the magnitude of the displacement times the cosine of the angle between them. This gives a scalar result.

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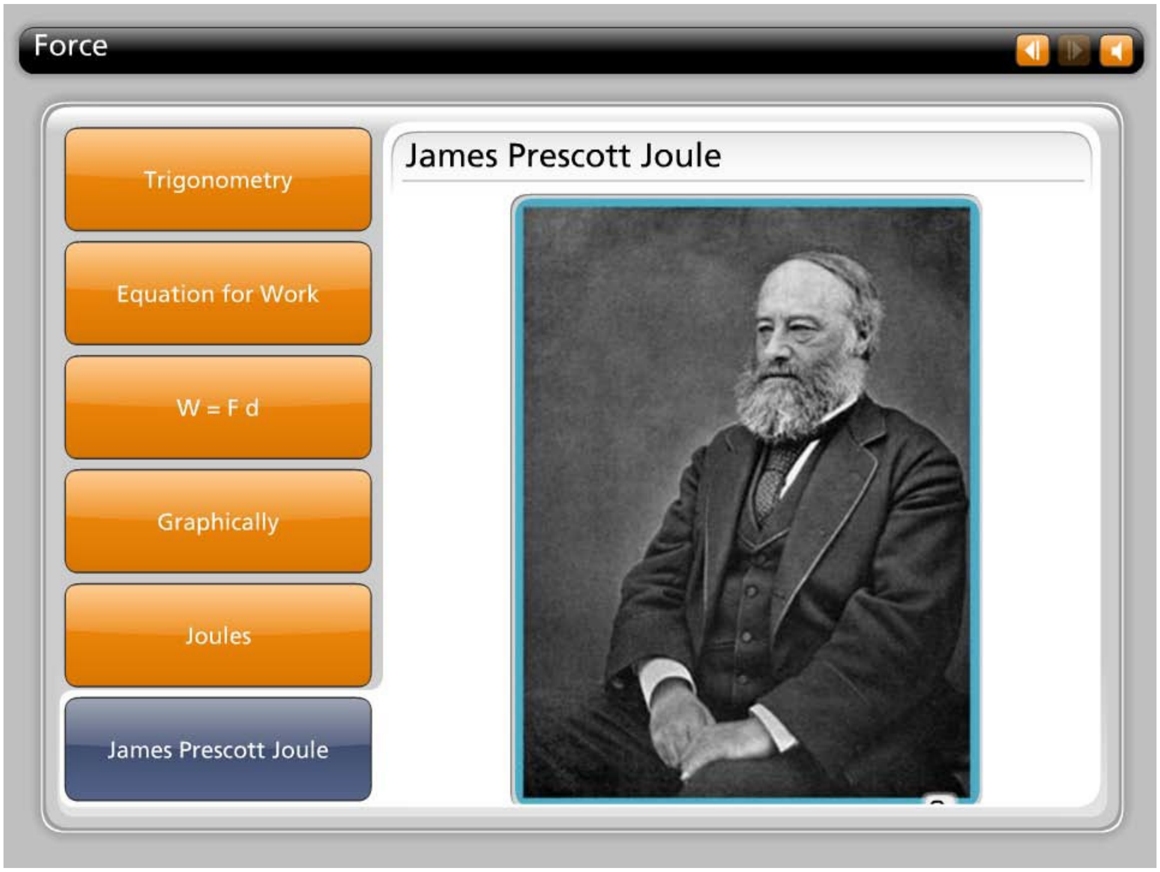
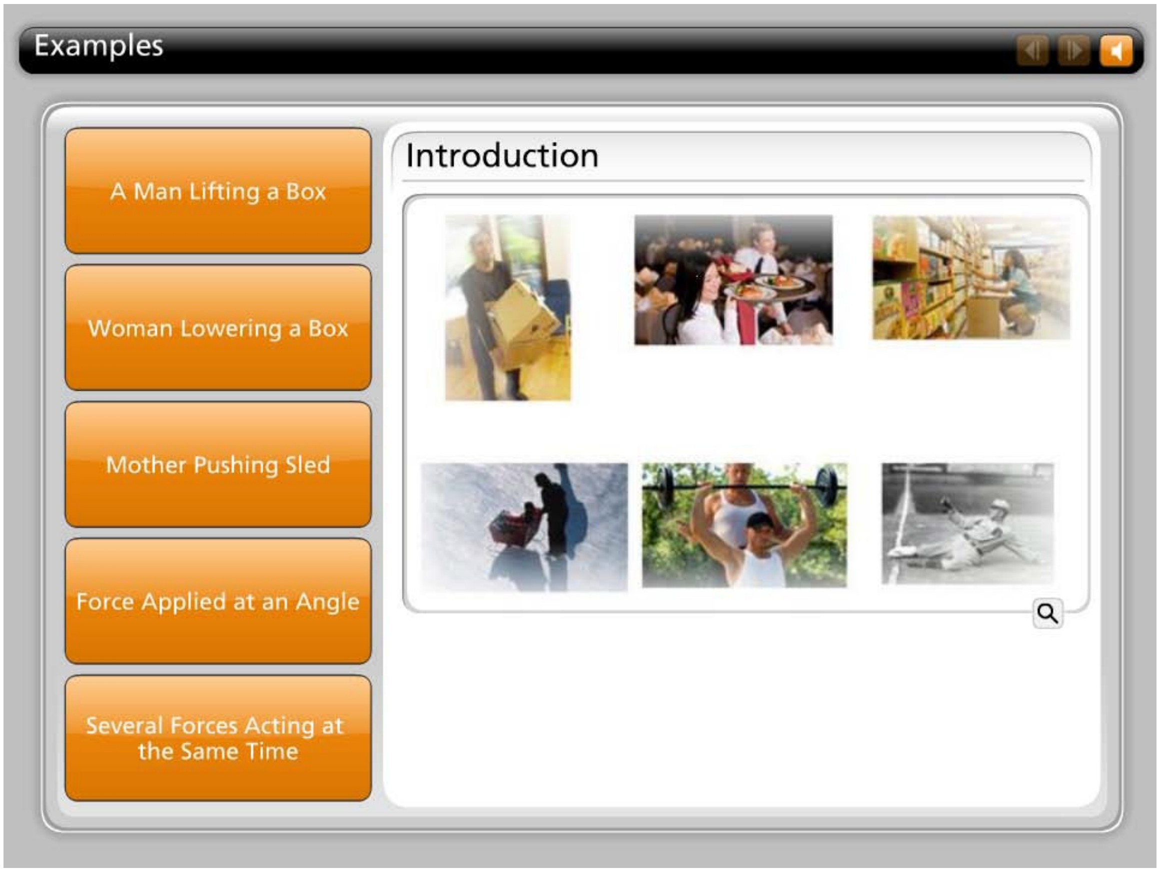
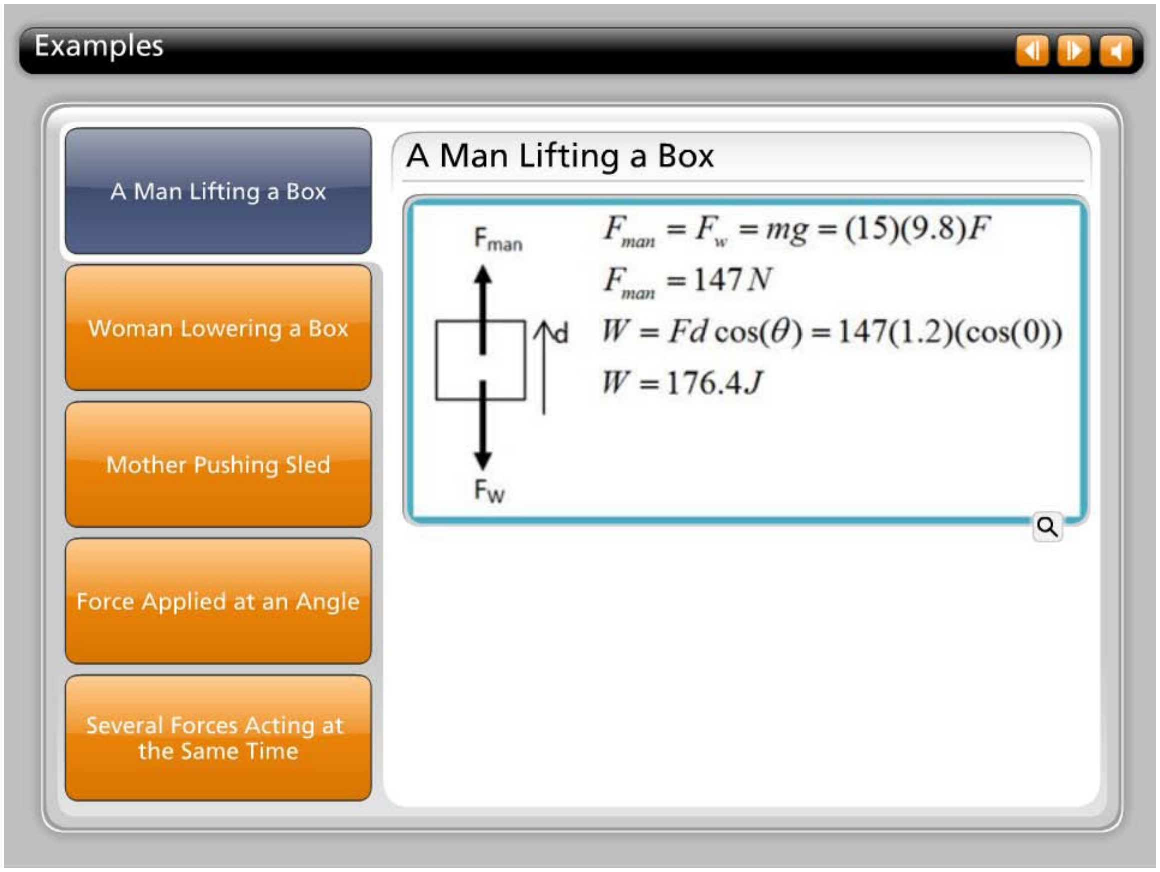


