

Momentum in Collisions

Momentum in collisions

Module 5: Impulse and Momentum

Topic 4 Content: Momentum in Collisions Presentation Notes

Momentum in Collisions

Before and After

Before	After
$p_1 + p_2$	$= p_1 + p_2$

Click on the segments of the diagram starting at the top or use the arrow at the top right of the interactivity to continue with this example.
Click on the magnifying glass on each image to enlarge it.

The diagram is a pyramid divided into four sections. The top section is blue and labeled 'Equation'. The middle section is divided into two blue sections labeled 'Elastic' and 'Inelastic'. The bottom section is divided into two black sections labeled 'Perfectly Inelastic Collision' and 'Objects Begin in Motion'.

Before and After

Since the change in momentum must be zero, this also means that the total momentum that exists before an interaction will be equal to the total momentum after the interaction. You can express this as an equation by writing $p_1 + p_2$ before equals $p_1 + p_2$ after, where p_1 and p_2 are the respective momentum of objects one and two, and before and after refer to before the interaction and after the interaction.

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Momentum in Collisions

Equation

Before = After
 $m_1 v_1 + m_2 v_2 = m_1 v_1 + m_2 v_2$

Equation

Elastic

Inelastic

Perfectly Inelastic Collision

Objects Begin in Motion

Equation

In all collisions, you can solve momentum problems by setting up an equation showing that the momentum before the collision equals the momentum after the collision. You write $m_1 v_1 + m_2 v_2$ before equals $m_1 v_1 + m_2 v_2$ after.

This equation will work for any collision, elastic or inelastic, since momentum is conserved. It will also work if both objects begin in motion or if one or both objects begin at rest.

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Momentum in Collisions

Elastic

Kinetic Energy is Conserved

Equation

Elastic

Inelastic

Perfectly Inelastic Collision

Objects Begin in Motion

Elastic

Interactions other than explosions are generally categorized as collisions. Collisions can be of several types. They can be elastic or inelastic. Elastic collisions are interactions in which kinetic energy is conserved.

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Inelastic

Kinetic Energy is Not Conserved
(Perfectly Inelastic: Objects stick together)

Equation

Elastic

Inelastic

Perfectly Inelastic Collision

Objects Begin in Motion

Inelastic

Inelastic collisions are interactions where kinetic energy is not conserved, as some kinetic energy is transformed into heat, sound or other mechanical vibrations, or is used as work deforming the object. Perfectly inelastic collisions are interactions where the interacting objects actually stick together.

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The screenshot shows a presentation window titled "Momentum in Collisions". Inside, there is a slide titled "Perfectly Inelastic Collision". The slide contains a diagram of a bullet (0.045 kg) hitting a block (1 kg) and sticking to it, moving together at 3.5 m/s. Below this is a calculation for the bullet's initial velocity v_1 using the conservation of momentum equation $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$, resulting in $v_1 = 81.3 \text{ m/s}$. To the right of the text is a pyramid diagram with four sections: "Equation" (top, blue), "Elastic" (middle-left, blue), "Inelastic" (middle-right, blue), and "Perfectly Inelastic Collision" (bottom-left, black) and "Objects Begin in Motion" (bottom-right, black). An arrow points from the calculation box to the "Perfectly Inelastic Collision" section of the pyramid.

