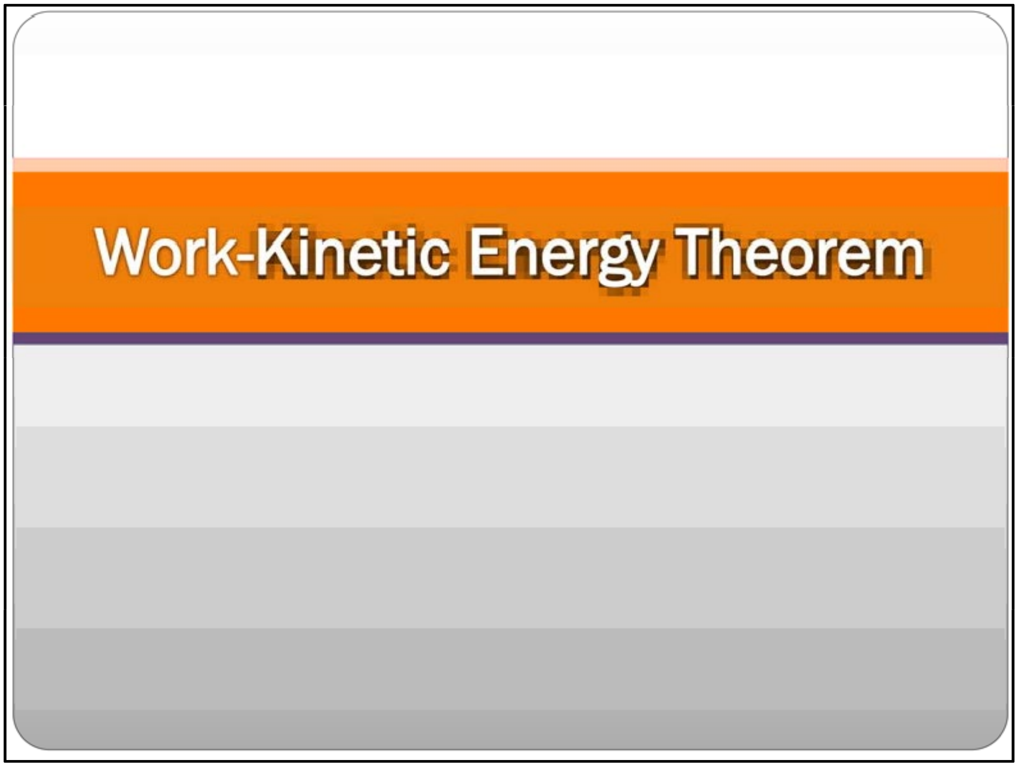
Module 4: Energy

Topic 3 Content: Projectiles at an Angle Practice Solutions



Work-Kinetic Energy Theorem