The Joule is named after physicist James Prescott Joule who first measured the relationship between mechanical energy and heat. This work, along with his study of heat generated in electric circuits led to the development of the law of conservation of energy. Joule also worked with Lord Kelvin to develop the absolute temperature scale.



You'll now look back at a few of our earlier examples, but you will add values so that you can calculate work.



A Man Lifting a Box

$$F_{man} = F_w = mg = (15)(9.8)F$$

$$F_{man} = 147N$$

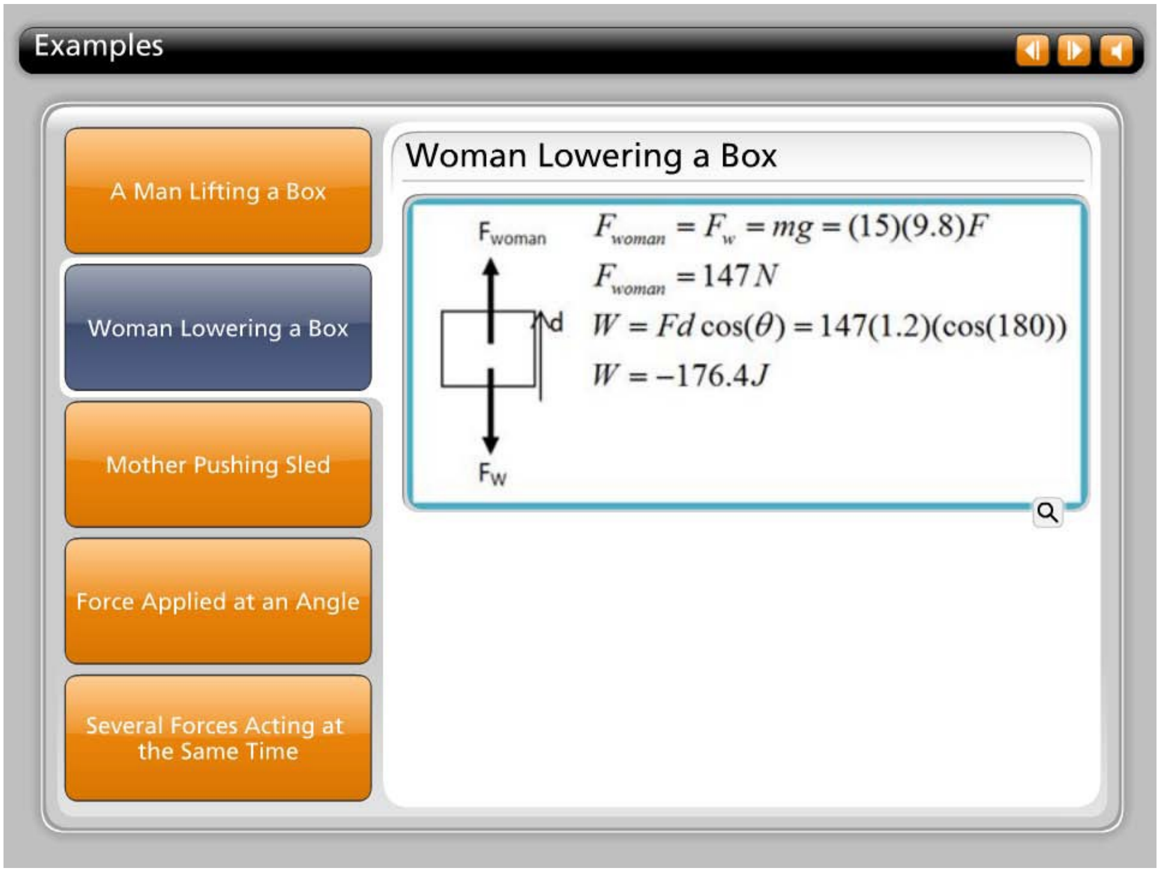
$$W = Fd\cos(\theta) = 147(1.2)(\cos(0))$$

$$W = 176.4J$$

Example one had a man lifting a box. Let's say that the box has a mass of fifteen kilograms and is lifted to a shelf that is one point two meters high.

The force the man must exert on the box to move it at a constant velocity is the same as the gravitational force on the box, or one hundred forty seven Newtons.