Perfectly Inelastic Collision

In all collisions, you can solve momentum problems by setting up an equation showing that the momentum before the collision equals the momentum after the collision. You write $m_1 v_1 + m_2 v_2$ before equals $m_1 v_1 + m_2 v_2$ after. This equation will work for any collision, elastic or inelastic, since momentum is conserved. It will also work if both objects begin in motion or if one or both objects begin at rest. Let's start by looking at perfectly inelastic collisions. If the objects stick together, you should recognize that their velocities after the collision will be the same. You can therefore simplify our equation by saying that the mass of the first object times its velocity plus the mass of the second object times its velocity is equal to the sum of the masses times the combined final velocity. In order to determine the speed of a bullet, it is shot into a piece of wood initially at rest. The mass of the bullet is zero point zero four five kilograms. It is shot into a one point zero kilogram block of wood. After the bullet lodges in the wood, they move at a speed of three point five meters per second. What was the initial velocity of the bullet? Using the special-case equation for conservation of momentum in an inelastic collision, you can substitute the mass of the bullet, the mass of the block, the initial velocity of the block and the final velocity of the block and bullet. Solving the equation you see that the initial velocity of the bullet was eighty one point three meters per second.

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The screenshot shows a presentation slide titled "Momentum in Collisions" with a sub-window titled "Objects Begin in Motion". The slide features a diagram of two bumper cars colliding. On the left, a car with mass 190 kg moves right at 3.2 m/s. On the right, a car with mass 215 kg moves left at 2.5 m/s. After the collision, the 190 kg car moves right at 1.69 m/s and the 215 kg car moves left at 0.5 m/s. The diagram is divided into three horizontal sections: "Equation" (top, blue), "Elastic" (middle, light blue), and "Perfectly Inelastic Collision" (bottom, dark blue/black). The "Equation" section contains the momentum conservation equation: $m_1 v_1 + m_2 v_2 = m_1 v_1 + m_2 v_2$ and the calculation: $(190)(-2.5) + (215)(3.2) = (190)v_1 + (215)(-0.5)$, resulting in $1.69 \text{ m/s} = v_1$. The "Elastic" section shows kinetic energy calculations: $KE \text{ Before} = \frac{1}{2} m v^2 + \frac{1}{2} m v^2$, $KE \text{ Before} = \frac{1}{2} (190)(2.5)^2 + \frac{1}{2} (215)(3.2)^2$, $KE \text{ Before} = 593.75 + 1100.8$, $KE \text{ Before} = 1694.55 \text{ J}$. The "Perfectly Inelastic Collision" section shows: $KE \text{ After} = \frac{1}{2} m v^2 + \frac{1}{2} m v^2$, $KE \text{ After} = \frac{1}{2} (190)(1.69)^2 + \frac{1}{2} (215)(0.5)^2$, $KE \text{ After} = 270.3 + 26.9$, $KE \text{ After} = 297.2 \text{ J}$.

Objects Begin in Motion

What about a situation in which both objects begin in motion?

Jane and Bobby head off to the amusement park and hop into the bumper cars. Jane and her bumper car have a combined mass of one hundred ninety kilograms and they are moving to the left at two point five meters per second. Bobby and his bumper car have a combined mass of two hundred fifteen kilograms and they are moving at three point two meters per second to the right as they hit Jane in a head-on collision.

After the collision, Bobby is moving to the left at zero point five meters per second. What is Jane's speed and direction after the collision?

Was the collision elastic or inelastic?

First, you should set up our general equation that equates momentum before and after the collision.

You will substitute Jane's values for m_1 and v_1 and Bobby's values for m_2 and v_2 and then solve for Jane's velocity after the collision. Velocities to the right will be positive and velocities to the left will be negative.

M_1 is Jane's mass, which is one hundred ninety kilograms. M_2 is Bobby's mass of two hundred fifteen kilograms. Before the collision, v_1 is Jane's velocity of negative two point five meters per second and v_2 is Bobby's speed of three point two meters per second. After the collision, you don't know v_1 , but v_2 is Bobby's speed of negative zero point five meters per second.

When you solve for v_1 , you see that Jane's speed after the collision is positive one point six nine meters per second, which means Jane is moving to the right.

To determine if the collision was elastic or inelastic, you have to compare the total kinetic energy before the collision with the total kinetic energy after the collision. Elastic collisions are those in which kinetic energy is conserved.