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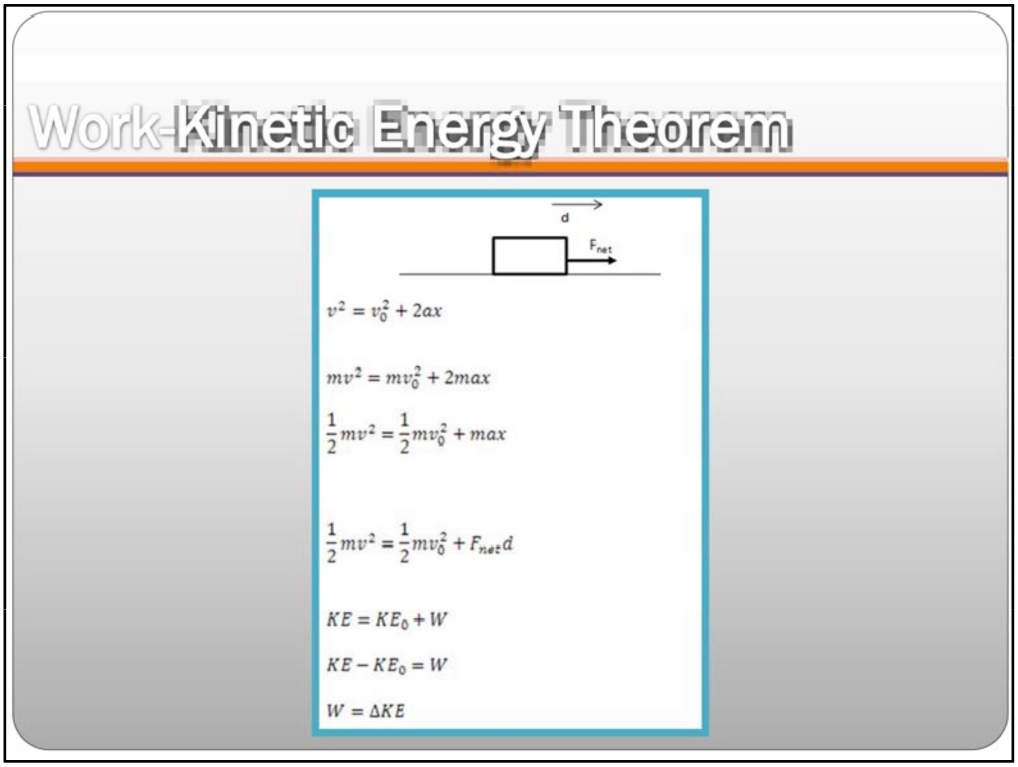
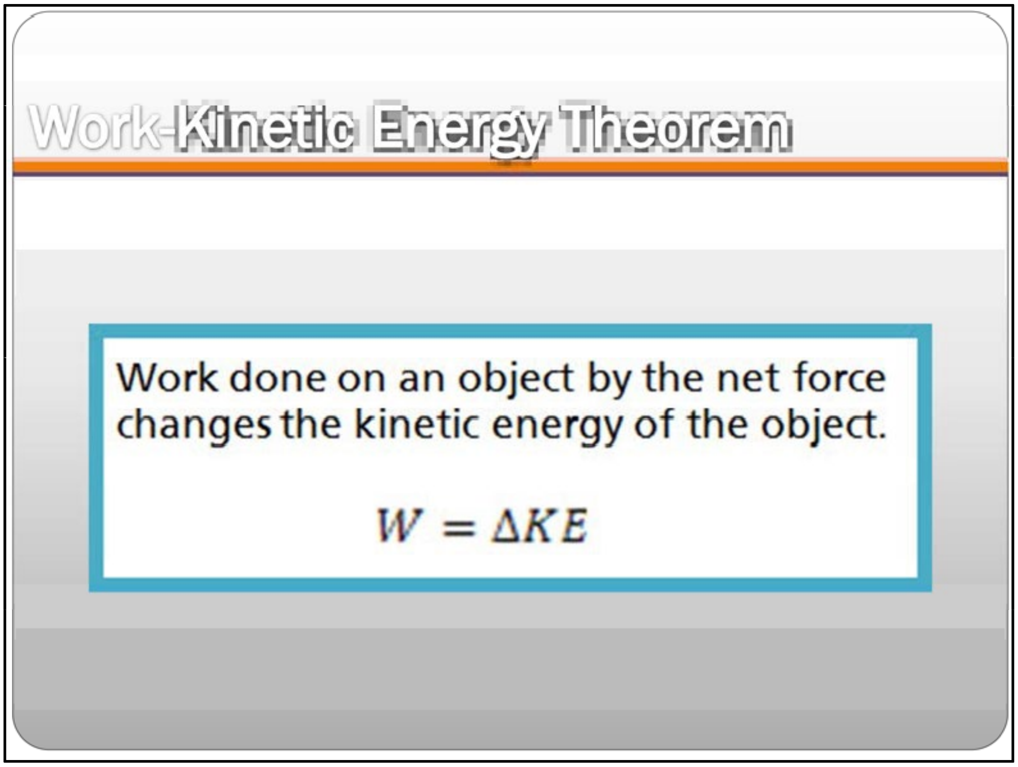
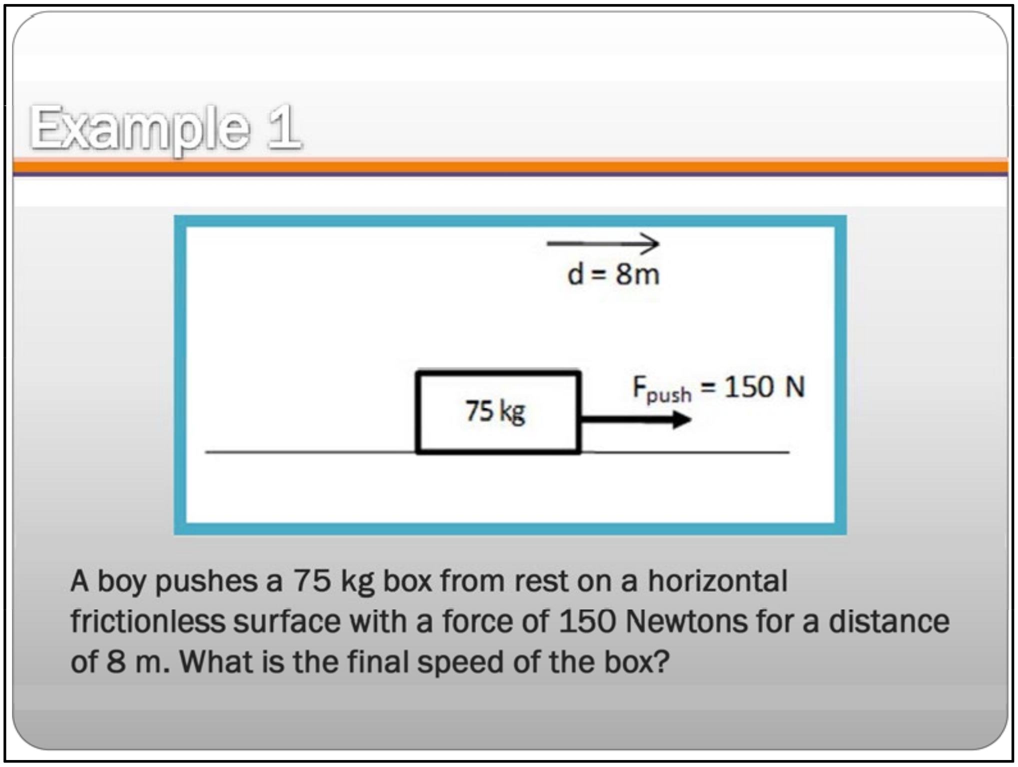