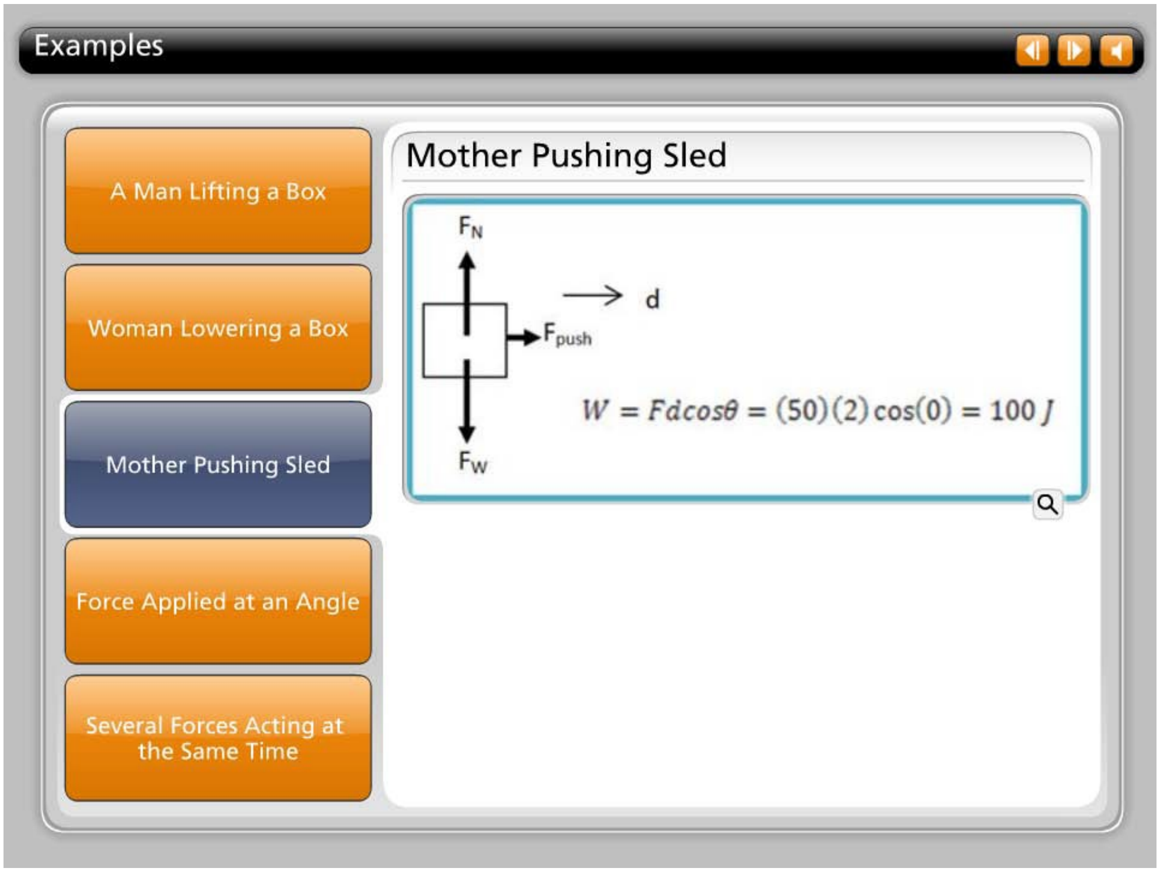
The work done by the man is the force times the distance times the cosine of the angle between the force and the displacement vectors. Since the force and the displacement are in the same direction, this angle is zero and the cosine of zero is one. So our equation basically simplifies to work is force times distance. Substituting and solving you see that the work done is one hundred seventy six point four Joules.



Woman Lowering a Box

$$F_{woman} = F_w = mg = (15)(9.8)F$$
$$F_{woman} = 147N$$
$$W = Fd\cos(\theta) = 147(1.2)(\cos(180))$$
$$W = -176.4J$$

You also had an example of the woman lowering the box to the floor. If you assume the same box and the same shelf, the woman still must provide an upward force of one hundred forty seven Newtons to keep the box from falling as he lowers it. The force is up but the displacement is down, with an angle of one hundred eighty degrees between them. The cosine of one hundred eighty degrees is negative one. The amount of work therefore works out to negative one hundred seventy six point four Joules.



Mother Pushing Sled

$$W = Fd\cos\theta = (50)(2)\cos(0) = 100\,J$$

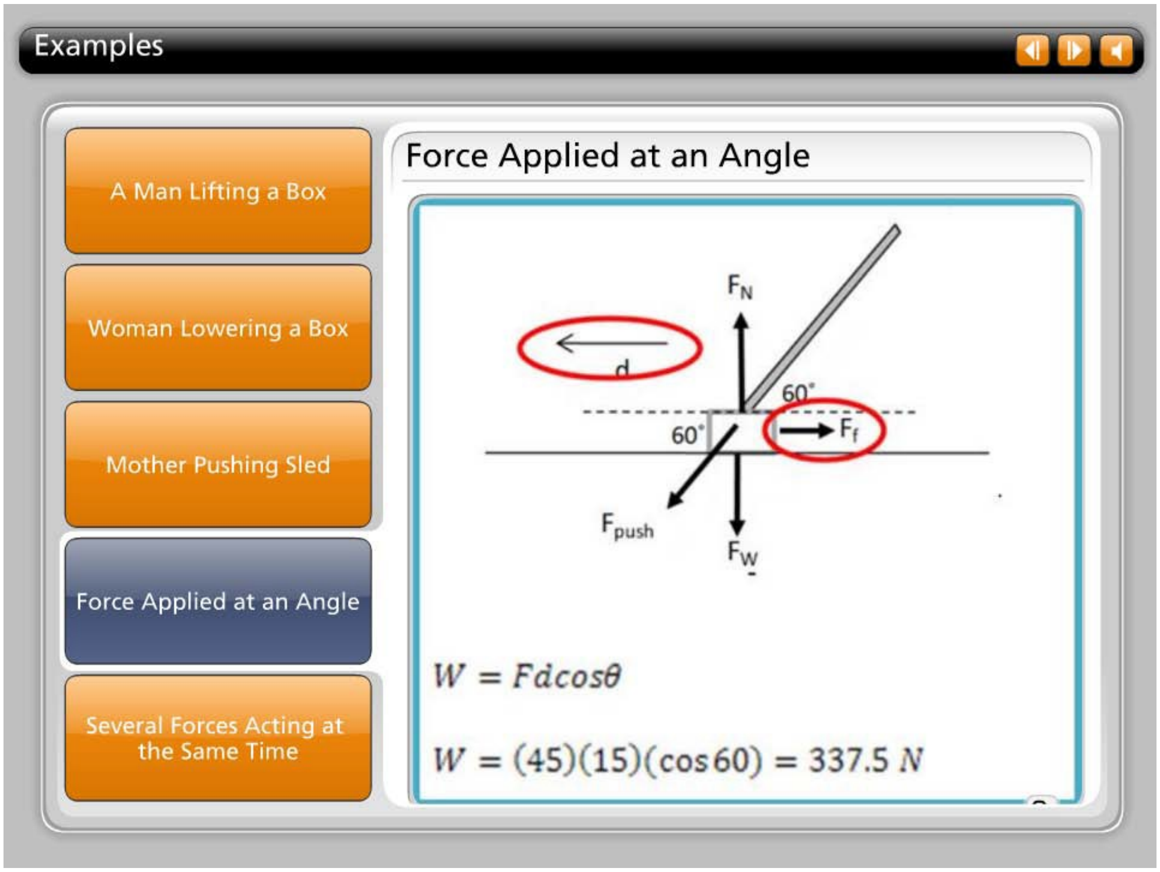
Another earlier example had a mother pushing a sled, to get it moving. Let's say that the sled has a mass of seventy five kilograms and the mother pushes with a force of fifty Newtons over a distance of two meters to get it moving.