You will have one half $m v^2$ for both Jane and Bobby before the collision and also after the collision. Substituting the masses and speeds, you find that kinetic energy before the collision was one thousand six hundred ninety five Joules,

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while the kinetic energy after the collision was only two hundred ninety seven Joules. This is a loss of over eighty percent of the kinetic energy, which is clearly an inelastic collision.

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Momentum in Collisions

Summary

- Momentum before = Momentum after
- When objects stick together, their velocity is the same
- Kinetic Energy is conserved only in elastic collisions
- Momentum is always conserved

The diagram is a pyramid divided into four sections. The top section is labeled 'Equation'. The middle section is split into 'Elastic' and 'Inelastic'. The bottom section is split into 'Perfectly Inelastic Collision' and 'Objects Begin in Motion'.

Summary

As you can see, adding up the momentum before the equation and setting it equal to the momentum after the collision provides you with a reliable way of solving momentum problems. Recognizing that when objects stick together, they move at the same speed after a collision provides you with a way to eliminate one unknown and more easily solve these problems.

Kinetic energy is conserved only in elastic collisions, but momentum is always conserved.

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DIRECTIONS:

Read the problem and type your answer in the blank provided below.

Alex, a 95 kg football player is running with the football at 6.2 m/s towards the goal. He is met just before the goal line by Bruce, a 115 kg player running the other direction at 5.7 m/s. The players collide and fall together to the field. Does Alex score?

Problem 1:

Alex, a ninety-five kilogram football player is running with the football at six point two meters per second towards the goal. He is met just before the goal line by Bruce, a one hundred fifteen kilogram player running the other direction at five point seven meters per second. The players collide and fall together to the field. Does Alex score?



Problem 1 Solution

Before	After
$p_{\text{total}} = p_{\text{total}}$	
$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$	
$m_1v_1 + m_2v_2 = (m_1 + m_2)v$	
$(95)(6.2) + (115)(-5.7) = (95 + 115)v$	
$v = -0.316 \text{ m/s}$	

Problem 1 Solution:

This is a conservation of momentum problem that involves a perfectly inelastic collision, one in which the objects stick together. In order to know if Alex scores, we need to know if the final velocity is in the same direction Alex was moving or if it is in the opposite direction. For this problem, we'll consider the direction Alex was initially moving to be positive, and the direction Bruce was moving to be negative.

Since momentum is conserved, we will start by setting momentum before the collision to momentum after the collision. Both objects are moving before the collision and both objects are moving after the collision, so we write $m_1v_1 + m_2v_2$ before equals $m_1v_1 + m_2v_2$ after.

Since Alex and Bruce move together after the collision, their velocities will be the same. We can therefore simplify the right side of the equation by combining the masses and having a single velocity.

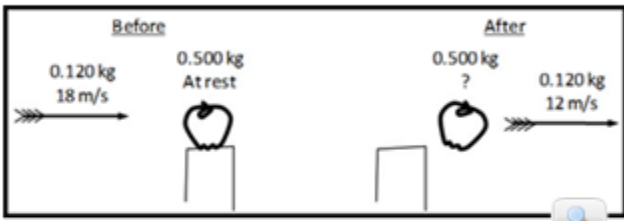
Now, we can substitute Alex's mass for m_1 and Bruce's mass for m_2 . We can also substitute both players' initial velocities for v_1 and v_2 before, keeping in mind that we set the direction Bruce was moving to be negative, so we write negative five point seven meters per second. We can now solve for v .

We find that their combined velocity after the collision is negative zero point three one six meters per second. Since they are moving in the negative direction, this means that Alex does not score, but is instead forced backwards in the collision.

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DIRECTIONS:
Read the problem and type your answer in the blank provided below.



A 500 g apple sits on a fencepost. A 120 g arrow is shot towards the apple at 18 m/s. The arrow passes through the apple and continues at a speed of 12 m/s. What is the speed of the apple after the arrow passes through it?