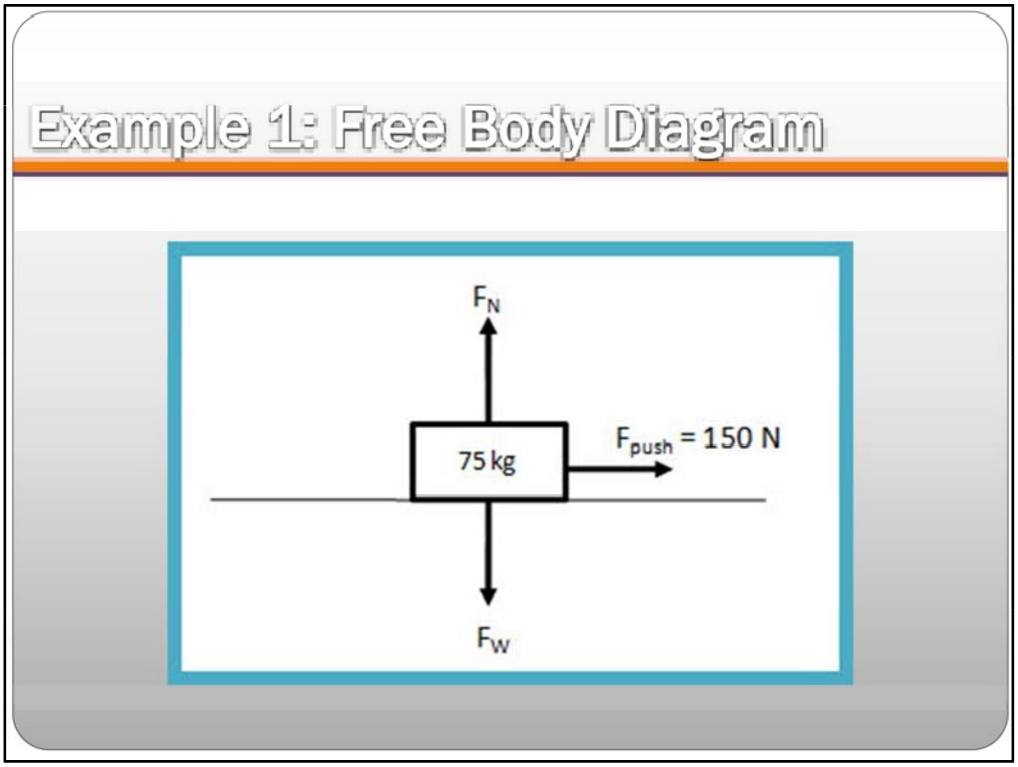
Earlier you defined energy as the capacity to do work. Similarly, when work is done on an object, by applying a force in the direction of motion, the energy of the object changes. If you apply a net force F to an object over a distance d, what happens to the speed of the object? If you recall, you had a kinematics equation that related the initial and final speed to the acceleration and the displacement. You can multiply all the terms in this equation by the mass of the object and then multiply each term by one half. The mass times the acceleration is simply the net force, and the displacement is the distance d, so you can substitute F d for m a x. You should now recognize that the force times the distance is the work done, and also recognize the terms for the initial and final kinetic energy. By subtracting the initial kinetic energy from both sides, you end up showing that the work done by the net force is equal to the change in kinetic energy.