In this case, there are three forces acting on the sled. The gravitational force pulls down, the normal force pushes up, perpendicular to the surface, and the mother's push acts to the right.

You are interested in the amount of work done by the mother, so you are only interested in the push force of fifty Newtons. This is also the only force acting in the direction of motion.

The normal force and the gravitational force are both perpendicular to the displacement. So no work is done by either of these forces. Only the fifty Newton force of the mother on the sled does any work.

The work done is the force times the distance times the cosine of the angle between them. The angle is zero, which makes the cosine equal to one. This results in one hundred Newtons of work being done by the mom.



$$W = Fd\cos\theta$$

$$W = (45)(15)(\cos 60) = 337.5\ N$$

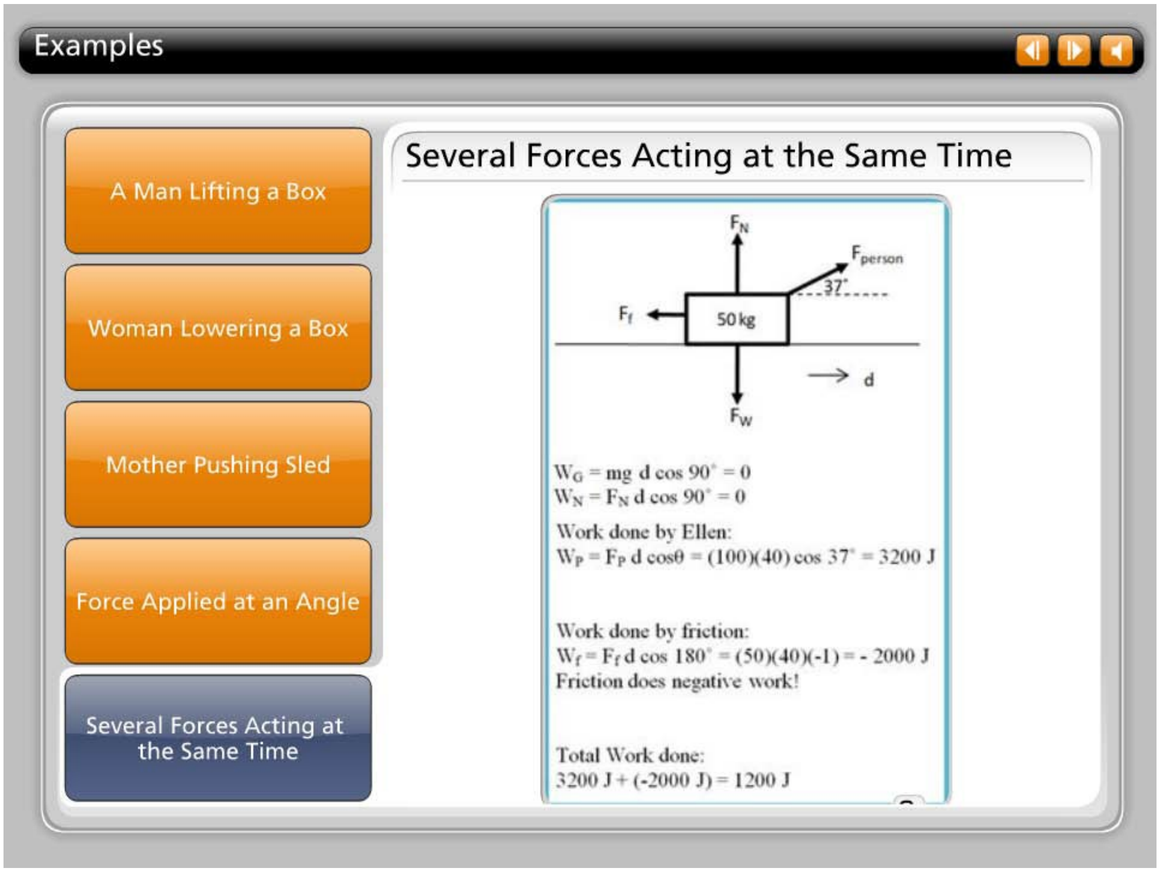
Now let’s see what happens when the force is applied at an angle to the direction of motion.

Jacob, a stage manager, pushes a broom across a fifteen meter wide stage at a constant speed by applying a forty five Newton force along the handle of the broom, which makes an angle of sixty degrees to the horizontal. How much work does Jacob do on the broom?

A free body diagram of the broom demonstrates that there are several forces acting on the broom. You are only interested in the force applied by Jacob.

This force is at an angle of sixty degrees to the displacement. When you substitute these values into the work equation you see that work equals three hundred thirty seven point five Joules.

This is the work done by Jacob, which answers the original question. But it is interesting to note that there are other forces also acting on the broom. The normal force and the gravitational force both are acting perpendicular to the displacement of the broom, so no work is done by them, but friction acts opposite the direction of motion and therefore will do negative work.



Several Forces Acting at the Same Time

$W_G = mg\ d \cos 90^\circ = 0$
$W_N = F_N\ d \cos 90^\circ = 0$

Work done by Ellen:
$W_P = F_P\ d \cos\theta = (100)(40) \cos 37^\circ = 3200\ J$

Work done by friction:
$W_f = F_f\ d \cos 180^\circ = (50)(40)(-1) = -2000\ J$
Friction does negative work!

Total Work done:
$3200\ J + (-2000\ J) = 1200\ J$

To see how you account for the work done by several forces acting at the same time, let's look at another example.

Suppose Ellen pulls a fifty kilogram crate forty meters along a horizontal floor by applying a constant force of one hundred Newtons at an angle of thirty seven degrees above the horizontal. If the constant force of friction is fifty Newtons, what is work done on crate by each force? What is the total work done on the crate?

There are four forces acting on the crate. Ellen's pull force, Friction, the normal force, and the gravitational force.

Both the normal force and gravitational force do no work as they are perpendicular to motion.

Ellen applies a force of one hundred Newtons to the crate at an angle of thirty seven degrees as it moves a horizontal distance of forty meters. This results in thirty two hundred Joules of work done by Ellen.

Friction applies a force of fifty Newtons directly opposite the direction of motion, or at an angle of one hundred eighty degrees. This gives us negative two thousand Joules of work done by friction.

The total work done to the crate is thirty two hundred plus negative two thousand, or twelve hundred Joules.