Problem 2:

A five hundred gram apple sits on a fencepost. A one hundred twenty gram arrow is shot towards the apple at eighteen meters per second. The arrow passes through the apple and continues at a speed of twelve meters per second. What is the speed of the apple after the arrow passes through it?

Before
 0.120 kg 18 m/s
 0.500 kg At rest

After
 0.500 kg ?
 0.120 kg 12 m/s

Problem 2 Solution

Before	After
$P_{\text{total}} = P_{\text{total}}$	
$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$	
$(0.120)(18) + (0.500)(0) = (0.120)(12) + (0.500)v_2$	
$v_2 = 1.44 \text{ m/s}$	
$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$	
$0 = m_1v_{1f} + m_2v_{2f} - (m_1v_{1i} + m_2v_{2i})$	
$0 = m_1v_{1f} + m_2v_{2f} - m_1v_{1i} - m_2v_{2i}$	
$0 = m_1v_{1f} - m_1v_{1i} + m_2v_{2f} - m_2v_{2i}$	
$0 = m_1(v_{1f} - v_{1i}) + m_2(v_{2f} - v_{2i})$	
$0 = m_1(\Delta v_1) + m_2(\Delta v_2)$	
$0 = \Delta p_1 + \Delta p_2$	
$\Delta p_1 = -\Delta p_2$	

Problem 2 Solution:

Again, we will use conservation of momentum to solve this problem. The total momentum before the interaction is the same as the total momentum after, so we start by writing $m_1 v_1 + m_2 v_2$ before equals $m_1 v_1 + m_2 v_2$ after. We will let the arrow be our first object and the apple be our second object.

We can substitute the mass of the arrow for m_1 and the mass of the apple for m_2 . v_1 before is eighteen meters per second, v_2 before is zero since the apple begins at rest. v_1 after is twelve meters per second and we're looking for v_2 after.

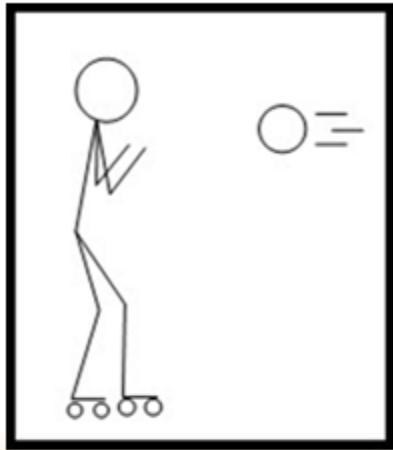
Solving for our unknown, we find that the apple has a velocity of one point four four meters per second. Another way to think of this is that since momentum is conserved, the momentum lost by the arrow must equal the momentum gained by the apple.

If we are careful to add additional subscripts to our velocities, with subscript i standing for initial and subscript f standing for final, we can rearrange our momentum equation to show that the change in momentum of object one plus the change in momentum of object two equals zero.

This could also be expressed as the change in momentum of the first object is equal in magnitude but opposite in direction to the change in momentum of the second object.

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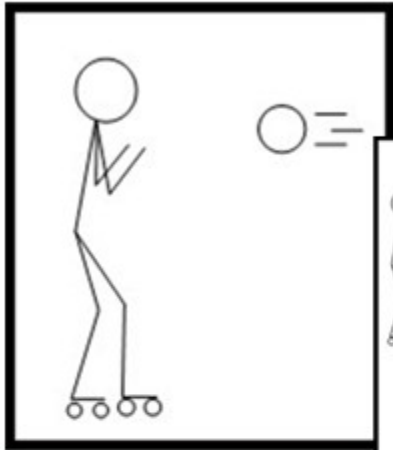
DIRECTIONS:

Read the problem and type your answer in the blank provided below.