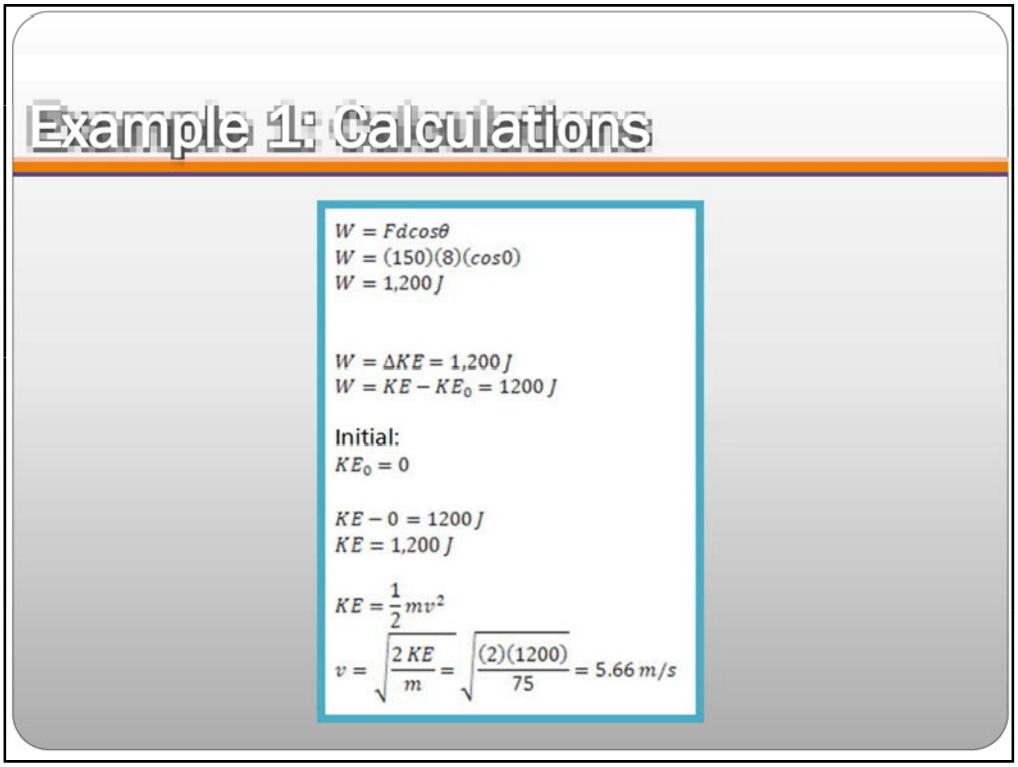
This relationship is called the Work-Kinetic Energy Theorem. The amount of work done by the net force on the object is equal to the change in the kinetic energy of the object.



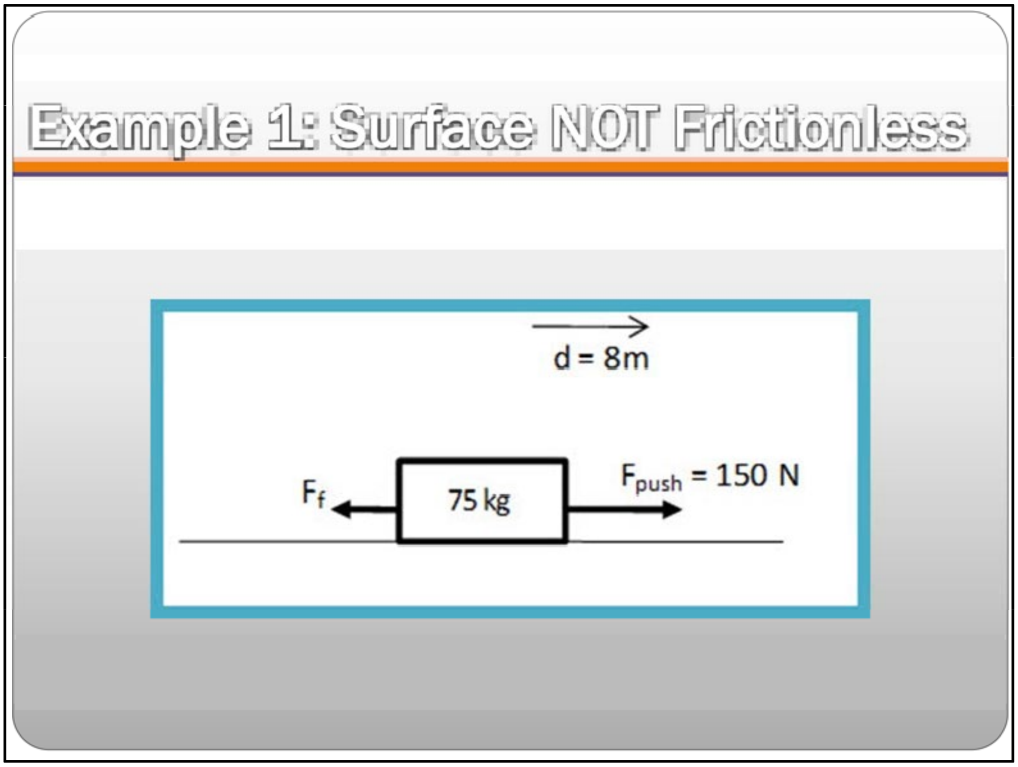
To see how this works, it helps to start with an example. A boy pushes a seventy five kilogram box from rest on a horizontal frictionless surface with a force of one hundred fifty Newtons for a distance of eight meters. What is the final speed of the box?