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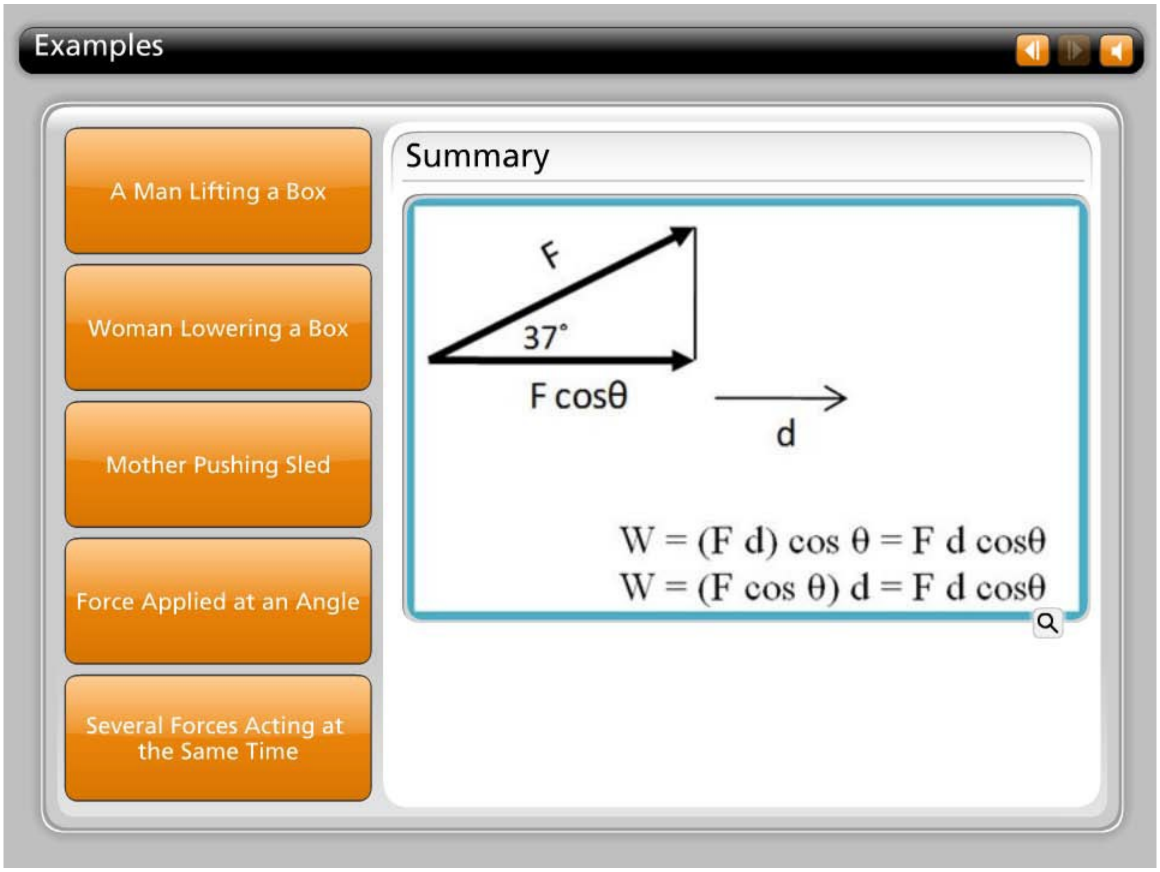
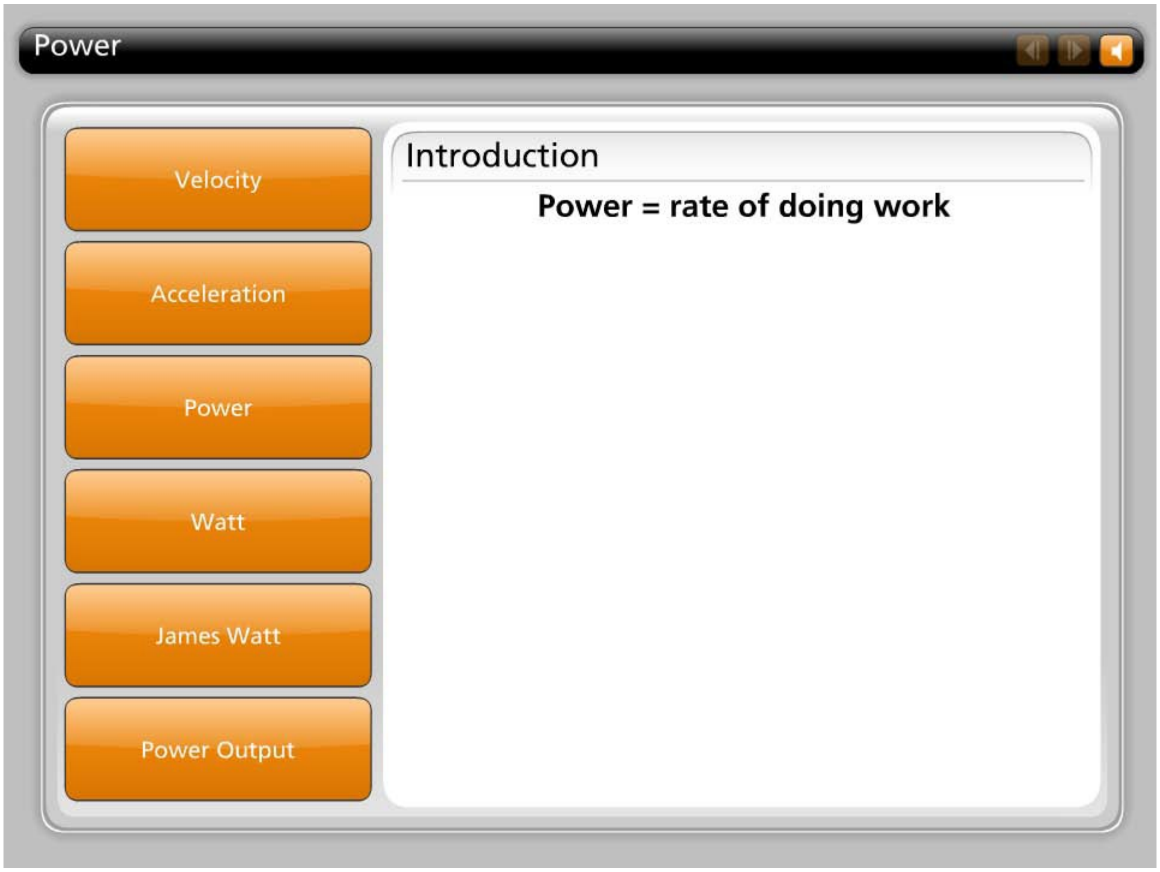


Summary

$$W = (F\ d) \cos \theta = F\ d \cos\theta$$
$$W = (F \cos \theta)\ d = F\ d \cos\theta$$