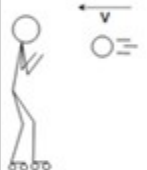
Steven, who has a mass of 68 kg is standing on roller skates at rest. A 5 kg bowling ball is thrown to him at 4 m/s. Steven catches the bowling ball, then throws it back the way it came at a speed of 3 m/s with respect to the ground. What is Steven's final speed after catching and throwing the bowling ball?

Problem 3:

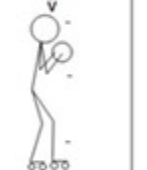
Steven, who has a mass of sixty eight kilograms is standing on roller skates at rest. A five kilogram bowling ball is thrown to him at four meters per second. Steven catches the bowling ball, then throws it back the way it came at a speed of three meters per second with respect to the ground. What is Steven's final speed after catching and throwing the bowling ball?



Problem 3 Solution



Before



After

$P_{\text{total}} = P_{\text{total}}$

$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$

$(68)(0) + (5)(-4) = (68)(v_1) + (5)(3)$

$v_1 = -0.515 \text{ m/s}$

Problem 3 Solution:

Again, this is a conservation of momentum problem, but this problem involves two steps. First, Steven catches the ball, which is a perfectly inelastic collision. Together, their whole mass will move to the left with the same momentum that the bowling ball had initially.

Next, we have an explosion, but both objects start in motion to the left. After the throw, the ball will move to the right, and Steven will move even quicker to the left.

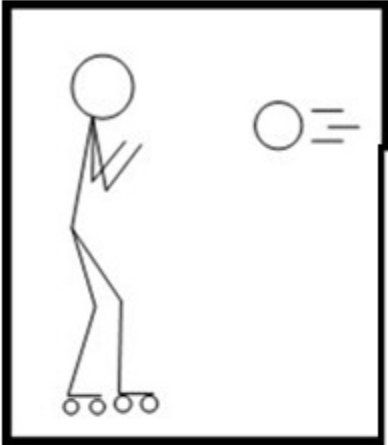
Overall, the total momentum for all three situations must be the same, and therefore the change in Steven's momentum in catching and throwing the ball must be of the same magnitude as the change in the bowling ball's momentum, but in the opposite direction.

So we will only look at the very beginning and the very end of the scenario.

Since momentum is conserved, we can use our conservation of momentum equation and write $m_1v_1 + m_2v_2$ before equals $m_1v_1 + m_2v_2$ after.

We will consider right positive and left negative. We'll let Steven's mass of sixty eight kilograms be m_1 and the ball's mass of five kilograms be m_2 . Steven's velocity is v_1 and the ball's velocity is v_2 . Before the catch and throw, Steven's velocity is zero and the ball's velocity is negative four meters per second. After the catch and throw the ball's velocity is positive three meters per second and we're looking for Steven's velocity.

As we substitute all the values and solve for v_1 , we find that Steven's velocity is negative zero point five one five meters per second, or zero point five one five meters per second to the left.



Problem 3 Alternate Solution

$$\Delta p_1 = - \Delta p_2$$
$$m_1(\Delta v_1) = - m_2(\Delta v_2)$$
$$(68)(\Delta v_1) = - (5)((-4)-3)$$
$$(68)(\Delta v_1) = - ((5)(7))$$
$$(68)(\Delta v_1) = - (35)$$
$$(68)(\Delta v_1) = -35$$
$$\Delta v_1 = -0.515 \text{ m/s}$$

Problem 3 Alternate Solution:

We could have also solved this problem by recognizing that Steven's change in momentum will be equal and opposite to the ball's change in momentum.

The ball initially had a velocity of negative four meters per second and ended with a velocity of positive three meters per second, which is a change in velocity of positive seven meters per second. The change in momentum of the ball therefore was positive thirty five kilogram meters per second.

Steven's change in momentum is therefore negative thirty five kilogram meters per second.

We solve for Steven's change in velocity by dividing by Steven's mass of sixty eight kilograms and find that his change in velocity is negative zero point five on five meters per second, which is consistent with our earlier analysis.