The work-energy theorem tells us that the work done on the box by the net force will change the kinetic energy of the box, so you first need to determine the net force. A free body diagram shows that you have three forces acting. The gravitational force, the normal force and the push force. The gravitational force is equal in magnitude to the normal force and in the opposite direction, so these forces are balanced. The net force, therefore, is simply equal to the push force of one hundred fifty Newtons.



Calculating the work done to the box by the push force is relatively straightforward. Using our equation for work, you see that the work done to the box equals twelve hundred Joules. The work done changes the kinetic energy, so you know that the change in kinetic energy is equal to twelve hundred Joules. Since the box was initially at rest, the initial kinetic energy was zero, so the final kinetic energy must be twelve hundred Joules. Using the equation for kinetic energy, and rearranging it for speed, you see that the speed of the box is five point six six meters per second.



Now, what if the surface was not frictionless? Let's set the coefficient of friction between the box and the floor at zero point one two. The boy still pushes with the same amount of force. But now, you also have the force of friction acting opposite the direction of motion.

Example 1: Net Force

$$F_f = \mu F_N$$

$$F_N = F_W = mg$$

$$F_f = \mu mg$$

$$F_f = (0.12)(75)(9.8)$$

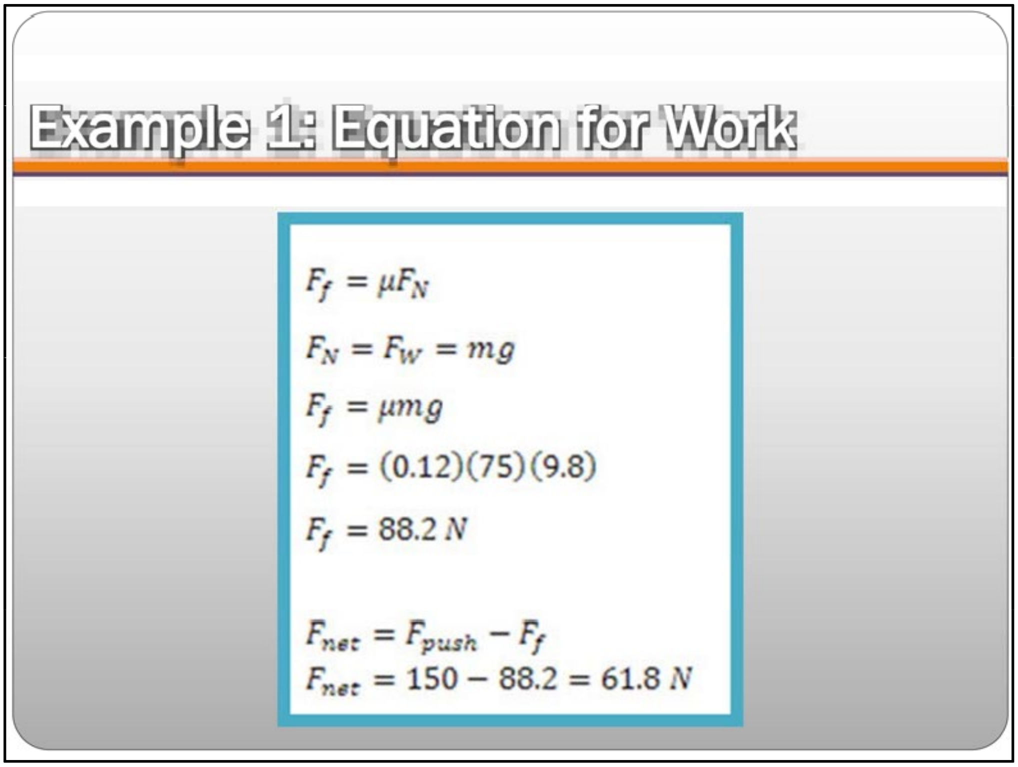
$$F_f = 88.2\ N$$

$$F_{net} = F_{push} - F_f$$

$$F_{net} = 150 - 88.2 = 61.8\ N$$

The force of friction is equal to the coefficient of friction times the normal force. The normal force in this situation is equal to the mass times the acceleration of gravity. So the force of friction will be equal to mu m g. The result of this calculation is eighty eight point two Newtons.

The net force, therefore is equal to one fifty minus eighty eight point two, or sixty one point eight Newtons.