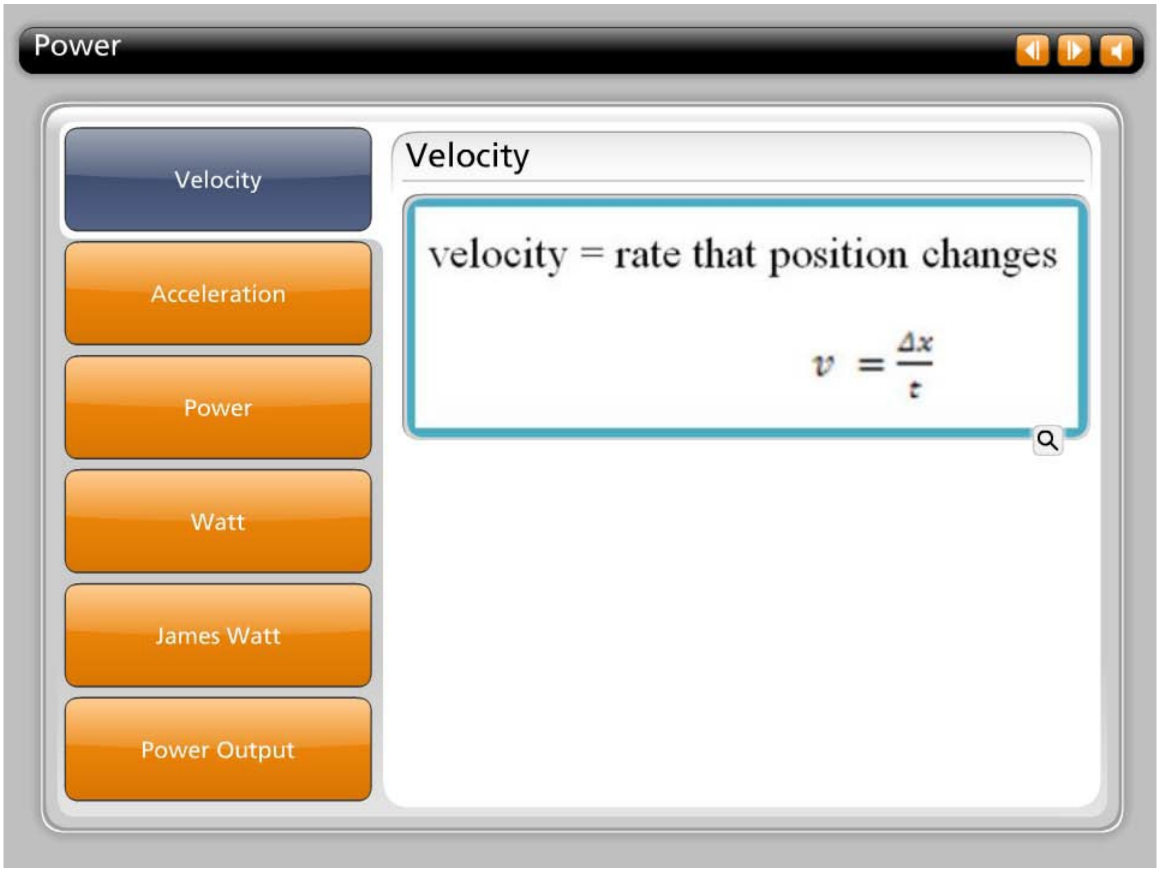
When looking at forces applied at an angle, you could also calculate the component of the force that acts in the direction of motion, then multiply this by the displacement. But whether you multiply force by displacement, then multiply by the cosine of the angle, or multiply the force by the cosine of the angle to get the horizontal component, then multiply by the displacement, you wind up with F d cosine theta.



Introduction

Power = rate of doing work

Another quantity you are interested in measuring is power. Power is the rate of doing work.

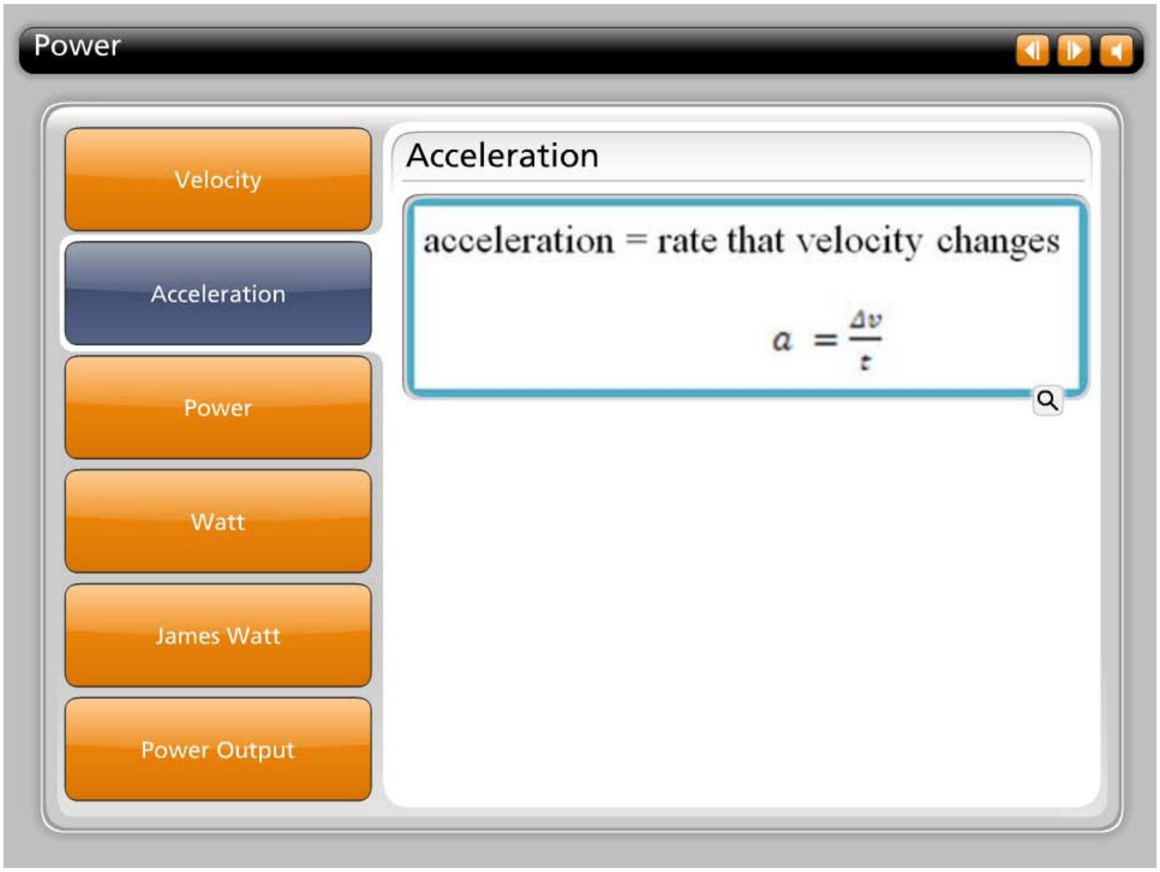


Velocity

velocity = rate that position changes

$$v = \frac{\Delta x}{t}$$

Remember that velocity is the rate of change in position. You write velocity as change in position divided by time.



Acceleration

acceleration = rate that velocity changes

$$a = \frac{\Delta v}{t}$$

Acceleration is the rate of change in velocity. You write acceleration as change in velocity divided by time.

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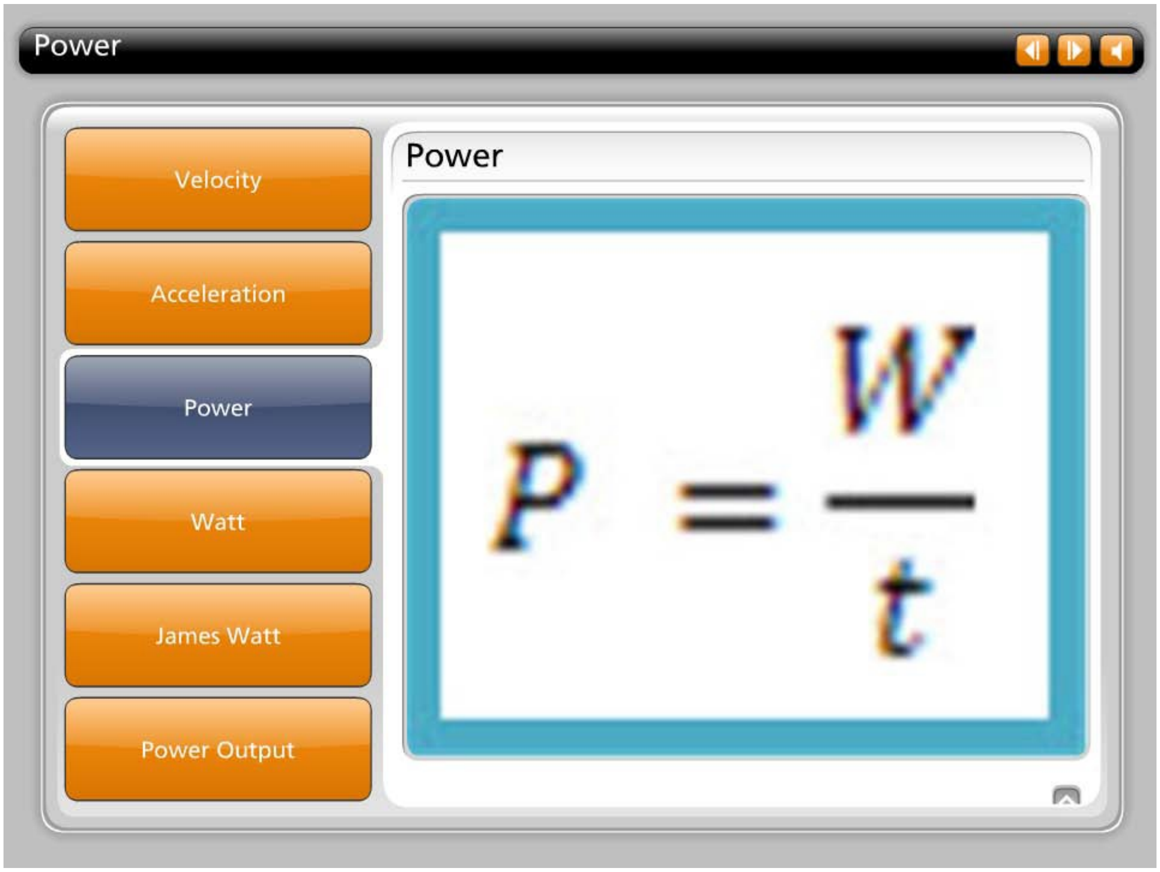

$$P = \frac{W}{t}$$

Anytime you talk about rate, you mean the change in that quantity divided by the time required for that change to happen. This is also true for power. Power is work divided by time.

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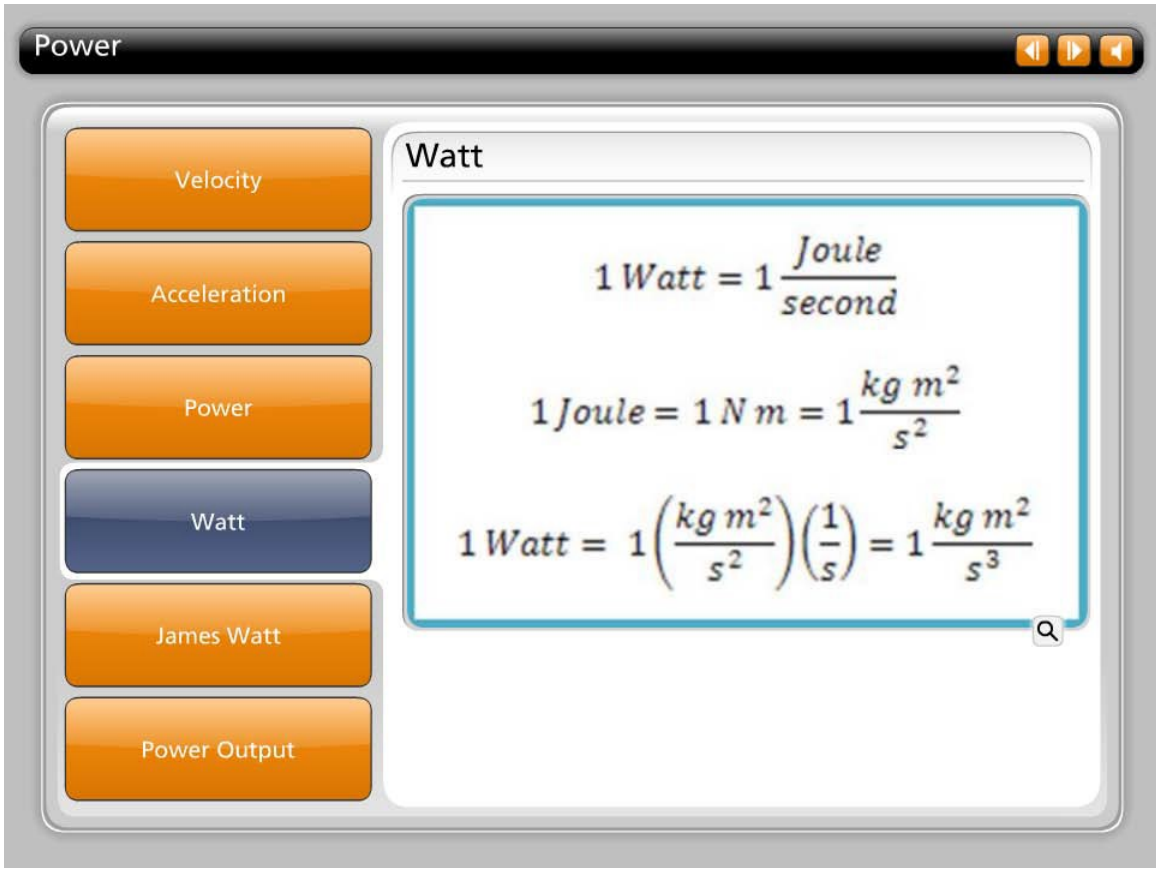

$$1\,Watt = 1\,\frac{Joule}{second}$$

$$1\,Joule = 1\,N\,m = 1\frac{kg\,m^2}{s^2}$$

$$1\,Watt = 1\left(\frac{kg\,m^2}{s^2}\right)\left(\frac{1}{s}\right) = 1\frac{kg\,m^2}{s^3}$$

A Watt is defined as one Joule per second. Power is expressed in Watts.

Since a Joule is equal to a kilogram meter squared per second squared, if you divide this again by seconds, you see that a Watt is a kilogram meter squared per second cubed in standard units.