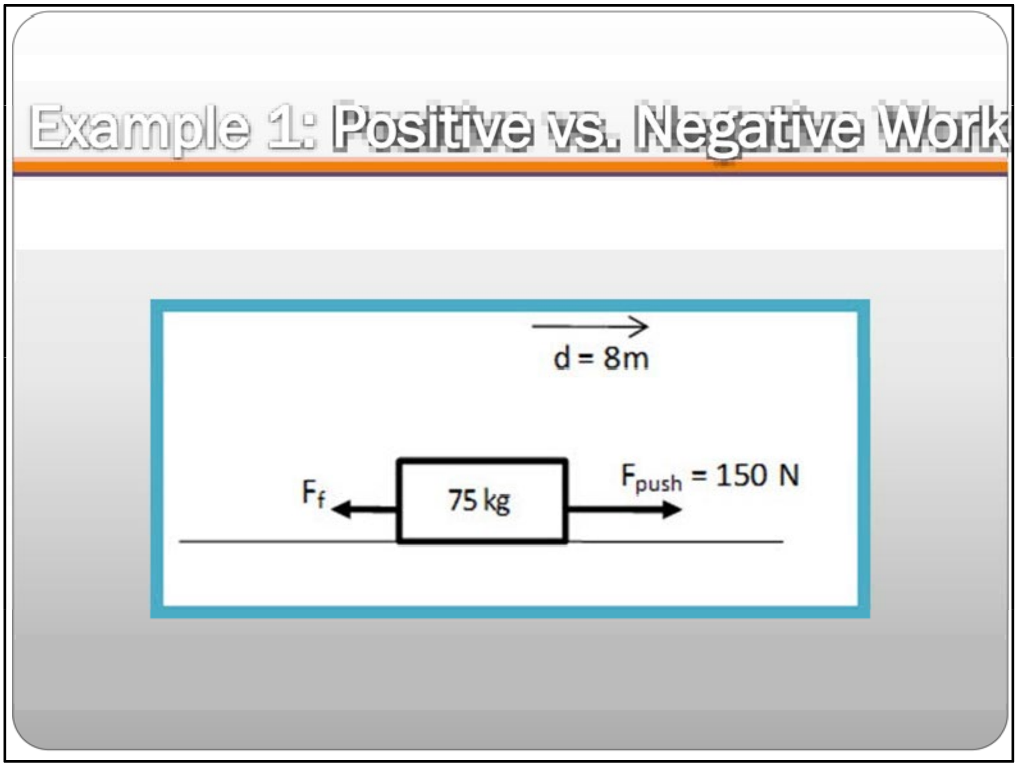
The work done by the net force is calculated using our equation for work and is equal to four hundred ninety four point four Joules.

Example 1: Speed

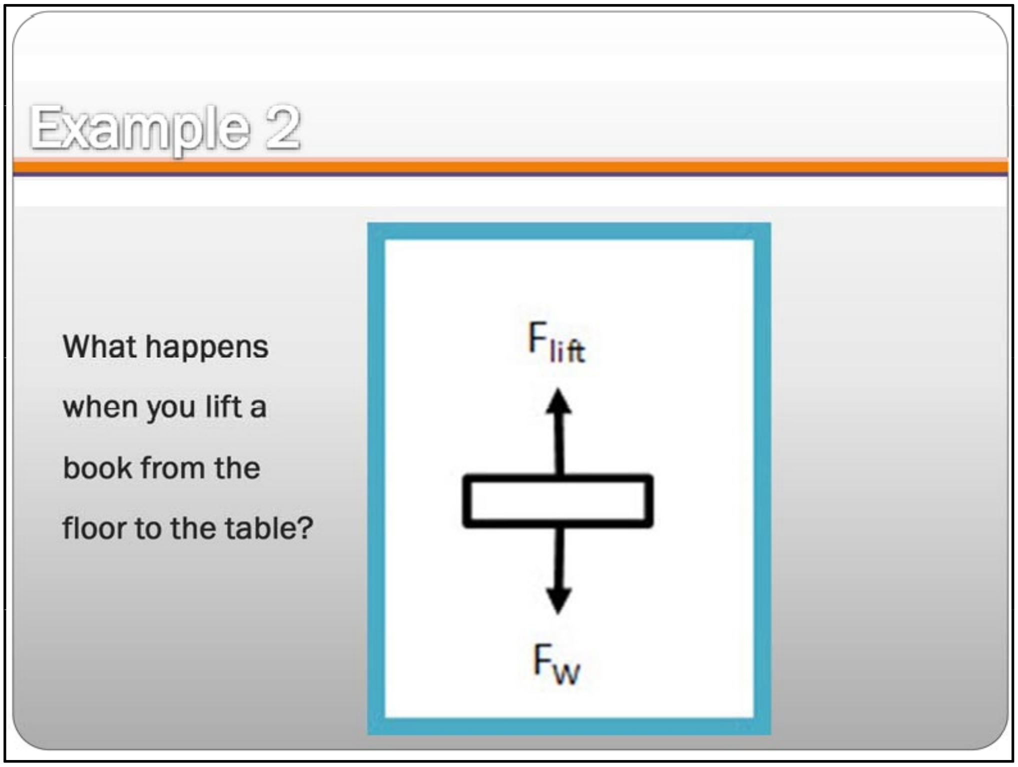
$$KE = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2\,KE}{m}} = \sqrt{\frac{(2)(494.4)}{75}} = 3.63\,m/s$$

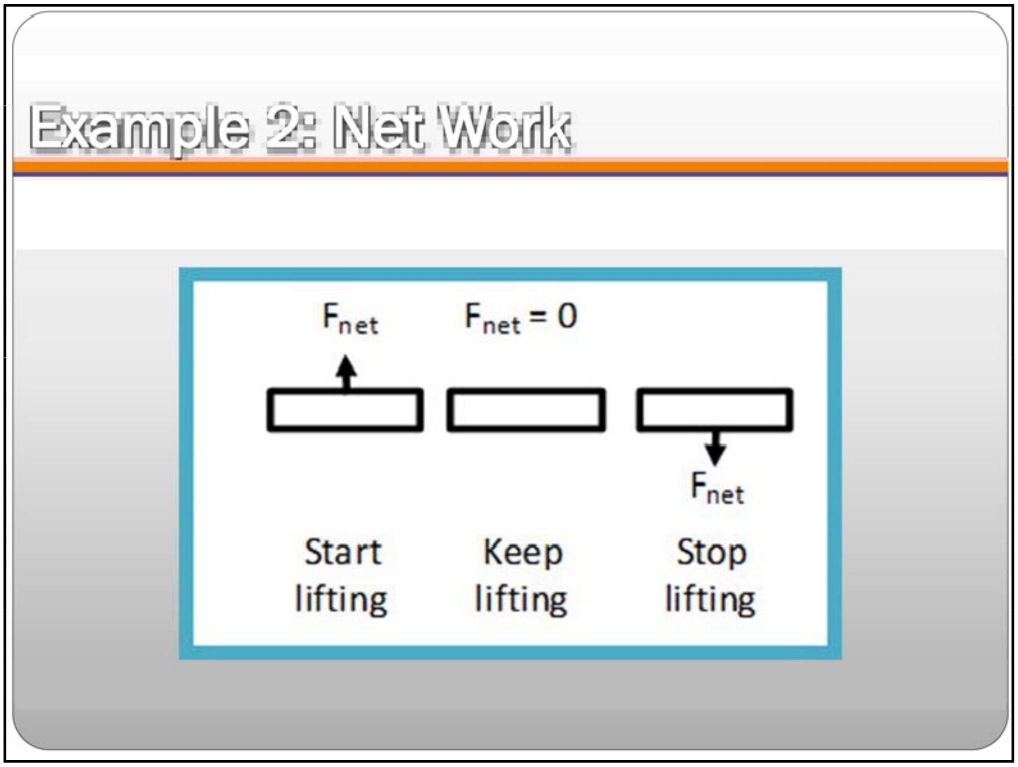
Now, as before, you see that since the work done by the net force changes the energy of the box, and it begins at rest, the kinetic energy is equal to four hundred ninety four point four Joules, and the speed calculates to be three point six three meters per second.



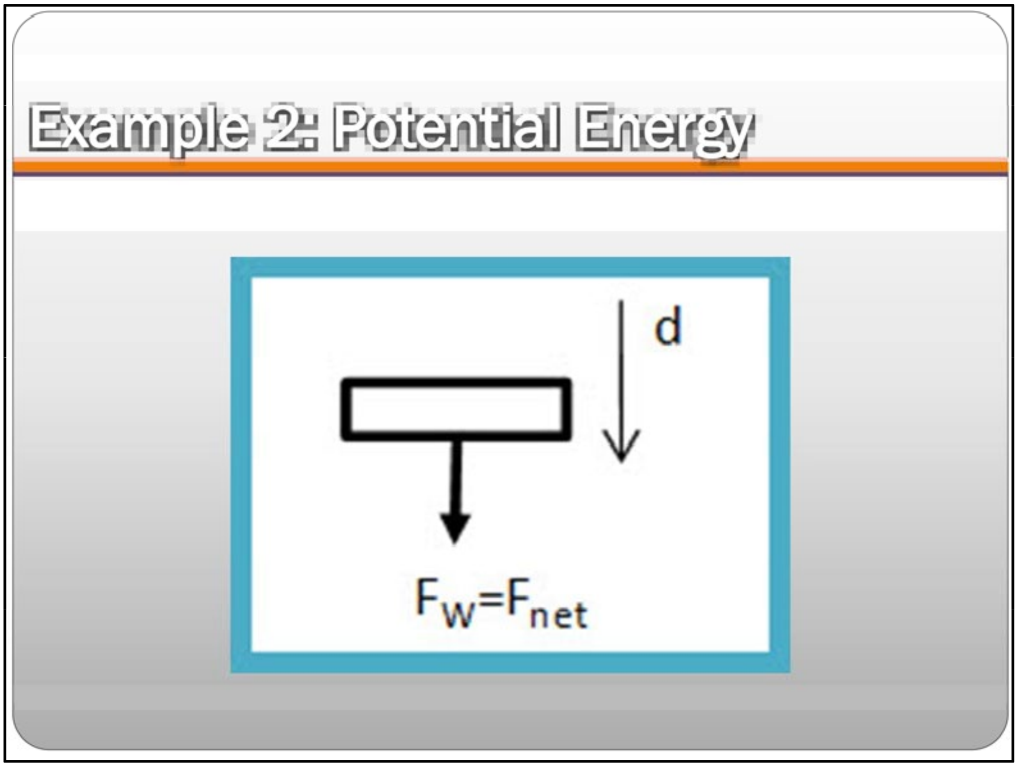
Another way to think about this is that the boy did positive work to the box and friction did negative work, since the frictional force acted opposite the direction of motion. The effect of the push force increased the kinetic energy of the box while the effect of the frictional force decreased the kinetic energy of the box.