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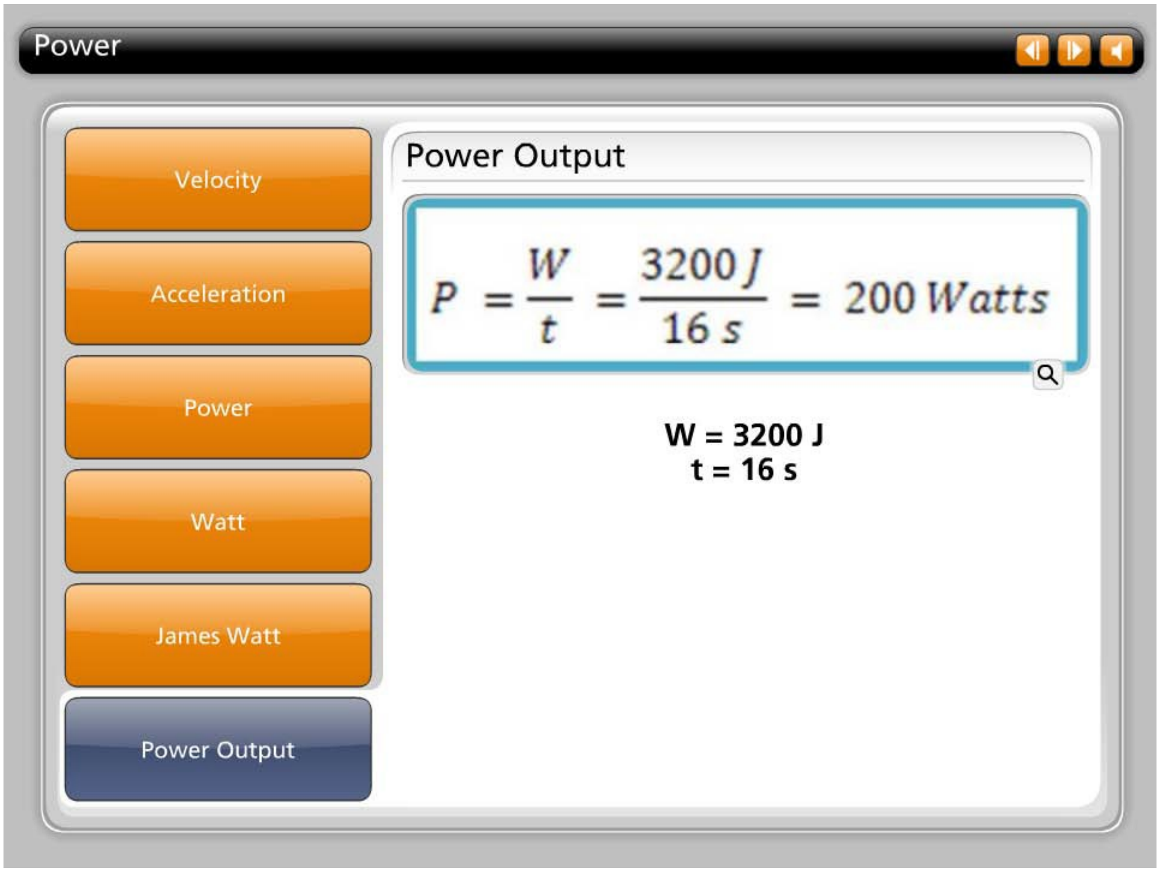
The Watt is named in honor of James Watt, a Scottish inventor and engineer whose improvement of the efficiency of the steam engine made it a powerful tool in the industrial revolution.

Image Source: http://en.wikipedia.org



Power Output

$$P = \frac{W}{t} = \frac{3200\,J}{16\,s} = 200\,Watts$$

W = 3200 J
t = 16 s

Looking back at our last example where Ellen did thirty two hundred Joules of work pulling the box, if this took sixteen seconds to accomplish, what was Ellen's power output?

Since power is work divided by time, you see that Ellen delivered two hundred watts of power.

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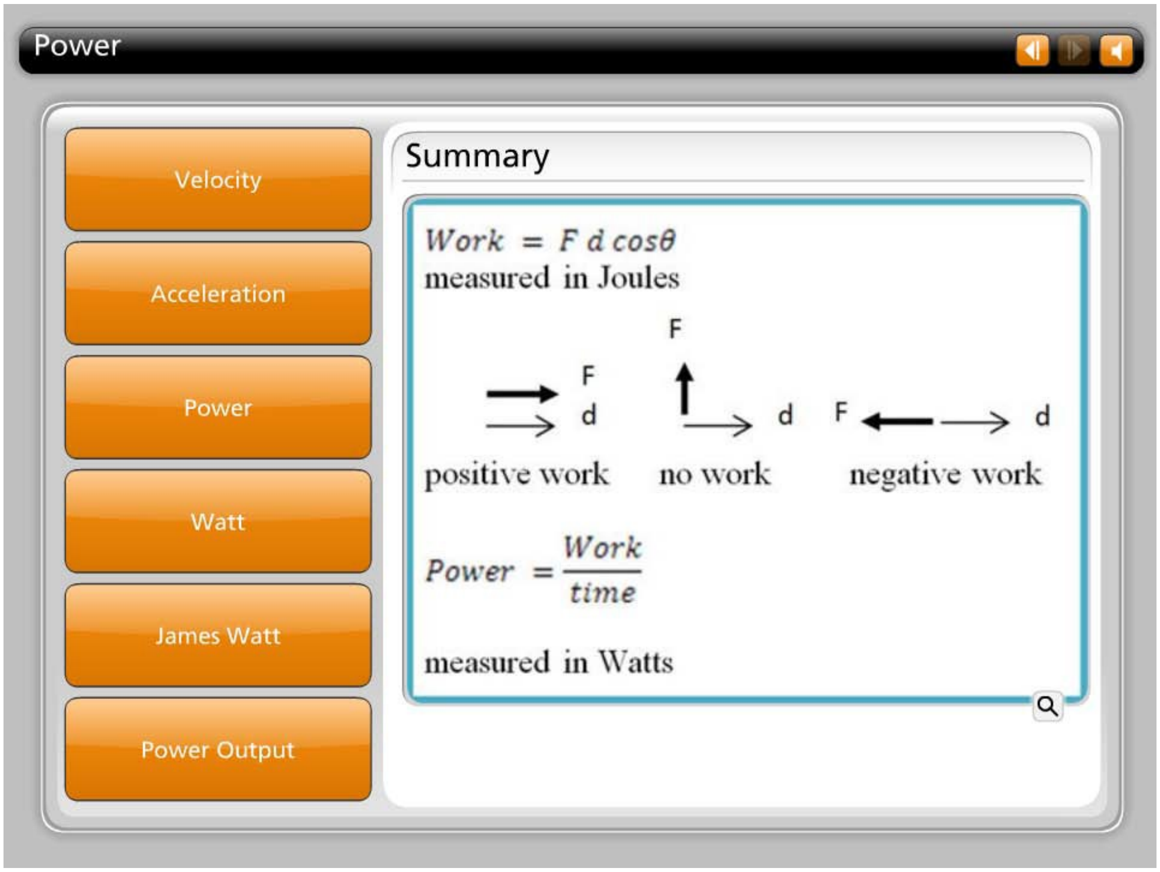

Summary

$Work = F\,d\cos\theta$

measured in Joules

positive work no work negative work

$$Power = \frac{Work}{time}$$

measured in Watts

In summary, work is the product of the force on an object and the distance the object moves in the direction of the force. Work is measured in Joules.

In the simplest case, when force and displacement are in the same direction work equals force times distance.

When force and displacement are not in the same direction, work equals force times distance times cosine theta.

Theta is the angle between the force and displacement vectors. If the force and displacement are in the same direction, then positive work is done. If the force is perpendicular to the displacement, no work is done. If the force points opposite the direction of motion, then negative work is done, and if the force is at an angle, the cosine will help you to calculate the work done.

Power is simply work divided by time. Power is measured in Watts.