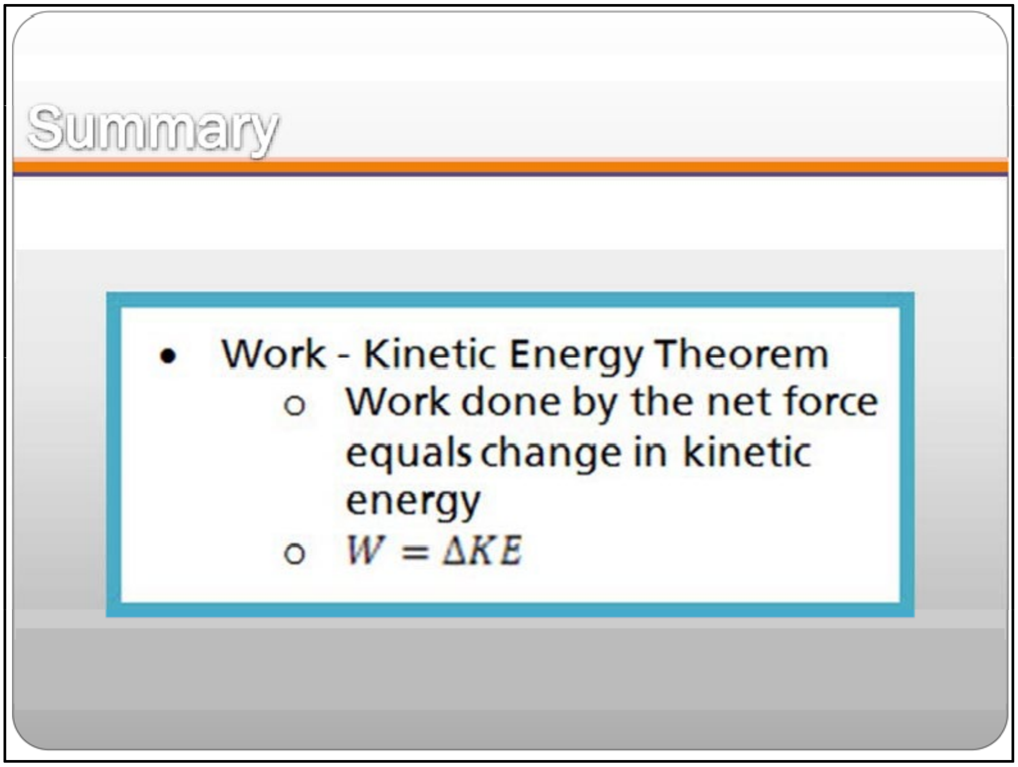
Now, what happens when you lift a book from the floor to the table? As you lift, there are two forces acting on the book. You are applying a force upwards, in the direction of motion, but gravity is applying a force downwards, opposite the direction of motion. As you lift the book at a constant speed, these forces are balanced and the net force is zero. Since the net force is zero, the work done by the net force is zero and the change in kinetic energy is also zero. According to the work kinetic energy theorem, this makes sense, since the book is moving at a constant speed.



Of course, to get the book moving, the net force had to be upwards, so a small amount of work was done resulting in a small increase in kinetic energy. However, when you slow the book down to place it on the table, the net force must point down, so an equal amount of negative work is done. Over all, no net work is done.



However, the book is now on the table. If it were to fall off the table, gravity would accelerate the book towards the floor. The force of gravity would not be balanced by an opposing lift force, and the force would point in the direction of motion. Gravity would do positive work on the book, and as its speed increased, its kinetic energy would increase. This is why you consider a book on the table to have potential energy. The book has the potential for gravity to accelerate it downwards, increasing its kinetic energy.



You can see that the work-kinetic energy theorem provides us with another tool to solve physics problems. The work-kinetic energy theorem states that the work done to an object by the net force equals its change in kinetic energy.